THE PRINCIPLES OF
ELECTRIC POWER TRANSMISSION
BY ALTERNATING CURRENTS

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BY

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AUTHOR'S PREFACE.

THERE appears to be scope for a suitable textbook for use by students pursuing a course of study in Electric Power Transmission at Universities and Technical Colleges. The present book is intended primarily to meet this need, but the practical side has been kept in view, and it is hoped that the subject-matter will also be of direct utility to designers and those engineers responsible for the operation of power transmission systems. It is not claimed that anything novel is presented herein, but rather that the work is a systematic exposition of the principles underlying the electrical design of transmission lines, and contains in a complete and concise form results hitherto recorded in a very scattered literature.

In the treatment of the subject familiarity with the use of complex quantities is presupposed, as a general course in alternating currents can hardly be considered complete without a discussion of this method of calculation. A knowledge of hyperbolic functions is an advantage when reading Chapter V, but the hyperbolic method of solution can be readily used for the solution of problems without this knowledge in the same way that tables of logarithms can be efficiently employed without understanding their theory or method of derivation. Wherever possible fully-worked numerical examples have been introduced in order to clarify the subject.

It is not suggested in connection with the exact mathematical methods of solution that these should be used for all line calculations. In actual practice charts and nomograms are often used to determine line constants, regulation, etc., and probably furnish results to the degree of accuracy warranted by the state of knowledge of the data of the problem. This, however, does not detract from the importance of the more rigorous methods, and there are many problems which cannot properly be solved without their assistance. As Miles Walker has said in connection with another branch of electrical

engineering, the art of the designer is to know when to calculate and when to use the more approximate methods.

The author wishes to thank the following individuals and firms: British Insulated Cables, Ltd., W. T. Henley's Telegraph Works Co., Ltd., and Metropolitan-Vickers Electrical Co., Ltd., for various data; the editor of *World Power* for permission to reprint the substance of an article on the "Hyperbolic Method of Line Solution," published by the author in the August and September (1924) issues of the journal; also the authors and publishers of the following books: *Electrical Characteristics of Transmission Circuits* (Westinghouse Technical Night School Press), by W. Nesbit; *Electrical Phenomena in Parallel Conductors* (John Wiley & Sons), by F. E. Pernot; *Electric Power Transmission* (McGraw-Hill Book Co.), by A. Still; *Constant-Voltage Transmission* (John Wiley & Sons), by H. B. Dwight; and *Protective Relays* (McGraw-Hill Book Co.), by V. H. Todd, for permission to reproduce various diagrams and material. The first-named book, in particular, is a mine of information, and should be on the bookshelf of every transmission line engineer.

In preparing the book full advantage has been taken of the information contained in original papers widely scattered throughout technical literature. References to the more important of these papers have been given in the hope that they will serve as a basis for the student's further reading.

As regards the illustrations practically all of these have been specially drawn for the book by Mr. A. Nash, Assoc.M.C.T., to whom the author is greatly indebted.

Great care has been taken with the numerical work, but it is too much to hope that no inaccuracies have crept in, and the author would be grateful if readers would bring any such errors to his notice.

H. WADDICOR.

WEMBLEY, MIDDLESEX,
January, 1928.

PREFACE TO FOURTH EDITION.

In preparing this edition the principal alterations found necessary have again been in connection with the section on Underground Cables. In particular, Chapter X, dealing with the thermal characteristics of cables, has been considerably revised in order to incorporate new data and new methods of calculation for the determination of current rating, these developments in practice being based substantially on work carried out during the last five years by the British Electrical and Allied Industries Research Association.

H. WADDICOR.

Wembley, Middlesex,
November, 1938.

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LIST OF SYMBOLS.

A	cross-sectional area of conductor.
	resistance to flow of heat from outside surface to surrounding air per cm. length of cable.
	auxiliary or general circuit constant.
A, A', A''	constants in solution for differential equations of line (Art. 41).
A_0	general circuit constant of network.
A_{01}	real component of A_0.
A_{02}	imaginary component of A_0.
A_1	real component of auxiliary constant A.
A_2	imaginary component of auxiliary constant A.
$A_\alpha, A_\beta, A_\gamma$	general circuit constants.
	distance between centres of conductors and axis of overhead line or cable.
	spacing between conductors B and C.
	co-ordinate of centre of circle in circle diagram.
	percentage to cover annual interest and depreciation on cost of conductors.
	a constant.
B	susceptance of conductor to neutral.
	magnetic flux density.
	auxiliary or general circuit constant.
B_0	general circuit constant of network.
B_{01}	real component of B_0.
B_{02}	imaginary component of B_0.
B_1	real component of auxiliary constant B.
	susceptance of receiving circuit.
B_2	imaginary component of auxiliary constant B.
B_t	primary exciting susceptance of transformer.
$B_\alpha, B_\beta, B_\gamma$	general circuit constants.
b	spacing between conductors A and C.
	co-ordinate of centre of circle in circle diagram.
	susceptance of conductor per unit length of line.
	barometric pressure of air.
	capacitance of conductor.
	auxiliary or general circuit constant.
C'	capacitance of bunched conductors to sheath in multi-core cable.
C_0	capacitance of conductor per mile.
	general circuit constant of network.
C_{01}	real component of C_0.
C_{02}	imaginary component of C_0.

C_0 .	= capacitance of single conductor to earth per mile.
	= real component of auxiliary constant C.
	= imaginary component of auxiliary c
	= capacitance of conductor A per cm.
C'_A .	= capacitance of conductor A per cm., including effect of earth.
C_n . .	= total capacitance of string of suspension insulators.
C_α, C_β, C_γ	= general circuit constants.
	= spacing between conductors A and B.
	= radius of circle in circle diagram.
.	= capacitance to earth of suspension-insulator disc.
	= capacitance between conductor and intersheath of graded cable.
.	= capacitance between adjacent suspension-insulator discs.
	= capacitance between intersheath and outer sheath of graded cable.
c_1, c_2, c_3 .	= see Art. 216.
c_0, c_s, .	= cable capacitances (Art. 116).
c_e . .	= equivalent capacitance of transformer circuit at frequencies greater than f_k.
c_g . .	= capacitance of transformer coils to earth per unit length.
c_s . .	= capacitance between transformer coils per unit length.
D . .	= electrostatic flux density.
	= auxiliary or general circuit constant.
D_0 . . .	= general circuit constant of network.
D_{01} . .	= real component of D_0.
D_{02} . .	= imaginary component of D_0.
D_A . .	= electrostatic flux density due to conductor A.
D_B . .	= electrostatic flux density due to conductor B.
D_α, D_β, D_γ .	= general circuit constants.
d . . .	= distance between centres of conductors.
	= conductor diameter.
	= $\sqrt{abc}$, equivalent delta spacing of line.
d' . .	= external diameter of lead sheath in S.L.-type cable.
E . .	= voltage to neutral or star voltage of line.
	= voltage of generator.
E' . . .	= voltage to neutral at terminals of condenser in middle-condenser method.
E_0 . . .	= disruptive critical voltage of line, or breakdown voltage of cable.
E_1, E_2 . .	= voltage across 1st and 2nd sections of graded cable.
E_1, E_3, E_5, etc.	= harmonic voltages.
E_a . . .	= flash-over voltage of insulator string.
E_c . . .	= voltage to neutral along line at point C.
E_m . .	= crest value of voltage to neutral.
E_m, E_n .	= voltage to neutral at intermediate points in series of networks.
E_r, E_{ro} . .	= voltage to neutral at receiving end of line under load and no-load conditions respectively.
E_s, E_{so} . .	= voltage to neutral at sending end of line under load and no-load conditions respectively.
E_s' . . .	= real component of E_s.
E_s'' . .	= imaginary component of E_s.
E_v . . .	= visual critical voltage of line.
E_p, E_{po} . .	= voltage to neutral along line at point P under load and no-load conditions respectively.
e . .	= e.m.f. of self-inductance.
	= instantaneous value of voltage wave.

e . . . = base of Napierian logarithms.
e_2 . . . = instantaneous value of transmitted voltage wave.
e_1 . . . = instantaneous value of reflected voltage wave.
e_1, e_2, e_m = voltage across 1st, 2nd, and mth unit of suspension-insulator string.
e_7 . . . = voltage across capacitance of circuit due to 7th harmonic.
e_a . . . = flash-over voltage of suspension-insulator disc.
e_{sh} . . . = voltage induced in open-circuited sheath of single-core cable.
F . . . = electrostatic force.
= mechanical force between conductors.
f . . . = frequency in cycles per second.
f_k . . . = critical frequency of transformer circuit.
G . . . = conductance of conductor to neutral.
= thermal resistance of ground path per cm. length of cable.
= weight per cm. length of conductor.
G' . . . = sum of conductances of conductors.
G_1 . . . = conductance of receiving circuit.
G_2 . . . = primary exciting conductance of transformer.
g . . . = conductance to neutral per unit length of line.
= potential gradient.
= thermal resistivity of soil.
= specific gravity of conductor material.
g_0 . . . = disruptive strength of air at 0° C. and 76 cm. barometric pressure.
g_{max} . . = maximum potential gradient (gradient at surface of conductor).
g_{min} . . = minimum potential gradient (potential gradient at inside of cable sheath).
g_v . . . = apparent disruptive strength of air at surface of conductor when corona begins.
H . . . = magnetic field intensity.
= heat generated in cable in watts.
h . . . = height of conductor above earth's surface.
= emissivity constant of outside surface of cable.
I . . . = current in each conductor.
= R.M.S. current during twelve months (Art. 150).
= R.M.S. current in conductor during short-circuit (Art. 184).
I' . . . = current inside conductor within radius x (Art. 18).
I_1 . . . = current in high-tension winding of transformer.
I_2 . . . = current in low-tension winding of transformer.
I_7 . . . = seventh harmonic of current wave.
I_A, I_B, I_C . = current in conductors A, B, and C respectively.
I_c . . . = charging or capacitance current of conductor.
= current along line at point C.
I_{cr}, I_{cs} . . = current consumed by receiving- and sending-end condenser respectively.
I_m, I_n . . = current at intermediate points in series of networks.
I_p . . . = active current at receiving end of line.
I_q, I_{qo} . . = reactive current at receiving end of line under load and no-load conditions respectively.
I_r, I_{ro} . . = current at receiving end of line under load and no-load conditions respectively.
I_r' . . . = see Art. 47.
I_s, I_{so} . . = current at sending end of line under load and no-load conditions respectively.

I_s' . . . = real component of I_s.
I_s'' . . . = imaginary component of I_s.
I_{sc} . . . = permanent short-circuit current.
I_{sc}' . . . = initial short-circuit current.
I_{sh} . . . = induced current flowing in short-circuited sheath of single-core cable.
I_α, I_β, I_γ . = current per conductor in single-phase, two-phase, and three-phase systems of transmission respectively (Arts. 8 and 9).
I_p, I_{po} . . = current along line at point P under load and no-load conditions respectively.
i . . . = instantaneous value of current wave.
i_0 . . . = instantaneous value of transmitted current wave.
i_1 . . . = instantaneous value of reflected current wave.
i_1, i_2, i_3 . . = see Art. 216.
j . . . = $\sqrt{-1}$, an operator.
K . . . = thermal resistivity of cable dielectric.
K_1 . . . = thermal resistivity of textile covering of cable.
K_d . . . = Luke's dissipation constant (Art. 153).
k . . . = reduction factor due to grouping of cables.
= $\dfrac{\text{mutual capacitance of suspension-insulator units}}{\text{earth capacitance of suspension-insulator units.}}$
k_0, k_1, k_2 . = constants (Arts. 147 and 148).
kVA_r . . = kilovolt-amperes per phase at receiving end of line.
kW_r . . = kilowatts per phase at receiving end of line.
kVA_s, kVA_{so} . = kilovolt-amperes per phase at sending end of line under load and no-load conditions respectively.
kW_s, kW_{so} . = kilowatts per phase at sending end of line under load and no-load conditions respectively.
L . . . = inductance of conductor.
= armature leakage inductance of alternator.
= inductance of transformer coils per unit length.
L_0 . . . = inductance of conductor per mile.
L_0' . . . = inductance of single conductor to earth per mile.
L_1 . . . = external inductance of conductor.
L_2 . . . = internal inductance of conductor.
L_e . . . = equivalent inductance of transformer circuit at frequencies less than f_k.
l . . . = length of line.
= depth of cable axis below surface of ground.
M . . . = mutual inductance between conductor and sheath of single-core cable.
m . . . = weight of conductor 1·0 sq. in. cross-section in lbs. per mile.
= $\sqrt{\dfrac{8\pi f l \times 10^{-9}}{R}}$ (Art. 59).
= an integer.
m_0, m_v . . = irregularity factors in corona formulæ.
N . . . = total number of wires in stranded conductor.
n . . . = number of phases or conductors.
= number of line sections.
= number of units in string of suspension insulators.
= number of layers in stranded conductor.
= fractional conductivity of copper.
= an integer.
n_1 . . . = number of turns on high-tension winding of transformer.
n_2 . . . = number of turns on low-tension winding of transformer.

P . . . = kW. per phase delivered at receiving end of line.
P_c . . . = corona power loss per phase per mile.
P_{max} . . = maximum kW. per phase delivered at receiving end of line.
P_1 . . . = annual charge on capital cost of line (or conductors).
P_2 . . . = annual cost of energy losses in line.
p . . . = power loss in line per phase.
= ratio between sheath losses and conductor losses in single-core cables.
p_1 . . . = cost of conductors in pence per lb.
p_2 . . . = cost of one kWH. of energy in pence.
Q . . . = electrostatic charge.
= reactive kVA. per phase delivered at receiving end of line.
Q_1 . . . = reactive kVA. of load.
Q_{pm}, Q_{pmo} . . = reactive kVA. taken by synchronous phase modifiers under load and no-load conditions respectively.
q . . . = electrostatic charge per cm. of conductor.
R . . . = resistance of conductor.
= resistance of conductor to flow of steady current (Art. 59).
= internal radius of lead sheath.
R' . . . = effective resistance of isolated conductor to flow of alternating current (Art. 59).
= insulation resistance of cable (bunched conductors to sheath).
R_0 . . . = resistance of conductor at 0° C.
R_1 . . . = external radius of lead sheath.
= resistance of receiving circuit.
R_2 . . . = overall radius of cable.
R_{eff} . . = effective resistance of single-core cable
R_m . . . = resistance of conductor 1·0 sq. in. cross-section in ohms per mile.
= $\dfrac{R + R_1}{2}$ = mean radius of lead sheath.
R_s . . . = resistance of lead sheath (Art. 112).
R_t . . . = resistance of conductor at t° C.
= equivalent resistance of transformer.
R_α, R_β, R_γ, . . = resistance of conductors in single-phase, two-phase, and three-phase systems of transmission respectively.
R_θ . . . = resistance of conductor at temperature corresponding to temperature rise of θ° C. (including skin and proximity effect).
r . . . = radius of conductor.
= resistance of conductor per unit length of line.
= resistance of conductor per cm. (Art. 184).
r_1 . . . = resistance of high-tension winding of transformer.
= external radius of 1st layer of insulation in graded cables.
r_2 . . . = resistance of low-tension winding of transformer.
= external radius of intersheath in graded cable.
S . . . = $E_s^2 - 2\,000\,(PR + QX)$ . . . (Art. 26).
= specific heat of conductor material.
= thermal resistance between conductors and outer surface per cm. length of cable.
S' . . . = thermal resistance between conductors and sheath per cm. length of cable.
S'' . . . = thermal resistance of bedding or serving per cm. length of cable.
s . . . = interaxial spacing of cable conductors.
T . . . = $\left(R^2 + X^2\right)\left(\dfrac{1\,000\,P}{\ }\right)^2$ . . . (Art. 26).
= conductor-insulation thickness.

t . . . = time in seconds or hours.
= belt-insulation thickness.
= temperature.

t' . . . thickness of bedding between sheath and armour in S.L.-type cable.

t_1 . . . C.

V . . .

V . . . $\pm\sqrt{xy}$ . . . (Art. 41).

V_m . . . crest value of voltage between conductors.

v . . . potential.
velocity of propagation of electric field.
modulus or size of line angle (Art. 49).

v_m . . . [illegible] insulator string.

W_d . . . = total dielectric losses of cable per phase.

W_h . . . = [illegible] per phase.

W_l . . . = leakage losses of cable per phase.

X . . . = reactance of conductor.

X_1 . . . = reactance of receiving circuit.

X_2 . . . = internal reactance of conductor with a steady current.

X_2' . . . = effective internal reactance of conductor with alternating currents (Art. 59).

X_e . . . = see Art. 112.

X_{eff} . . . = [illegible] of single-core cable.

$X_m = \omega M$. = mutual reactance between conductor and sheath of single-core cable.

X_t . . . = equivalent reactance of transformer.

x . . . = a distance.
= reactance of conductor per unit length of line.
= iron loss of transformer.
= argument of sinh x, etc.
= interaxial spacing of grouped cables.

x_1 . . . = reactance of high-tension winding of transformer.

x_2 . . . = reactance of low-tension winding of transfor

x_l . . . = inherent or leakage reactance of alternator.

x_r . . . = component of synchronous reactance of alternator due to armature reaction.

x_s . . . = synchronous reactance of alternator.

Y . . . = admittance of conductor to neutral.

Y_0 . . . = surge or natural admittance of line.

Y_1 . . . = admittance of receiving circuit.

Y_m . . . = see Art. 112.

Y_t . . . = primary exciting admittance of transformer.

Y_{tr}, Y_{ts} . . = primary exciting admittance of receiving- and sending-end transformer respectively.

y . . . = admittance of conductor per unit length of line.
= copper loss of transformer at full load.

Z . . . = impedance of conductor.

Z_0 . . . = surge or natural impedance of line.

Z_0' . . . = surge or natural impedance of terminal circuit.

Z_1 . . . = impedance of receiving circuit.

Z_e, Z_m . . = see Art. 112.

Z_{eff} . . . = effective impedance of single-core cable

Z_t . . . = equivalent impedance of transformer.

Z_{tr}, Z_{ts} . . = equivalent impedance of receiving- and sending-end transformer respectively.

z . . . . = impedance of conductor per unit length of line.
α . . . . = real component of complex angle.
= temperature coefficient of resistance of copper.
$\alpha, \alpha_1, \alpha_2$ . . = impulse ratio of spark gaps.
$\alpha_0, \alpha_t, \alpha_{t_1}$. = temperature coefficient of resistance of copper at 0°, t°, and t_1° C. respectively.
β . . . . = imaginary component of complex angle.
β, β_1, β_2 . . = setting factor of spark gaps.
δ . . . . = air density factor in corona formulæ.
δ_c . . . . = position angle along line at point C.
δ_r, δ_{r0} . . = position angle at receiving end of line under load and no-load conditions respectively.
δ_s . . . . = position angle at sending end of line
δ_p, δ_{p0} . . = position angle along line at point P under load and no-load conditions respectively.
ϵ . . . . = permittivity.
$\epsilon_1, \epsilon_2, \epsilon_3$ . . = permittivity of 1st, 2nd, and 3rd layers of dielectric in graded cable.
η . . . . = efficiency of transmission.
θ . . . . = $\sqrt{ZY}$ = complex angle of line.
= $2\pi ft$ (Art. 56).
= temperature rise of conductors in deg. C.
= $\tan^{-1}\dfrac{R}{X}$ . . . (Art. 24).
θ_1 . . . . = temperature rise of cable conductors when running alone.
θ_r, θ_{r0} . . = complex angle of receiving circuit under load and no-load conditions respectively.
θ_s . . . . = temperature rise of cable sheath in deg. C.
θ_v . . . . = argument or slope of line angle (Art. 49).
θ_x . . . . = temperature rise of cable conductors due to effect of neighbouring cable or cables.
μ . . . . = permeability.
ν_1, ν_2, ν_3 . . = dielectric strength of 1st, 2nd, and 3rd layers of dielectric in graded cable.
π . . . . = 3·14159 . . . .
ρ . . . . = resistivity of conductor material.
= resistivity of cable dielectric.
ρ_0, ρ_t . . . = resistivity of cable dielectric at 0° and t° C. respectively.
σ . . . . = impedance load to earth at receiving end of line (Art. 54).
Φ . . . . = magnetic flux.
Φ_1 . . . . = magnetic flux surrounding a conductor (external flux).
Φ_2 . . . . = magnetic flux inside cross-section of conductor (internal flux).
Φ_1, Φ_2, Φ_3 . . = magnetic flux surrounding each cm. of conductor A due to A, B, and C in initial condition and after 1st and 2nd transposition respectively.
Φ_A, Φ_B, Φ_C . . = magnetic flux between conductor A and neutral per cm. length of line due to A, B, and C respectively.
ϕ . . . . = phase-angle.
ϕ_d . . . . = angle of advance of charging current of cable.
ϕ_r . . . . = receiving-end phase-angle.
ϕ_s, ϕ_{s0} . . = sending-end phase-angle under load and no-load conditions respectively.
$\phi_p \phi_{p0}$ . . . = phase-angle along line at point P under load and no-load conditions respectively.
ψ . . . . = dielectric flux.
ω . . . . = $2\pi f$ = electrical angular velocity.

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CHAPTER I.

ELEMENTARY ECONOMIC AND ELECTRICAL PRINCIPLES.

1. **Electrical Transmission of Power.**—Compared with other forms of power, electric power possesses unique advantages for power transmission as it may be carried in far greater quantities, and to far greater distances, at a high efficiency. Furthermore, the methods of control are extremely elastic and convenient. Electric power, in fact, is the highest grade of power, since it can be changed to other forms—heat, light, mechanical power, or chemical action—with unparalleled directness and simplicity.

Power transmission schemes may be roughly divided into two types:—

1. Schemes arising from the exploitation of water-power plants.
2. Schemes brought into being by the linking up of generating stations and neighbouring supply networks.

The first type generally necessitates the straight transmission of power over long distances, as water-power sources of any magnitude are in most cases situated at a considerable distance from industrial centres. Hydro-electric plants usually involve a large initial capital outlay, but the running expenses are extremely small and thus permit a large economic radius of transmission providing that the line costs can be kept to a reasonable figure. For this purpose large amounts of power to be transmitted, and high line voltages, are essential features.

The concentration of generating plant in large or super-power

stations favourably situated as regards fuel supplies, etc., is responsible for another type of power transmission which ultimately develops into a complete network of transmission lines. The lines in this case play the part of interconnectors, the flow of power varying in magnitude and direction from time to time according to the operating conditions of the network. In this case considerable savings can be effected owing to:—

1. The reduced capital and running costs of a few large, as compared with a large number of small, generating units. In an interconnected system the bulk of the load can be supplied by the large efficient machines which are kept running more or less continuously at their maximum economic rating.

2. The lower percentage of reserve plant required on an interconnected system.

3. The possibility of tying together stations having different types of prime movers, thus enabling each station to be operated in the most economic manner.

2. Systems of Transmission.—Although energy can be transmitted electrically either by means of alternating currents or continuous currents, practically all existing transmission schemes make use of three-phase alternating currents. It will be shown later that for economy in the transmission of power, high line voltages must be employed, and the longer the distance of transmission the higher the economic voltage of the system. With alternating currents, no matter what the generating-station voltage, it can be stepped up by transformers with ease and efficiency to a high line voltage. At the receiving end of the line pressure transformations can be again effected to the voltage suitable for distribution. In fact the system is extremely flexible, and the distribution of power in scattered districts at any voltage desired by the consumer is a very simple matter.

The popularity of the three-phase system of transmission rather than the somewhat simpler single-phase system must without doubt be largely ascribed to the superior operating characteristics and more economical design obtainable with polyphase plant. When applied to electric traction work, however, the three-phase system possesses a serious drawback in that two insulated conductors have to be provided—the rails being employed as the third phase conductor. The line construction at junctions and crossings is thus somewhat complicated. A further disadvantage of the three-

phase system is that induction motors are not suitable for a wide range of speed variation. Single-phase power transmission has accordingly found a limited sphere of application in connection with electric traction work. By its adoption the construction of the line is simplified, and it is possible to take full advantage of the variable-speed characteristic of the single-phase commutator motor.

As regards continuous currents, although their use would have important advantages for transmission work, the chief obstacle has been the difficulty of generating sufficiently high voltages, and of utilising the high-voltage energy at the receiving end of the line. The few installations in existence using this method of transmission operate on the Thury constant-current system which whilst possessing several unique features can only be considered a partial solution of the problem. Further attention has been focussed on the subject by the invention of the "transverter" by Calverley and Highfield,[1] and more recently by the rapid development of grid-controlled arc rectifiers of the mercury-vapour or hot-cathode type.[2] The successful development of efficient converting plant would probably lead to the adoption of continuous currents in cases where large amounts of power have to be transmitted for extremely long distances.

3. Typical Alternating-current Power Scheme.—Fig. 1 gives a single-line diagram of a typical alternating-current power supply scheme. Generation is usually carried out at pressures of the order of 6 600 or 11 000 volts, though 33 000 volts has been utilised in one or two schemes, and has advantages where this voltage is also suitable for the transmission line. In most cases, however, the generating pressure is stepped up by transformers to the higher pressure required for the transmission line itself —this may be anything up to a couple of hundred kilovolts, depending to a great extent on the length of the line and the amount of power to be transmitted. The other end of the line terminates at the receiving station, this being located if possible near the centre of gravity of the various loads to be supplied. At the receiving station the voltage is reduced by step-down transformers to a value suitable for supplying the various substations dotted up and down the district. From these substations radiate the distributing mains to which the actual consumers are connected. In the majority of cases there are two distinct sets of distributing mains in use:—

1. A high-pressure distribution system operating at about 3 300 or 6 600 volts for power supply to large consumers.

2. A low-pressure distribution system from transformer stations installed near groups of consumers. Supplies of this character are now standardised at 400 / 230 volts, using the line voltage for power and the star voltage for lighting loads.

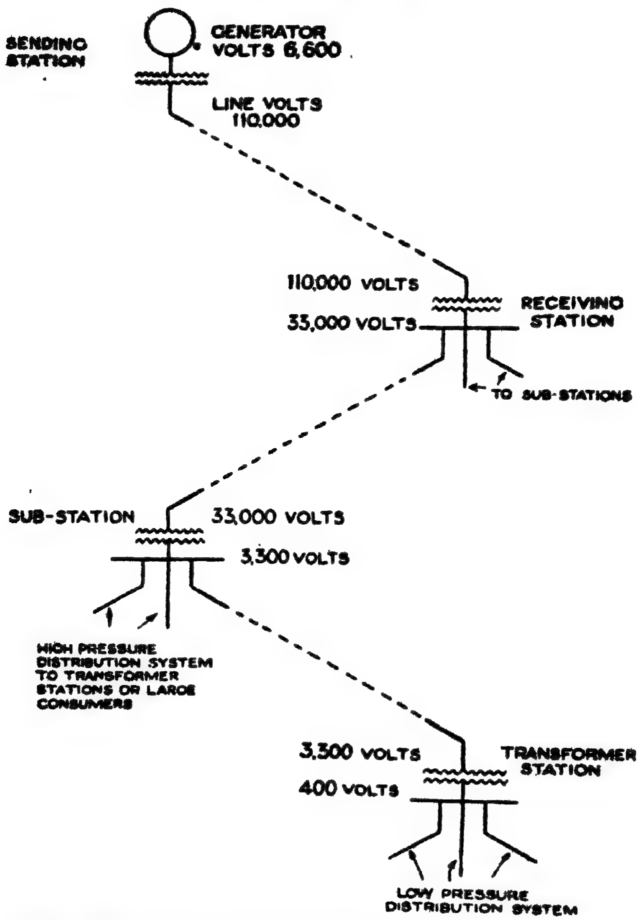

FIG. 1.—Typical alternating-current power transmission scheme.

4. **Overhead *versus* Underground Lines.**—An underground system is more expensive than an overhead system designed for the same operating voltage, consequently lines for power transmission are placed overhead when the conditions are suitable.

Thus overhead lines are used in the following cases:—

1. For the extremely high pressures which must be used for transmitting power over long distances, where this method of construction must be used in order to provide adequate insulation for the conductors.

2. For the lower pressures (where underground cables would also be technically suitable), when the line route runs through open country.

In city areas, however, underground cables are generally employed, partly to eliminate the danger to human life which would be present with overhead conductors, and partly to avoid the unsightly appearance and inconvenience of pole lines running down the main thoroughfares.

Hitherto the principal use of cables has been in connection with city distribution systems at comparatively low pressures, and only to a very limited extent for the straight transmission of power. During the last few years, however, great improvements have been made in the art of cable making, and cables designed for pressures up to 220 000 volts are now being employed. New types of cable have been developed which can be operated at higher electrostatic stresses and current densities, thus reducing the disparity between the cost of overhead and underground lines. It seems probable, therefore, that more use will be made of underground transmission in the future, particularly in those cases where the installation costs can be kept down by using mechanical excavating and cable-laying machines.

5. Influence of Voltage on Amount of Conductor Material Required.—In most schemes of power transmission the cost of the conductor material is a large item in the capital expenditure and has to be considered when fixing the line pressure.

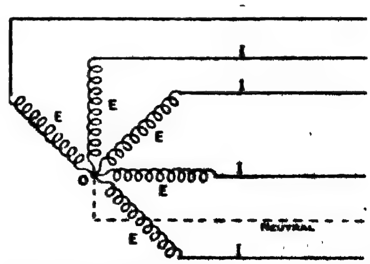

FIG. 2.—Polyphase system of transmission.

In general, in transmitting a given amount of power at a given loss over a given distance, the amount of copper required in the conductors is inversely proportional to the square of the pressure employed. For if the pressure is raised m times

the current in the line required to transmit the same amount of power is reduced m times, and the line losses which are proportional to the product of the square of the current and the line resistance are reduced m^2 times. So to transmit with the same losses as before the resistance of the line can be increased m^2 times, that is the amount of copper in the conductors can be decreased m^2 times.

This fact can also be shown very clearly as follows:—

Let E be the voltage to neutral of an n-phase system of transmission represented diagrammatically in Fig. 2. I is the current, lagging behind the voltage by ϕ degrees on each phase of the system.

Then the total power transmitted is

$$P = \frac{EI \cos \phi}{1\,000} \text{ kW. per phase.} \qquad (1)$$

If R is the resistance of each line conductor, the power loss will be

$$p = \frac{I^2 R}{1\,000} \text{ kW. per phase.} \qquad (2)$$

Substituting for I^2 from equation (1) gives

$$p = \frac{1\,000 P^2 R}{E^2 \cos^2 \phi}. \qquad (3)$$

The volume of copper per phase equals lA; where l is the length of the line, and A is the cross-sectional area of the conductors. If ρ is the resistivity of the copper,

$$R = \frac{l\rho}{A}.$$

Substituting for R in (3) gives

$$p = \frac{1\,000 P^2 l\rho}{AE^2 \cos^2 \phi},$$

or

$$A = \frac{1\,000 P^2 l\rho}{pE^2 \cos^2 \phi}.$$

Thus the volume of copper in each line conductor is

$$lA = \left(\frac{1\,000 P^2 l^2 \rho}{p}\right) \frac{1}{E^2 \cos^2 \phi}; \qquad (4)$$

the quantity in brackets being a constant for any given line and transmission efficiency and given amount of power to be delivered.

The above formula, besides showing the great saving of copper possible by employing a high line voltage, indicates also the importance of working as near unity power factor as possible. The effect of operating at, say, 0·8 power factor instead of unity, is to increase the amount of copper used in the ratio of $1 : \frac{1}{0{\cdot}8^2}$, or an increase of 56 %. A very considerable economy can be effected in the conductor material if a high load power factor can only be guaranteed.

6. Practical Limits of Line Voltage.—From formula (4) it might appear advisable to use the highest possible obtainable line pressures for the transmission of energy as by so doing the expenditure on conductors would be reduced to a minimum. On each particular transmission, however, there is a superior limit fixed for the pressure beyond which there is nothing to be gained in the matter of economy. This limit is reached when the saving in cost of conductor material due to the higher voltage is more than counterbalanced by (1) the increased cost of insulating the conductors, and (2) the increased cost of transformers, switchgear and other terminal apparatus, which is greater for the higher voltages.

The determination of the best line voltage is thus essentially an economic problem and will be discussed more fully in a later chapter.

7. Basis of Comparison of Systems as regards Copper Efficiencies.—From a technical point of view it is evident that in long-distance transmission lines the pressure is only limited by the problem of insulating the conductors against disruptive discharge, and hence when comparing the respective amounts of copper required in various systems the proper comparison is on the basis of equality of the maximum potential difference; that is equal maximum stress on the dielectric. The comparison voltage may be either the potential difference between any two conductors of the system, or the potential difference between any conductor and earth, depending on the character of the circuit.

In an overhead system it is easy to avoid a discharge between the conductors at points intermediate to the line supports by suitably spacing the conductors. At the supporting towers themselves, the insulation between conductors is indirectly provided by insulating each conductor from earth (represented by the tower cross-arm). Thus the disruptive stress is from conductor to earth, and the comparison of systems in this case has to be made on the

basis of maximum potential difference between conductor and earth It is usually assumed that the neutral point of the system is earthed, but whether or not if the system is of any considerable extent the neutral will be maintained at earth potential by the capacitance currents flowing.

With underground transmission by means of single-core cables or multi-core cables of the screened or S.L. type, the disruptive stress is from conductor to earth. Hence equality of the maximum potential difference between conductor and earth must again be taken as the basis of comparison of systems.

In an underground transmission using multi-core belted-type cables the chief stress on the insulation is usually between conductors. Systems employing these cables must therefore be compared on the basis of maximum potential difference between conductors.

8. Copper Efficiencies of Systems on the Basis of Equal Maximum Potential Difference between any Conductor and Earth.—In comparing the relative amounts of conductor material necessary for different systems of transmission, similar conditions will be assumed in each case, *viz.* (1) equal amounts of power transmitted; (2) equal lengths of line; and (3) equal power losses in the line.

Case 1. *Continuous-current.* (Fig. 3 (*a*).)—Let E_m be the maximum potential difference between any conductor and earth, and I the current. Assuming that the middle point of the system is earthed the line voltage is $2E_m$, the power transmitted is $2E_mI$, and if R is the resistance of each conductor the power loss is $2RI^2$.

Case 2. *Single-phase.* (Fig. 3 (*b*).)—E_m again being the maximum potential difference to earth the R.M.S. voltage to earth is

$$\frac{E_m}{\sqrt{2}},$$

and between conductors its value is

$$\frac{2E_m}{\sqrt{2}} \text{ or } \sqrt{2}E_m.$$

The amount of power to be transmitted is the same as before, *viz.* $2E_mI$, so the line current at a power factor $\cos \phi$ is

$$I_a = \frac{2E_mI}{\sqrt{2}E_m \cos \phi} = \frac{\sqrt{2}I}{\cos \phi}.$$

If R_a stands for the resistance of each conductor, the power loss in transmission is

$$2R_aI_a^2 = \frac{4R_aI^2}{\cos^2\phi},$$

and for equality of losses

$$\frac{4R_aI^2}{\cos^2\phi} = 2RI^2 \quad \text{or} \quad \frac{R_a}{R} = \frac{\cos^2\phi}{2}.$$

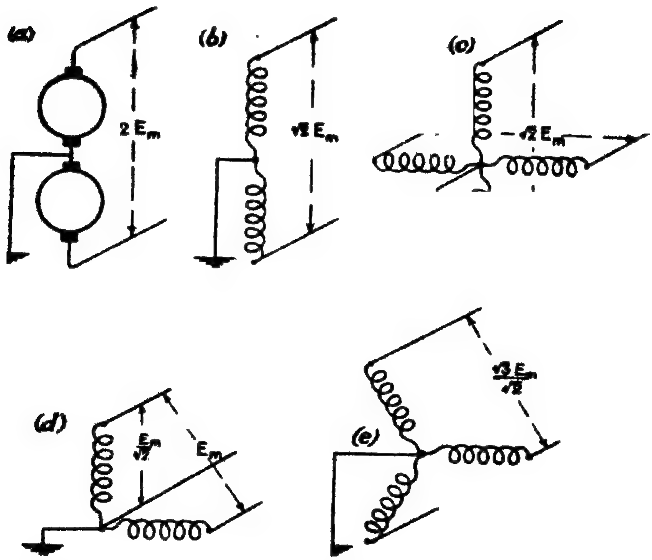

FIG. 3.—Transmission systems with same maximum potential above earth.

The cross-section of each conductor and the total amount of copper required must therefore be

$$\frac{2}{\cos^2\phi}$$

of the corresponding values in the continuous-current case.

Case 3. *Two-phase, Four-wire.* (Fig. 3 (*c*).)—This system can be considered as two independent single-phase systems each transmitting one-half the total power. The line conductors can thus be reduced to one-half the size required in the single-phase case, but as there are now four conductors instead of two the

total amount of copper required will still be the same, or $\frac{2}{\cos^2 \phi}$ times the quantity used in the continuous-current transmission.

Case 4. Two-phase, Three-wire. (Fig. 3 (*d*).)—The R.M.S. voltage between each outgoing conductor and the common return is

$$\frac{E_m}{\sqrt{2}},$$

and if I_β is the current in each of the outgoing wires the total power transmitted is $\sqrt{2}E_m I_\beta \cos \phi$. Comparing this with the continuous-current case $\sqrt{2}E_m I_\beta \cos \phi$ must equal $2E_m I$, or

$$I_\beta = \frac{\sqrt{2}I}{\cos \phi},$$

and if R_β is the resistance of the outgoing wires, the power loss in each of these is $R_\beta I_\beta^2$. Since the common return carries $\sqrt{2}$ times the current in each of the others, the cross-section must be increased in the same ratio to maintain the same current density. The resistance of this common wire is therefore

$$\frac{R_\beta}{\sqrt{2}},$$

and the power loss in it is

$$\frac{R_\beta}{\sqrt{2}}(\sqrt{2}I_\beta)^2 = \sqrt{2}R_\beta I_\beta^2.$$

The total power loss in the system is thus

$$2R_\beta I_\beta^2 + \sqrt{2}R_\beta I_\beta^2 = (2 + \sqrt{2})R_\beta I_\beta^2.$$

Substituting in this for I_β gives the total loss as

$$\frac{2(2 + \sqrt{2})R_\beta I^2}{\cos^2 \phi}.$$

Equating this to the power loss in the continuous-current case gives

$$\frac{2(2 + \sqrt{2})R_\beta I^2}{\cos^2 \phi} = 2RI^2, \text{ or } \frac{R_\beta}{R} = \frac{\cos^2 \phi}{2 + \sqrt{2}}.$$

The ratio of the amount of copper required in this case to that required in the continuous-current system is therefore

$$\left(\frac{2+\sqrt{2}}{2}\right)\frac{2+\sqrt{2}}{\cos^2\phi} = \frac{5{\cdot}82}{\cos^2\phi}.$$

Case 5. *Three-phase.* (Fig. 3 (*e*).)—The voltage to neutral (R.M.S.) is

$$\frac{E_m}{\sqrt{2}},$$

and if I_Y denotes the current in each conductor, the power transmitted is

$$\frac{3E_m}{\sqrt{2}} I_Y \cos\phi,$$

and for equality with the continuous-current system

$$\frac{3E_m}{\sqrt{2}} I_Y \cos\phi = 2E_m I, \text{ or } I_Y = \frac{2\sqrt{2}I}{3\cos\phi}.$$

If R_Y is the resistance of each conductor, the total power loss is $3R_Y I_Y^2$, and substituting for I_Y this gives

$$\frac{8R_Y I^2}{3\cos^2\phi}.$$

For the same transmission loss as before

$$\frac{8R_Y I^2}{3\cos^2\phi} = 2RI^2, \text{ or } \frac{R_Y}{R} = \frac{3\cos^2\phi}{4}.$$

Since, however, there are three conductors in the three-phase system as against two in the continuous-current system, the amount of copper necessary in the former case will be

$$\frac{4}{3\cos^2\phi} \times \frac{3}{2} = \frac{\quad}{\cos^2\phi}$$

times the amount required by the latter.

9. Copper Efficiencies of Systems on the Basis of Equal Maximum Potential Difference between any Two Conductors.—Equal distances and loads transmitted, and equal line losses will again be assumed.

Case 1. *Continuous-current.* (Fig. 4 (*a*).)—If V_m denotes the maximum potential difference between the conductors it will also be the working voltage in this case. R and I being respectively

the resistance of each conductor and current flowing, the power transmitted is $V_m I$, and the loss in transmission $2RI^2$.

Case 2. Single-phase. (Fig. 4 (*b*).)—In the case of the single-phase system the R.M.S. line voltage is

$$\frac{V_m}{\sqrt{2}},$$

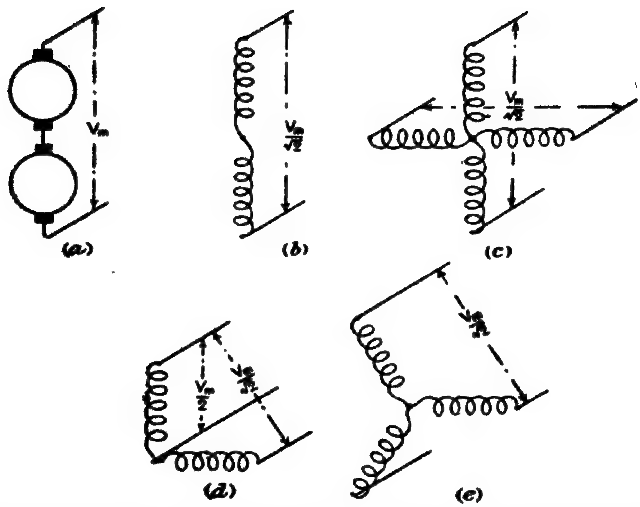

FIG. 4.—Transmission systems with same maximum potential between conductors.

the corresponding current

$$\frac{\sqrt{2}I}{\cos\phi},$$

and if R_a denotes the resistance of each conductor the transmission loss will be

$$\frac{4R_aI^2}{\cos^2\phi}.$$

For equality of loss

$$\frac{4R_aI^2}{\cos^2\phi} \text{ must equal } 2RI^2, \text{ or } \frac{R_a}{R} = \frac{\cos^2\phi}{2}.$$

Thus, the single-phase system requires $\frac{2}{\cos^2 \phi}$ times as much copper as the continuous-current system.

Case 3. *Two-phase, Four-wire.* (Fig. 4 (*c*).)—A two-phase system with its phases entirely independent is clearly on the same footing as the single-phase system and will also require $\frac{2}{\cos^2 \phi}$ times as much copper as would be necessary on a continuous-current system.

Case 4. *Two-phase, Three-wire.* (Fig. 4 (*d*).)—The maximum voltage, V_m, here occurs between the two outgoing conductors; the maximum voltage per phase is therefore

$$\frac{V_m}{\sqrt{2}},$$

and the R.M.S. voltage per phase

$$\frac{V_m}{2}$$

If I_β denotes the current in each of the two outgoing wires the total power transmitted is $V_m I_\beta \cos \phi$. Hence $V_m I_\beta \cos \phi$ must equal $V_m I$, or

$$I_\beta = \frac{I}{\cos \phi}.$$

Supposing the common return to be worked at the same current density as the other two conductors (each of which has a resistance R_β), its resistance is

$$\frac{R_\beta}{\sqrt{2}}$$

and the current flowing in it is

$$\frac{\sqrt{2} I}{\cos \phi}.$$

Hence the total loss in transmission is

$$\frac{2R_\beta I^2}{\cos^2 \phi} + \frac{\sqrt{2} R_\beta I^2}{\cos^2 \phi} = \frac{(2 + \sqrt{2}) R_\beta I^2}{\cos^2 \phi}$$

and for equality of loss

$$\frac{(2 + \sqrt{2}) R_\beta I^2}{\cos^2 \phi} = 2RI^2, \text{ or } \frac{R_\beta}{R} = \frac{2 \cos^2 \phi}{2 + \sqrt{2}}.$$

Thus the ratio of the amount of copper required in this case to that required in the continuous-current system is

$$\frac{(2+\sqrt{2})\left(\frac{2+\sqrt{2}}{2\cos^2\phi}\right)}{2} = \frac{2{\cdot}91}{\cos^2\phi}.$$

Case 5. Three-phase. (Fig. 4 (*e*).)—Finally considering the case of the three-phase system, since the R.M.S. voltage between conductors is

$$\frac{V_m}{\sqrt{2}}$$

the total power transmitted is

$$\frac{\sqrt{3}}{\sqrt{2}}V_m I_Y \cos\phi$$

where I_Y is the line current. Hence

$$\frac{\sqrt{3}}{\sqrt{2}}V_m I_Y \cos\phi$$

must equal $V_m I$, or

$$I_Y = \frac{\sqrt{2}I}{\sqrt{3}\cos\phi}.$$

If R_Y is the resistance of each conductor the loss in transmission is

$$3R_Y I_Y^2 = \frac{2R_Y I^2}{\cos^2\phi},$$

so that for equality of power loss

$$\frac{2R_Y I^2}{\cos^2\phi} = 2RI^2, \text{ or } \frac{R_Y}{R} = \cos^2\phi.$$

As three conductors are required in the three-phase system, as against two in the continuous-current case, the ratio of the amounts of copper required is

$$\frac{1{\cdot}5}{\cos^2\phi}.$$

10. Results of Comparison of Systems.—The results obtained above are summarised in Table 1:—

TABLE I.—*Copper Efficiencies of Various Systems of Transmission.*

System.	Amount of Conductor Material required.	
	On Basis of Maximum P.D. between Conductor and Earth.	On Basis of Maximum P.D. between any Two Conductors.
Continuous-current	Unity	Unity
Single-phase	$\frac{2}{\cos^2\phi}$ units	$\frac{2}{\cos^2\phi}$ units
Two-phase, Four-wire	$\frac{2}{\cos^2\phi}$ "	$\frac{2}{\cos^2\phi}$ "
Two-phase, Three-wire	$\frac{5\cdot82}{\cos^2\phi}$ "	$\frac{2\cdot91}{\cos^2\phi}$ "
Three-phase	$\frac{2}{\cos^2\phi}$ "	$\frac{1\cdot5}{\cos^2\phi}$ "

The great economy of conductor material possible by adopting the continuous-current system is clearly shown, particularly when it is remembered that the power factor of an alternating-current system is usually considerably less than unity.

The two-phase three-wire system is obviously quite unsuitable for long-distance transmission and needs no further consideration.

Considering the other alternating-current systems among themselves, it will be noted that there is a decided saving in copper with the three-phase system in the cases compared on the basis of equal maximum potential difference between conductors. In the case where the maximum potential difference between conductor and earth is the criterion, the three-phase system has no superiority over the single-phase or the two-phase four-wire system.

As previously pointed out the single-phase system is rarely used for general power supply purposes owing to the greater convenience and efficiency of polyphase plant, and comparing the two-phase four-wire and three-phase systems amongst themselves the latter has the advantage of only requiring three conductors instead of four.

II. Transmission Voltages in Use.—In Great Britain there are no water-powers of any considerable magnitude except tidal energy, and the latter is still of problematical utility. Energy is generated in coal-burning stations, and areas of supply are small as most of the industrial areas are situated on the coalfields. This

fact to a very large extent obviates the necessity for long-distance transmission of power. There is also no doubt that the growth of overhead transmission lines has been hindered by legislative restrictions, and the difficulty and delay experienced in securing suitable rights of way. The most important development has been the Government superpower scheme which has involved the construction of a primary high-tension network or "grid" covering the whole country, and operating at 132 kV. A fairly extensive secondary system of lower voltage lines to work in conjunction with this network has also been brought into being.

In the British Dominions overseas there are many high-voltage transmission systems in operation. Canada, Australia, New Zealand, South Africa, and India furnish instances of lines employing pressures of 80 to 132 kV., and the Irish Free State is utilising the water-power of the Shannon which involves transmission at 100 kV.

On the continent of Europe and in many other parts of the world, transmission schemes—often associated with water-power—are too numerous to mention, and line voltages range chiefly from 60 to 150 kV. In several countries, Government schemes have been actively developed for the co-ordination of electricity supply stations by means of interconnected high-tension lines. Examples of these superpower schemes are to be found in Germany, France, Italy, and Sweden where many new lines have been constructed for operation at 220 kV., the line design, in one case at least, being such that an ultimate pressure of 380 kV. can be used if necessary.

Undoubtedly the most striking examples of power transmission are to be found in the United States. This vast country, with its enormous industrial demands situated at long distances from equally large water-powers, can be considered as the home of power transmission schemes. A large portion of this country is one immense network of high-voltage overhead lines operating at all pressures up to 275 kV.

Speaking generally, the trend is towards higher line pressures owing to the larger amounts of power being transmitted. As previously pointed out the choice of voltage is primarily an economic consideration, but recent improvements in the design and construction of insulators and high-tension transformers have undoubtedly materially assisted this development. In this connection it is interesting to note that a pressure of 1 000 kV. has been produced in the laboratory and a short experimental transmission effected at

this voltage. It has been established that the empirical laws of corona and spark-over discharge found satisfactory at the ordinary transmission voltages apply with very little modification to pressures of this magnitude. The problems to be solved are mainly in connection with the mechanical design of the line having regard to the fact that large diameter tubular conductors at considerably wide spacings would be necessary. Sufficient has been done, however, to prove that line pressures very much higher than any of those employed at the present day can be used when the conditions demand them.[3]

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CHAPTER II.

INDUCTANCE AND CAPACITANCE OF CONDUCTORS.

12. Electric Field between Two Round Parallel Conductors.—The lines of magnetic force due to a single round conductor (that is, with the return conductor assumed at an infinite distance), consist of a series of concentric circles, and the lines of dielectric force are straight lines radiating from the conductor.

In an ordinary two-wire circuit, however, the influence of the return wire is not negligible, and the lines of magnetic and dielec-

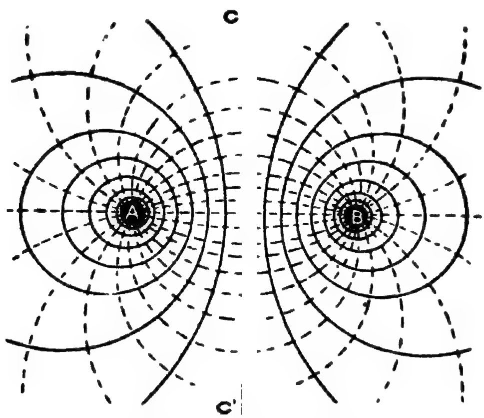

FIG. 5.—Electric field between two round parallel conductors.

tric force are crowded together between the conductors. The former become circles with centres on the line passing through the centres of the conductors, and the latter are also circles intersecting in two points in the conductors as shown in Fig. 5.

13. Inductance of Round Parallel Conductors.—Fig. 6 represents two parallel conductors A and B, carrying equal currents I c.g.s. units but in opposite directions, and forming part of an ordinary two-wire circuit.

If Φ is the magnetic flux associated with each wire the inductance of the wire by definition is

$$L = \frac{\Phi}{I}. \tag{5}$$

Let r = radius of each wire in centimetres,
and d = distance between centres in centimetres.

The magnetic field intensity at the point P distant x centimetres from A and $d - x$ centimetres from B is

$$H = \frac{2I}{x} + \frac{2I}{d - x} \quad \text{dynes,}$$

and the flux density is

$$B = H = \frac{2I}{x} + 2I \quad \text{lines per sq. cm.;}$$

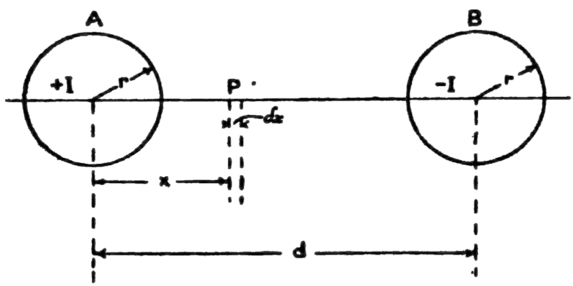

Fig. 6.—Inductance of parallel conductors.

the flux in a section of width dx and length 1 centimetre is

$$d\Phi = Bdx \quad \text{lines.}$$

The flux between the wires per centimetre length produced by the current flowing is

$$\int_r^d Bdx = \int_r^d \left(\frac{2I}{x} + \frac{2I}{d - x}\right)dx$$

$$= 4I \log_e \frac{d}{r};$$

the flux surrounding each wire being one-half this quantity is given by

$$\Phi_1 = 2I \log_e \frac{d}{r},$$

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and therefore the inductance of each wire per centimetre length is

$$L_1 = \frac{\Phi_1}{I} = 2 \log_e \frac{d}{r}. \quad . \quad . \quad . \quad (6)$$

This quantity is the external inductance of the conductor.

The flux inside the conductor has been neglected so far, but its inductive effect can be calculated very easily if it is assumed that the current is distributed uniformly over the section of the wire. In Fig. 7 the section of wire A is shown enlarged.

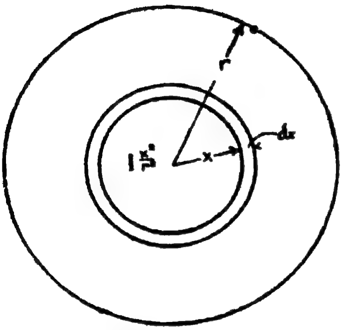

FIG. 7.—Flux inside a conductor.

If μ is the permeability of the conductor material the flux density at a distance x centimetres from the centre of A is

$$B = \mu H = \frac{2\mu I'}{x}$$

where I' is the current inside the radius x centimetres and its value is

$$I' = I\frac{x^2}{r^2} \quad \text{c.g.s. units.}$$

The flux in the ring of radius x, width dx, and length 1 centimetre is

$$d\Phi' = \frac{2\mu I x}{r^2} dx \quad \text{lines;}$$

this flux surrounds only the current I' and is equivalent to a smaller flux surrounding the current I, of value

$$d\Phi = d\Phi' \frac{x^2}{r^2} = \frac{2\mu I x^3}{r^4} dx;$$

the flux equivalent to the flux inside the whole section is

$$\Phi_2 = \int_0^r d\Phi = \int_0^r \frac{2\mu I x^3}{r^4} dx$$

$$= \frac{\mu I}{2} \quad \text{lines,}$$

and the inductance per centimetre length due to the flux inside the conductor is

$$L_2 = \frac{\Phi_2}{I} = \frac{\mu}{2} \quad \text{c.g.s. units,} \qquad . \quad (7)$$

which is the internal inductance of the conductor.

The total inductance of each wire per centimetre is therefore

$$L_1 + L_2 = 2 \log_e \frac{d}{r} + \frac{\mu}{2} \quad \text{c.g.s. units.} \; . \qquad . \quad (8)$$

For the usual non-magnetic conductors used in practice $\mu = 1$, and changing into practical units, the inductance of each wire is

$$L_0 = 2{\cdot}54 \times 12 \times 5\,280\left(2 \times 2{\cdot}303 \log_{10} \frac{d}{r} + \frac{1}{2}\right)10^{-9} \quad \text{henries per mile}$$

$$= 0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{d}{r} \quad \text{millihenries per mile.} \qquad . \quad (9)$$

14. **Capacitance of Round Parallel Conductors.**—As before A and B (Fig. 8) represent two parallel conductors each of radius

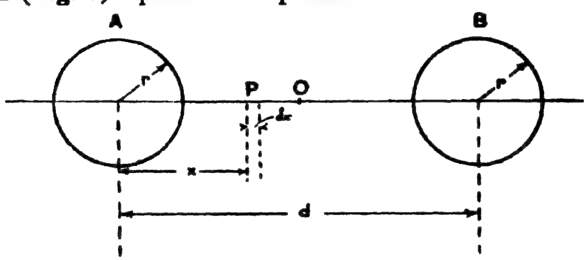

FIG. 8.—Capacitance of parallel conductors.

r centimetres, suspended in air at a distance of d centimetres between centres.

When a potential difference V is applied between them a dielectric flux ψ passes across from A to B and a positive charge

$$Q \quad \frac{}{4\pi}$$

appears on A and an equal negative charge

$$-Q = -\frac{\psi}{4\pi}$$

on B.

The electrostatic force at any point P on the line joining the centres of conductors, distant x centimetres from A and $d - x$

centimetres from B, is then the resultant of the forces exerted by the charges on A and B acting independently.

Assuming that the electrostatic charges are distributed uniformly over the surfaces of the conductors their effect at any external point can be calculated as though they were concentrated on the lines forming the axes of the conductors.

Let q and $-q$ be the charges per centimetre length on A and B respectively.

Then from each centimetre length of A, $4\pi q$ lines of dielectric flux extend out normally and produce at the point P a flux density

$$D_A = \frac{4\pi q}{2\pi x} = \frac{2q}{x} \quad \text{lines per square centimetre.}$$

Similarly, the charge on B produces at P a flux density

$$D_B = \frac{2q}{d - x} \quad \text{lines per square centimetre,}$$

and the resultant flux density at P is

$$D = D_A + D_B = \frac{2q}{x} + \frac{2q}{d - x} \quad \text{lines per square centimetre.}$$

The electrostatic force at the point P is

$$F = D = \frac{2q}{x} + \frac{2q}{d - x} \quad \text{dynes} \qquad . \quad . \quad (10)$$

and acts from A to B.

The work done in moving a unit positive charge from B to A is equal to the potential difference between the two conductors and is

$$V = \int_r^{d-r} \left(\frac{2q}{x} + \frac{2q}{d - x}\right) dx$$

$$= 4q \log_e \frac{d - r}{r}, \qquad . \quad . \quad . \quad . \quad . \quad (11)$$

but since r is always small compared with d in overhead lines

$$V = 4q \log_e \frac{d}{r}. \qquad . \quad . \quad . \quad (12)$$

The capacitance of the two conductors per centimetre length is therefore

$$\frac{q}{V} = \frac{q}{4q \log_e \frac{d}{r}}$$

$$= \frac{1}{4 \log_e \frac{d}{r}} \text{ electrostatic units.}$$

It is more convenient, however, to separate the circuit capacitance into the capacitance of the two conductors forming it. The potential of the point O, midway between the two wires, is zero, and the potential of the wire A is

$$E = \frac{V}{2} = 2q \log_e \frac{d}{r}; \qquad . \quad . \quad . \quad (13)$$

therefore the capacitance of A per centimetre length between the wire and the neutral or point of zero potential is

$$C_A = \frac{q}{E}$$

$$= \frac{1}{2 \log_e \frac{d}{r}} \text{ electrostatic units,} \quad . \quad . \quad (14)$$

and the capacitance of each wire is

$$C_0 = 2{\cdot}54 \times 12 \times 5\,280 \left(\frac{1}{2 \times 2{\cdot}303 \log_{10} \frac{d}{r}} \right) \frac{1}{9 \times 10^{n}} \text{ farads per mile}$$

$$= \frac{0{\cdot}0388}{\log_{10} \frac{d}{r}} \text{ microfarads per mile.} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

15. Potential and Potential Gradient between Parallel Conductors.—The potential at the point P in Fig. 8 is the work done in moving unit positive charge from the neutral point O to the point P; it is

$$v = \int_x^{\frac{d}{2}} \left(\frac{2q}{x} + \frac{2q}{d - x} \right) dx$$

$$= 2q \log_e \frac{d - x}{x}, \quad . \quad . \quad . \quad . \quad (16)$$

but $$q = \frac{V}{4 \log_e \frac{d - r}{r}} \quad \text{from equation (11)}$$

and substituting for q in (16) gives

$$\therefore v = \frac{V}{2 \log_e \frac{d - r}{r}} \log_e \frac{d - x}{x} \qquad . \quad (17)$$

The potential of conductor A is

$$\frac{V}{2} = E$$

and that of B is

$$-\frac{V}{2} = -E.$$

The potential gradient at any point P is equal to the intensity of the electrostatic field at that point and is given by

$$\begin{aligned} g = \frac{dv}{dx} &= \frac{2q}{x} + \frac{2q}{d - x} \\ &= 2q\left\{\frac{d}{x(d - x)}\right\} \\ &= \frac{V}{2 \log_e \frac{d - r}{r}}\left\{\frac{d}{x(d - x)}\right\}, \end{aligned} \qquad . \quad . \quad (18)$$

or expressed in terms of the potential to neutral it is

$$g = \frac{E}{\log_e \frac{d - r}{r}}\left\{\frac{d}{x(d - x)}\right\}. \qquad . \quad (19)$$

For small values of x and r compared with d this can be written

$$g = \frac{E}{x \log_e \frac{d}{r}} \qquad . \quad . \quad . \quad . \quad . \quad (20)$$

The maximum gradient occurs at the surface of the conductor and is

$$g_{max} = \frac{E}{r \log_e \frac{d}{r}} \qquad . \quad . \quad . \quad . \quad . \quad (21)$$

All these values of potential gradient are in volts per centimetre if V and E are expressed in volts.

Assuming the same potential difference applied between the two conductors, the distribution of potential around them is unaffected by the value of the permittivity ϵ of the medium in which the conductors are placed.

Fig. 9 shows the values of potential and potential gradient a

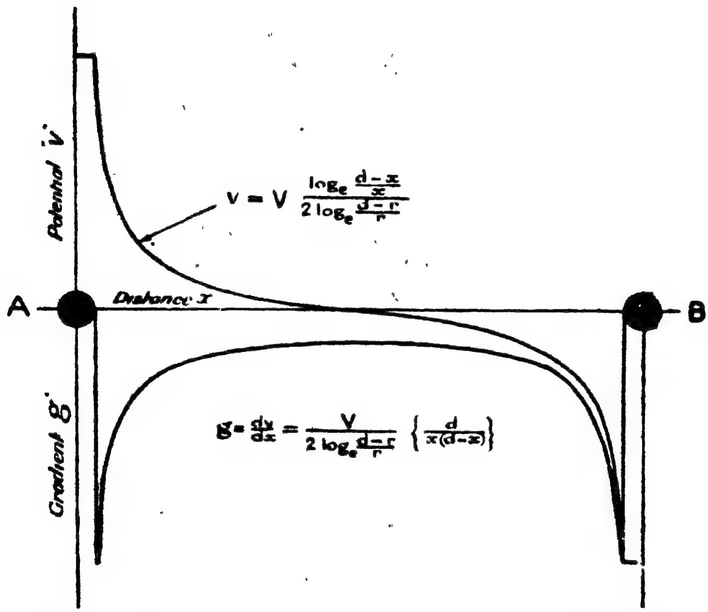

FIG. 9.—Potential and potential gradient between parallel conductors.

points along the line joining the centres of conductors as plotted from equations (17) and (18).

16. Inductance and Capacitance of a Single Conductor to Earth.—Referring to Fig. 5 it will be seen from considerations of symmetry that the centre line CC' is a line of magnetic force, and since the lines of electrostatic force intersect it at right angles, it is also a dielectric equipotential line. Hence if it were replaced by a conducting plane of perfect conductivity this would exert no effect on the magnetic or dielectric fields between the conductors A and B.

If, then, the earth's surface is assumed to be a perfectly conducting plane, the electric field between the earth and a single overhead conductor will be exactly similar to that between A and CC' in Fig. 5. In other words, if the conductor is at a potential E and carrying a current I the magnetic and dielectric flux distribution is exactly similar to that which would be produced on the assumption of an image conductor having a potential $-E$ and carrying a current $-I$ placed at the same distance below the ground as the conductor is above ground. That is, the formulæ obtained for the inductance and capacitance of a wire with return wire at distance d can be immediately applied to the inductance and capacitance of a wire to earth by using, as distance d, twice the distance of the wire from ground.

Thus if h is the height of the wire above the earth's surface, the inductance is

$$L'_0 = 0 \cdot 080 + 0 \cdot 741 \log_{10} \frac{2h}{r} \text{ millihenries per mile,} \qquad (22)$$

and the capacitance is

$$C'_0 = \frac{0 \cdot 0388}{\log_{10} \frac{2h}{r}} \text{ microfarads per mile.} \quad . \quad . \quad (23)$$

In practice the earth's surface is not a perfect conductor, and a perfectly conducting plane electrically equivalent to the earth would, in general, be situated at some depth below the actual surface. Hence L'_0 would be slightly higher, and C'_0 slightly lower, than the values given by the above formulæ.

17. Effect of Earth on Capacitance of Line.—Although a considerable capacitance exists between earth and a single overhead wire, the effect of the earth's presence on a transmission line consisting of outgoing and return wires is very small.

Considering the case of a single-phase line, image wires A' and B' must be assumed for both the wires A and B, as represented in Fig. 10, and the capacitance of each wire is then calculated as follows:—

Let h be the height of the wires from the plane electrically equivalent to the ground, d their distance apart from centre to centre, and r the radius of the wires, all in centimetres.

If A carries a charge of $+q$ units per centimetre length, A' will carry $-q$ units, B $-q$ units, and B' $+q$ units.

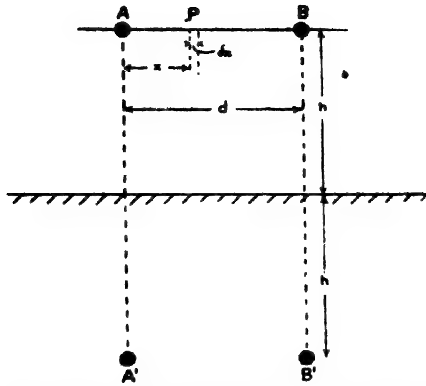

FIG. 10.—Effect of earth on capacitance of single-phase transmission line.

Then at a point P lying between A and B and distant x centimetres from A the electrostatic force is

$$\frac{2q}{x} \text{ dynes due to } A,$$

$$\frac{2q}{d-x} \text{ dynes due to } B,$$

$$-\frac{2qx}{4h^2+x^2} \text{ dynes due to } A',$$

$$-\frac{2q(d-x)}{4h^2+(d-x)^2} \text{ dynes due to } B'.$$

The resultant force at P is

$$\frac{2q}{x}+\frac{2q}{d-x}-\frac{2qx}{4h^2+x^2}-\frac{2q(d-x)}{4h^2+(d-x)^2} \text{ dynes.}$$

The work done in moving unit positive charge from B to A is equal to the potential difference between the two wires; it is

$$V = \int_{r}^{d} \frac{2q}{x} dx + \int_{r}^{d} \frac{2q}{d-x} dx - \int^{d} \frac{2qx}{4h^2 + x^2} dx - \int^{d} \frac{2q(d-x)}{4h^2 + (d-x)^2} dx$$
$$= 2q \log_e \frac{d}{r} + 2q \log_e \frac{d}{r} - q \log_e \frac{4h^2 + d^2}{4h^2} + q \log_e \frac{4h^2}{4h^2 + d^2}$$
$$= 4q \log_e \frac{d}{r} + 2q \log_e \frac{4h^2}{4h^2 + d^2}.$$

Midway between the two wires the potential is zero, and the potential of A is

$$E = \frac{V}{2} = 2q \log_e \frac{d}{r} + q \log_e \frac{4h^2}{4h^2 + d^2}$$
$$= 2q \log_e \frac{d}{r} + 2q \log_e \frac{2h}{\sqrt{4h^2 + d^2}}, \qquad (24)$$

and therefore the capacitance of A between the wire and neutral is

$$C'_A = \frac{q}{E} = \frac{1}{2 \log_e \frac{d}{r} + 2 \log_e \frac{2h}{\sqrt{4h^2 + d^2}}}$$
$$= \frac{1}{2 \log_e \frac{d}{r}\left(\frac{2h}{\sqrt{4h^2 + d^2}}\right)} \text{ electrostatic units per cm.} \qquad (25)$$

The formula given in Art. 14 for the capacitance of a wire as measured between wire and neutral was

$$C_A = \frac{1}{2 \log_e \frac{d}{r}} \text{ electrostatic units per cm.} \qquad (14)$$

Hence the effect of the earth's proximity is to increase slightly the capacitance of an overhead line.

Taking as an average case

$$h = 30' = 360'',$$
$$d = 8' = 96'',$$
and
$$r = 0{\cdot}2'',$$

we find the capacitance is changed by less than 0·2 °/₀, so that the effect of the earth in increasing the capacitance of an overhead system is negligible.

18. Inductance and Capacitance of Three-phase Overhead Line.—In order to understand the derivation of the formulæ for the inductance and capacitance of a three-phase system consider

first of all that the three phases are kept entirely separate along the line, six conductors being used. In Fig. 11 (a) A, B, and C are the three outgoing wires occupying the vertices of an equilateral triangle and A', B', and C' respectively, the return wires running along near the neutral axis of the line.

The system is thus divided into three circuits or loops symmetrically placed, and assuming the same load on each phase it is obvious that all calculations may be based on the phenomena occurring in one of these loops, say AA

It is therefore the inductance and capacitance of AA' that is required for the purpose of line calculations, but in obtaining these values the effect of the loops BB' and CC' has also to be taken into consideration.

Now the three return wires A', B', and C' being at the same potential may be amalgamated into one common return wire with

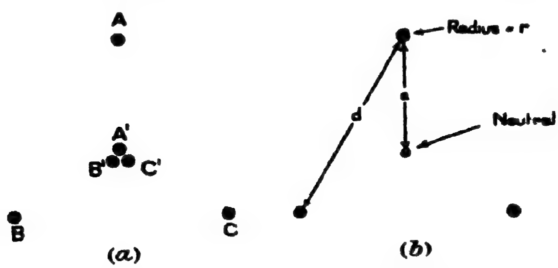

FIG. 11.—Inductance of three-phase transmission line.

its centre on the neutral axis of the line without any difference being made to the flux distribution of the system.

Also, since in a balanced three-phase system the sum of the currents in the three outgoing wires is zero, the current in the common return wire will also be zero, and the wire may be (and in practice always is) omitted without affecting the magnetic and dielectric fields between the other three wires.

The inductance and capacitance of a three-phase line is thus the inductance and capacitance between any one wire and the neutral axis of the line.

Referring to Fig. 11 (b) let

d = distance between centres of wires in centimetres,

a = distance between centres of wires and neutral in centimetres,

and r = radius of wires in centimetres.

The currents flowing in the wires have phase differences of 120 degrees so if the current in wire A is I c.g.s. units, the current in B will be

$$\left(-0.5 + \frac{\sqrt{3}}{2}j\right)I$$

$$= (-0.5 + 0.866j)I \quad \text{c.g.s. units,}$$

and in wire C

$$(-0.5 - 0.866j)I \quad \text{c.g.s. units.}$$

Assuming the positive direction of the current along the line as away from the reader it is evident that the fluxes due to B and C threading the loop formed by wire A and the neutral are in opposition to the flux produced by A itself.

Therefore the magnetic flux between A and neutral per centimetre length of line is

$$\Phi_A = \frac{I}{2} + 2I \log_e \frac{a}{r} \quad \text{due to } A,$$

$$\Phi_B = (-0.5 + 0.866j)2I \log_e \frac{d}{a} \quad \text{due to } B,$$

and

$$\Phi_C = (-0.5 - 0.866j)2I \log_e \frac{d}{a} \quad \text{due to } C.$$

The resultant flux is

$$\Phi = \Phi_A - \Phi_B - \Phi_C$$

$$= \frac{I}{2} + 2I \log_e \frac{a}{r} + I \log_e \frac{d}{a} - 1.732jI \log_e \frac{d}{a} + I \log_e \frac{d}{a} + 1.732jI \log_e \frac{d}{a}$$

$$= \frac{I}{2} + 2I \log_e \frac{a}{r} + 2I \log_e \frac{d}{a}$$

$$= \frac{I}{2} + 2I \log_e \frac{d}{r}.$$

The inductance between wire and neutral is

$$\tfrac{1}{2} + 2 \log_e \frac{d}{r} \quad \text{c.g.s. units per centimetre,}$$

and changing into practical units the inductance of each wire is

$$L_0 = 0.080 + 0.741 \log_{10} \frac{d}{r} \quad \text{millihenries per mile.}$$

Similarly, the line capacitance may be represented by condensers connected between each wire and the neutral as shown in Fig. 12

and after obtaining the dielectric flux distribution in an analogous manner to the above calculation the capacitance of each wire is found to be

$$C_0 = \frac{0\cdot 0388}{\log_{10}\frac{d}{r}} \text{ microfarads per mile.}$$

On comparing these results with those derived in Arts. 13 and 14, it is seen that the same formulæ apply for the inductance and capacitance of an overhead circuit whether single-phase or three-phase, providing these quantities are measured between conductor and neutral.

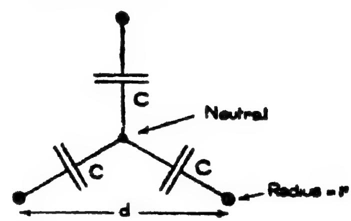

FIG. 12.—Capacitance of three-phase transmission line.

19. Inductance and Capacitance with Irregular Spacing of Conductors.—It has been assumed in the above discussion that the three line conductors were symmetrically arranged, each occupying the corner of an equilateral triangle. In many overhead transmissicn systems, however, the conductors are irregularly spaced, and the above formulæ are not directly applicable.

When the wires of a three-phase system are spaced so that they are not equidistant, the inductances and capacitances are not the same in the different phases. More important still, there is an interchange of power between the phases so that there may be wide differences between the apparent resistances of the conductors. It is the practice with such lines to interchange or transpose the wires at intervals along the line so that each of the three wires occupies a particular position relatively to the other wires for one-third the total length of transmission. This results in completely eliminating any out-of-balance effect.

The average inductance of an irregularly-spaced three-phase line in which the wires are transposed at regular intervals may be calculated as follows:—

Let Fig. 13 represent the actual spacing of the wires A, B, and C of an overhead line, and the currents in the wires be represented by the vectors OP, OQ, and OR respectively.

sequence of currents, and methods of transposition, if any. Morgan and Whitehead[1] have examined all the various combinations in detail, and given formulæ for calculating the inductance, and from these the corresponding formulæ for capacitance can also be written down.

REFERENCE.

[1] P. D. Morgan and S. Whitehead, 'The Impedance and Power Losses of Three-phase Overhead Lines,' *Jour. I.E.E.*, Vol. 68, p. 367 (1930).

CHAPTER III.

PERFORMANCE OF SHORT TRANSMISSION LINES.

21. Effect of Line Inductance and Capacitance on Transmission of Alternating Currents.—In consequence of the necessary spacing between the conductors of a transmission circuit, each conductor is surrounded by a considerable magnetic flux which generates in it an e.m.f. of self-inductance. This e.m.f. is in quadrature with the current flowing in the conductor and has the value

$$\begin{aligned} e &= 2\pi fLI \\ &= IX \text{ volts;} \end{aligned} \qquad (28)$$

where f = frequency of supply in cycles per second,
L = inductance per conductor in henries,
I = current in amperes,
and X = reactance per conductor in ohms.

The voltage impressed on the circuit at the sending end has thus to provide components to overcome the resistance pressure drop IR, which is in phase with the current, and the reactance pressure drop IX, which is in quadrature with the current.

The precise effect on the receiving-end voltage depends on the difference of phase between the current and the voltage at this end of the line. When the load has a lagging or unity power factor the receiving-end voltage is less than the sending-end voltage, but with a load of leading power factor there may be a rise of pressure at the receiving end of the line.

The effect of the line capacitance is to produce a current—usually called the charging current—which is in quadrature with the voltage. This component of current in the conductor has its maximum value at the sending end of the circuit and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

The value of the charging current at the sending end is given by

$$\begin{aligned} I_c &= 2\pi fCE \\ &= EB \text{ amperes;} \end{aligned} \qquad (29)$$

where C = capacitance per conductor in farads,
E = voltage to neutral,
and B = susceptance per conductor in mhos.

It should be noted that both the reactance drop and the charging current of the circuit are proportional to the frequency, and thus have a far greater influence on the performance of a 50-cycle than on a 25-cycle line. Also that with overhead lines reactance effects are relatively important due to the wide spacing of conductors which must be adopted, whilst in the case of transmission by underground cables reactance effects are small and the effect of capacitance predominates.

In the case of 50-cycle overhead lines not exceeding, say, 50 miles in length, the effect of capacitance on the line performance is so small that it may generally be neglected.

22. Treatment of Polyphase Systems.—A three-phase transmission circuit carrying an equal load on each phase is a particular example of a balanced symmetrical polyphase system, and in making calculations and drawing diagrams of such a system it is only necessary to take into account one of the star circuits of the system.

For instance, the polyphase system represented in Fig. 2 at balanced load can be considered as consisting of the equal single-phase systems: 0 – 1; 0 – 2; 0 – 3; . . . 0 – n lying between the conductors and the neutral axis.

In investigating a polyphase system it is only necessary to deal with the phenomena occurring in one of these constituent circuits, which carries $\frac{1}{n}$th the total load on the system, and has impressed on it the star voltage or voltage to neutral of the system. In a three-phase system each of the star circuits carries one-third of the total load and has a voltage equal to $\frac{1}{\sqrt{3}}$ times the voltage between conductors.

23. Vector Diagram for Short Line.—Fig. 14 is the vector diagram showing the relation between the various elements in a

short transmission circuit, the effect of capacitance being neglected. Both the diagram itself, and the upper part of the figure which illustrates the circuit schematically, are drawn for one phase only, in accordance with the rules laid down in the preceding article.

The various quantities are represented as follows:—

OD is the current.

OR is the voltage to neutral at the receiving end of the line drawn ϕ_r degrees in advance of *OD*: cos ϕ_r being the load power factor (assumed lagging in this case).

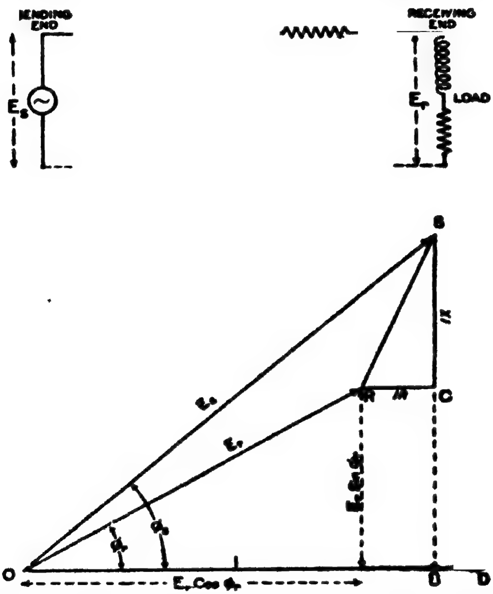

FIG. 14.—Vector diagram for short transmission line.

RC drawn parallel to *OD* is the voltage consumed in the resistance of the conductor.

CS drawn 90 degrees ahead of *OD* is the voltage consumed in the reactance of the conductor.

OS is the voltage to neutral at the sending end.

ϕ_s is the phase-angle at the sending end: cos ϕ_s being the power factor at this end.

The component of the sending-end voltage in phase with the current is

$$OB = E_r \cos \phi_r + IR,$$

and the component in quadrature ahead of the current is

$$BS = E_r \sin \phi_r + IX,$$

and therefore the sending-end voltage is

$$E_s = \sqrt{(E_r \cos \phi_r + IR)^2 + (E_r \sin \phi_r + IX)^2}. \quad . \quad (30)$$

Expressed in another form

$$E_s = E_r\sqrt{\left(\cos \phi_r + \frac{IR}{E_r}\right)^2 + \left(\sin \phi_r + \frac{IX}{E_r}\right)^2}, \quad . \quad (31)$$

which is rather more convenient for purposes of calculation as the quantity under the radical sign is then of the order of unity.

The sending-end phase-angle is

$$\phi_s = \tan^{-1} \frac{\sin \phi_r + \frac{IX}{E_r}}{\cos \phi_r + \frac{IR}{E_r}}. \quad . \quad . \quad . \quad (32)$$

Hence, knowing the conditions existing at one end of the line, those at the other end can easily be obtained by means of the above formulæ. Problems may also be solved graphically by drawing the vector diagram to scale, but owing to the small size of the impedance triangle compared with the rest of the diagram no great accuracy can be obtained. It is advisable, therefore, to calculate all quantities trigonometrically, using the graphical method as a rough check.

24. Regulation.—The regulation of a transmission line can be defined as the change of voltage at the receiving end when full load is thrown off, the sending-end voltage being kept constant. The percentage regulation is the regulation expressed as a percentage of the full-load receiving-end voltage.

When the load is thrown off, the only current in the conductors is the charging current of the system, and if this is of negligible magnitude the receiving-end voltage will equal the constant impressed voltage at the sending end of the line. Hence in the case of short lines, the regulation is equal to $E_s - E_r$, the arithmetical difference between the sending-end and receiving-end voltages.

The regulation for any given load power factor can be graphically determined by means of Kapp's regulation diagram, which is a slight modification of the fundamental impedance diagram of the line. In Fig. 15 if OD is kept fixed and ϕ_r varied, S lies on a

circle with O as centre. RC and CS remain constant in length and move parallel to themselves, consequently R lies on an equal circle with centre at O' where OO' is equal and parallel to SR.

These circles are drawn in the figure. If OR is produced to meet the other circle in E, then $OE = OS$, and therefore

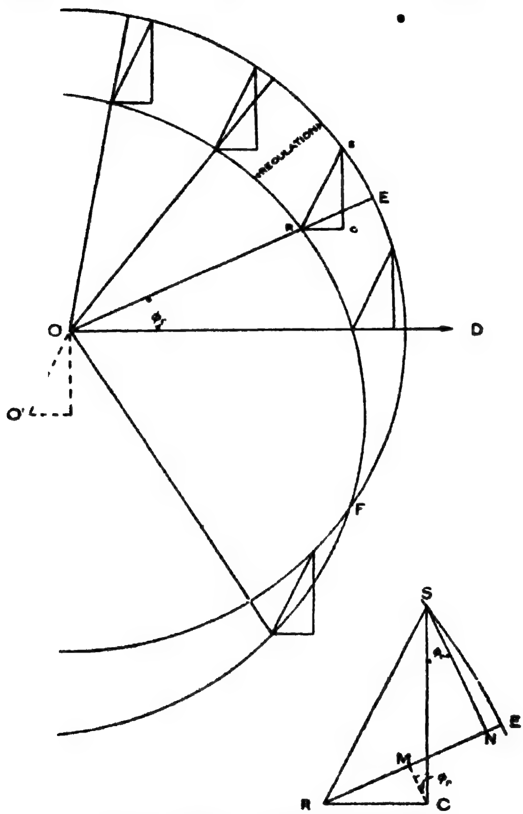

FIG. 15.—Kapp's regulation diagram.

$RE = OS - OR$, the pressure drop along the line and also the regulation.

All that is necessary then, in order to determine the regulation at any power factor $\cos \phi_r$ is to draw a line making an angle ϕ_r with OD, and measure the length of the intercept between the two circles.

An inspection of the diagram shows that as the angle of lag increases, the regulation increases, reaching a maximum value when OR is in line with OO', *i.e.* when

$$\phi_r = \tan^{-1}\frac{X}{R}.$$

For still greater angles of lag the regulation diminishes again.

With leading power factor the regulation diminishes as the angle of lead increases, becomes zero when $\phi_r = \angle DOF$, and beyond this has a negative value, *i.e.* the receiving-end pressure falls when the load is thrown off.

A formula giving the regulation directly may be found by referring to the part of the diagram shown enlarged in Fig. 15. RS is usually small in comparison with OS, and so the arc SE may be replaced with very little error by the perpendicular SN from S on to OR produced. If CM is drawn at right angles to RE we have

$$\begin{aligned}\text{Regulation} &= \overline{RM} + \overline{MN}\\ &= IR\cos\phi_r + IX\sin\phi_r, \qquad (33)\end{aligned}$$

or expressed in another form

$$\begin{aligned}\text{Regulation} &= IZ\left(\frac{IR}{IZ}\cos\phi_r + \frac{IX}{IZ}\sin\phi_r\right)\\ &= IZ(\sin\theta\cos\phi_r + \cos\theta\sin\phi_r)\\ &= IZ\sin(\phi_r + \theta), \qquad (34)\end{aligned}$$

where the angle $\theta = \tan^{-1}\frac{R}{X}$.

The regulation is a maximum when $\sin(\theta + \phi_r) = 1$, or $\phi_r = 90 - \theta$, that is, at a load power factor

$$\cos\phi_r = \sin\theta = \frac{R}{Z},$$

and is equal to the impedance drop IZ.

25. Numerical Example of Short Line Solution.—When the voltage, current, and power factor either at the sending or receiving end are given, the conditions at the other end of the line can easily be found by applying formulæ (31) and (32). To illustrate the method of solution the following example will be taken:—

Problem 1.—Find the characteristics of the load at the sending

end of a three-phase line 30 miles long, consisting of three hard-drawn, copper conductors, spaced in a four-foot delta. Temperature taken as 20° C. Load conditions at receiving end assumed as 8 000 kVA. (6 400 kW. at 80 % power factor lagging), 30 000 volts, 50 cycles; transmission loss to be approximately 10 %.

At the receiving end of the line

$$kVA_r = \frac{8\ 000}{3} = 2\ 667 \quad \text{kVA. per phase.}$$

$$kW_r = \frac{6\ 400}{3} = 2\ 133 \quad \text{kW. per phase.}$$

$$E_r = \frac{30\ 000}{\sqrt{3}} = 17\ 320 \quad \text{volts to neutral.}$$

$$\cos \phi_r = 80\ \% \quad \text{lagging.}$$

$$I = \frac{2\ 667}{17\ 320} \times 10^3 = 154 \quad \text{amperes.}$$

For a transmission loss of 10 % the resistance of each conductor will be

$$R = \frac{2\ 133 \times 10^3}{10 \times (154)^2}$$
$$= 8{\cdot}99 \quad \text{ohms,}$$

or 0·300 ohms per mile.

From wire tables [1] the nearest standard size of wire is found to be No. 5/0 S.W.G., which has a resistance of 0·302 ohms per mile, and a diameter of 0·432 inch.

The resistance of each conductor is therefore

$$R = 0{\cdot}302 \times 30 = 9{\cdot}06 \quad \text{ohms.}$$

The inductance of each conductor by formula (9) is

$$L = \left(0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{48}{0{\cdot}216}\right) 10^{-3} \times 30$$
$$= 0{\cdot}0546 \quad \text{henries,}$$

and the reactance is

$$X = 2\pi \times 50 \times 0{\cdot}0546$$
$$= 17{\cdot}1 \quad \text{ohms.}$$

We then have

$$IR = 154 \times 9{\cdot}06 \quad 1\,395 \quad \text{volts (resistance drop).}$$
$$IX = 154 \times 17{\cdot}1 \quad 2\,633 \quad \text{volts (reactance drop).}$$

$$E_s = 17\,320\sqrt{\left(0{\cdot}8 + \frac{1\,395}{17\,320}\right)^2 + \left(0{\cdot}6 + \frac{2\,633}{17\,320}\right)^2}$$
$$= 17\,320\sqrt{(0{\cdot}8805)^2 + (0{\cdot}7520)^2}$$
$$= 20\,060 \quad \text{volts.}$$

$$\phi_s = \tan^{-1}\frac{0{\cdot}\,7520}{0{\cdot}\,8805}$$
$$= 40{\cdot}5^\circ$$
$$\cos\phi_s = 76{\cdot}04\ \%\quad \text{lagging.}$$

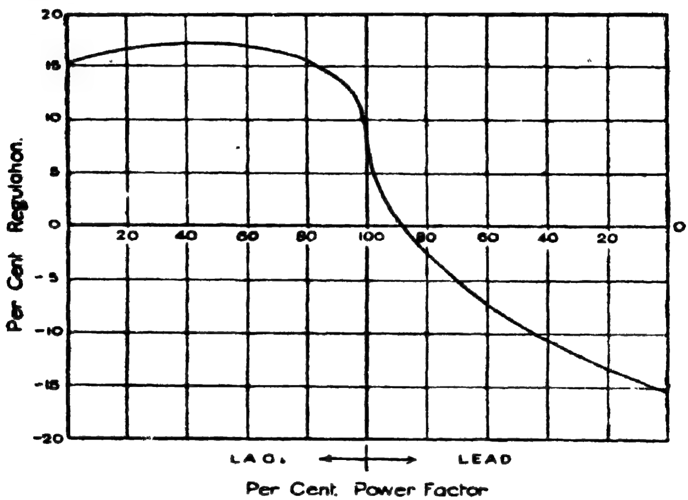

FIG. 16.—Regulation curve for short transmission line.

$$\text{Loss} = \frac{(154)^2 \times 9{\cdot}06}{1\,000}$$
$$= 214{\cdot}9 \text{ kW.}$$

$$\text{Efficiency} = \frac{2\,133 \times 100}{2\,133 + 214{\cdot}9}$$
$$= 90{\cdot}84\ \%.$$

$$\text{Regulation} = 20\,060 - 17\,320$$
$$= 2\,740 \quad \text{volts.}$$

$$\text{Percentage Regulation} = \frac{2\,740}{17\,320} \times 100\ \%$$
$$= 15{\cdot}82\ \%.$$

The percentage regulation derived from formula (33) is

$$\frac{IR \cos \phi_r + IX \sin \phi_r}{E_r} \times 100\ \%$$

$$= \frac{154 \times 9{\cdot}06 \times 0{\cdot}8 + 154 \times 17{\cdot}1 \times 0{\cdot}6}{17\,320} \times 100\ \%$$

$$= 15{\cdot}6\ \%,$$

which agrees fairly well with the accurate value.

The complete regulation curve for the line has also been calculated by the above formula and is shown in Fig. 16. The curve gives the percentage regulation of the line at all power factors, the current being the same in each case, *viz.* 154 amperes.

26. Mixed Sending- and Receiving-end Conditions.—Frequently in practice the case occurs where the voltage at the sending end is fixed, and also the power and power factor at the receiving end. In such a case the solution becomes more complicated. The following method of treatment is due to P. de Bancarel :—[2]

Referring to Fig. 14 the following equations can be written down :—

$$E_r \cos \phi_r + RI = E_s \cos \phi_s, \qquad (35)$$
$$E_r \sin \phi_r + XI = E_s \sin \phi_s, \qquad (36)$$

and also

$$E_r \cos \phi_r \times RI = 1\,000PR, \qquad (37)$$
$$E_r \sin \phi_r \times XI = 1\,000QX, \qquad (38)$$

where P = the power in kW. per phase delivered at the receiving end of the line,

and Q = the reactive kVA. per phase delivered at the receiving end of the line.

Squaring equations (35) and (36) and adding gives

$$E_r^2 + 2RIE_r \cos \phi_r + 2XIE_r \sin \phi_r + (R^2 + X^2)I^2 = E_s^2,$$

whence by substituting for the second and third terms from equations (37) and (38)

$$E_r^2 + 2\,000(PR + QX) + (R^2 + X^2)I^2 = E_s^2. \qquad (39)$$

Substituting

$$\frac{1\,000P}{E_r \cos \phi_r} = I,$$

and multiplying throughout by E_r^2 gives

$$E_r^4 - [E_s^2 - 2\,000(PR + QX)]E_r^2 + (R^2 + X^2)\left(\frac{1\,000P}{\cos \phi_r}\right)^2 = 0. \qquad (40)$$

Let

$$S = E_s^2 - 2\,000(PR + QX),$$

and

$$T = (R^2 + X^2)\left(\frac{1\,000P}{\cos\phi_r}\right)^2,$$

then equation (40) can be written

$$E_r^4 - SE_r^2 + T = 0, \quad . \quad . \quad . \quad (41)$$

whence

$$E_r = \sqrt{\frac{S \pm \sqrt{(S^2 - 4T)}}{2}} \quad . \quad . \quad . \quad . \quad (42)$$

In general, two values of the receiving-end voltage can be found to satisfy the conditions laid down, depending on whether the + or − sign is used in the formula. The solution giving the higher value of E_r and consequently higher transmission efficiency is the one required in practice, and thus the + sign should be taken.

The following example will illustrate the use of the formula given above :—

Problem 2.—Find the characteristics of the load at the receiving end of a three-phase line ten miles long, consisting of three hard-drawn, No. 1/0 copper conductors, spaced in a thirty-inch delta. Temperature taken as 20° C. Load at receiving end assumed as 1 500 kVA. (1 200 kW. at 80 °/ₒ power factor lagging). Voltage at sending end given as 11 000 volts, 50 cycles.

From wire tables the resistance per mile of conductor is found to be 0·531 ohms, and the diameter is 0·324 inch.

Hence the resistance of each conductor is

$$R = 0{\cdot}531 \times 10 = 5{\cdot}31 \quad \text{ohms.}$$

The inductance of each conductor is

$$L = \left(0{\cdot}080 + 0{\cdot}741 \log_{10} \frac{30}{0{\cdot}162}\right)10^{-3} \times 10 \quad . \quad (9)$$

$$= 0{\cdot}0176 \quad \text{henries,}$$

and the reactance is

$$X = 2\pi \times 50 \times 0{\cdot}0176$$
$$= 5{\cdot}54 \quad \text{ohms.}$$

There is also given

$$P = \frac{1\,200}{3} = 400 \quad \text{kW. per phase,}$$

$$Q = P \tan \phi_r$$
$$= 400 \times 0{\cdot}75$$
$$= 300 \quad \text{kVA. per phase,}$$
$$\cos \phi_r = 80\ \% \quad \text{lagging,}$$
$$E_s = \frac{11\,000}{\sqrt{3}} = 6\,351 \quad \text{volts to neutral.}$$

Hence

$$S = E_s^2 - 2\,000(PR + QX)$$
$$= 6\,351^2 - 2\,000(400 \times 5{\cdot}31 + 300 \times 5{\cdot}54)$$
$$= 3{\cdot}2763 \times 10^7,$$

and

$$T = (R^2 + X^2)\left(\frac{1\,000P}{\cos \phi_r}\right)^2$$
$$= (5{\cdot}31^2 + 5{\cdot}54^2)\left(\frac{4 \times 10^5}{0{\cdot}8}\right)^2$$
$$= 1{\cdot}4722 \times 10^{13}.$$

The receiving-end voltage

$$E_r = \sqrt{\frac{S + \sqrt{(S^2 - 4T)}}{2}} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (42)$$

$$= \sqrt{\frac{3{\cdot}2763 \times 10^7 + \sqrt{(1{\cdot}0734 \times 10^{15} - 5{\cdot}8888 \times 10^{13})}}{2}}$$

$= 5\,684$ volts to neutral,

and the current

$$I = \frac{500 \times 1\,000}{5\,684}$$
$$= 88{\cdot}0 \quad \text{amperes,}$$

also the sending-end power factor

$$\cos \phi_s = \frac{E_r \cos \phi_r + RI}{E_s}$$
$$= \frac{5\,684 \times 0{\cdot}8 + 5{\cdot}31 \times 88{\cdot}0}{6\,351}$$
$$= 79{\cdot}0\ \% \quad \text{lagging.}$$

27. Graphical Solution for Mixed Sending- and Receiving-end Conditions.—If preferred, a semi-graphical solution of this problem may be employed. Referring to Fig. 17,

OS = voltage to neutral at sending end of line,
OR = unknown voltage at receiving end,
and OD = unknown current.

Then the point R lies on an arc of a circle passing through O and S, the centre of which is at G, and the

$$\text{angle } GSO = \frac{\pi}{2} + \phi_r - \tan^{-1}\frac{X}{R}.$$

R also lies on a circle of radius

$$\overline{FR} = \sqrt{\frac{E_s^2}{4} - 1\,000(PR + QX)}, \quad . \quad . \quad (43)$$

the centre of which is at F the mid-point of OS.

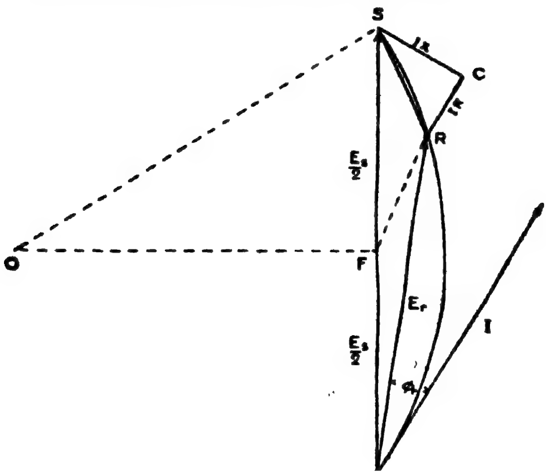

FIG. 17.—Vector diagram for mixed sending- and receiving-end conditions.

For a proof of this solution reference should be made to Fig. 18 in which a circle is drawn on OS as diameter, OR is produced to cut the circle at L, and FR produced both ways to cut the circle at H and K. RC drawn parallel to OD, and CS drawn at 90 degrees to OD, are respectively the ohmic and reactive pressure drops in the conductors.

Consider first of all the circle of which ORS is an arc. The angle ORS at the circumference is half the re-entrant angle OGS at the centre standing on the same chord OS.

The angle $ORS = 2\pi$ − angle ORC − angle CRS

$$= 2\pi - (\pi - \phi_r) - \tan^{-1}\frac{X}{R}$$

$$= \pi + \phi_r - \tan^{-1}\frac{X}{R},$$

hence the re-entrant angle $OGS = 2\pi + 2\phi_r - 2\tan^{-1}\frac{X}{R},$

angle $FGS = -\phi_r + \tan^{-1}\frac{X}{R},$

and angle $GSO \quad \pi + \phi_r - \tan^{-1}\frac{X}{R}.$

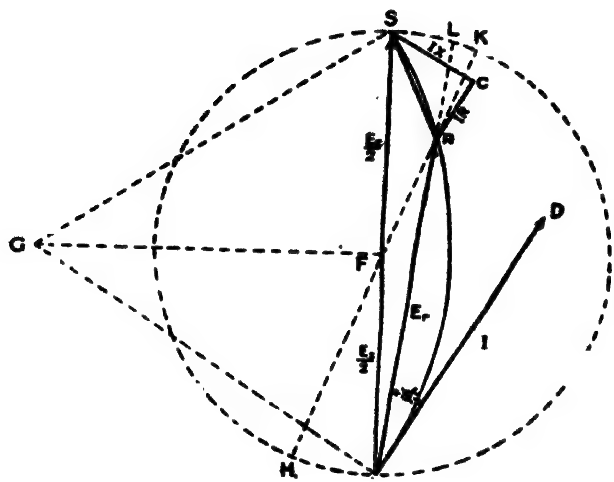

FIG. 18.—Derivation of formula for mixed sending- and receiving-end conditions.

Also in the triangle ORS

$$\overline{OS}^2 = \overline{OR}^2 + \overline{RS}^2 + 2\overline{OR}\,.\,\overline{RL},$$

or

$$E_s^2 = E_r^2 + (R^2 + X^2)I^2 + 2\overline{OR}\,.\,\overline{RL},$$

and comparing with equation (39) it is seen that

$$\overline{OR}\,.\,\overline{RL} = 1\,000(PR + QX).$$

Therefore $\overline{OR}\,.\,\overline{RL}$ expresses the power of point R in relation to the circle of diameter $OS = E_s$.

This quantity is constant so

$$\overline{OR}\,.\,\overline{RL} = 1\,000(PR + QX) = \overline{HR}\,.\,\overline{RK},$$

whence $$(\overline{HF} + \overline{FR})(\overline{FK} - \overline{FR}) = 1\,000(PR + QX),$$

or $$\frac{E_s^2}{4} - (FR)^2 = 1\,000(PR + QX),$$

and $$\overline{FR} = \sqrt{\frac{E_s^2}{4} - 1\,000(PR + QX)}.$$

28. Effect of Transformers.—In the foregoing discussion no account has been taken of the effect of transformers included in the circuit. Transmission lines usually have raising transformers installed at the sending end to step up from the generating to the line pressure, and lowering transformers at the receiving end to step down to the distribution pressure, and it is often required to

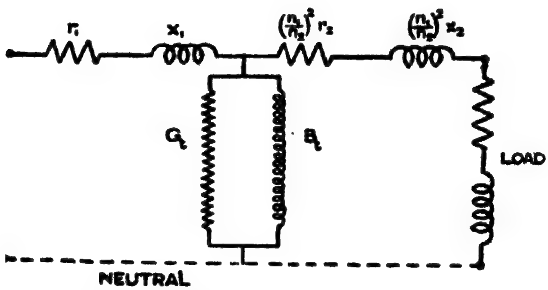

FIG. 19.—Equivalent circuit of transformer.

make calculations on the complete circuit from the generator terminals to the low-pressure busbars of the receiving station. Such calculations are conveniently made in terms of the high-voltage circuit and if results are desired in terms of the low-voltage side of the transformers they may be obtained from the ratio of transformation.

Fig. 19 represents the equivalent circuit of a transformer on load in which r_1 and x_1 are the resistance and reactance of the high-voltage winding, r_2 and x_2 the resistance and reactance of the low-voltage winding, and n_1 and n_2 the number of turns on the high-voltage and low-voltage windings respectively. $Y_t = G_t - jB_t$ is the primary exciting admittance of the transformer.

In power transformers the exciting current is of the order of 4 to 8 % of the full-load current, but it is nearly in quadrature with the voltage so that when the transformer is on load, the current I_1 in the high-voltage winding can be regarded as equal to $\frac{n_2}{n_1}$ times the current I_2 in the low-voltage winding. Hence, neglecting the effect of Y_t, the composite circuit of a transmission line and connected transformers can be represented as in Fig. 20. The equivalent resistance of the transformer in terms of the high-voltage circuit is

$$R_t \quad r_1 + \left(\frac{n_1}{n_2}\right)^2 r_2, \tag{44}$$

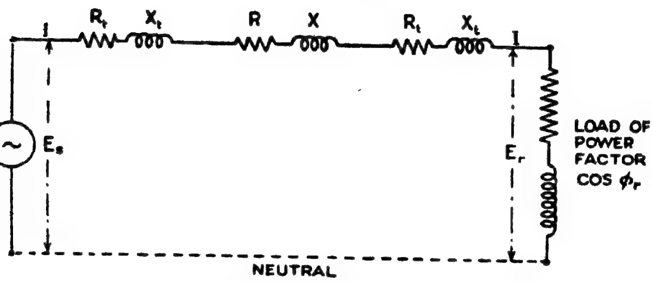

FIG. 20.—Short transmission line including effect of transformers.

and the equivalent reactance is

$$X_t = x_1 + \left(\frac{n_1}{n_2}\right)^2 x_2. \tag{45}$$

In many cases these values are not known but can be obtained from other known data.

For instance, take the case of an 8 000-kVA. transformer to be used in conjunction with the 30-mile transmission, particulars of which were given in Art. 25. The assumed performance is as follows :—

Efficiency at full load = 98 %.
Efficiency at one-fourth load = 96 %.
Regulation at full load 80 % power factor = ·4 %.

The equivalent resistance and reactance of the transformer (to neutral) can then be found by the method outlined below. At full load the input is

$$\frac{8\,000}{3 \times 0{\cdot}98} = 2\,721{\cdot}09 \quad \text{kW. per phase,}$$

so that the total loss per phase at full load is 54 420 watts.

At one-fourth load the input is

$$\frac{2\,000}{3 \times 0{\cdot}96} = 694{\cdot}44 \quad \text{kW. per phase,}$$

so that the total loss per phase is 27 770 watts.

The iron loss is constant at all loads, but the copper loss varies approximately as the square of the load, so that if x = the iron loss, and y = the copper loss at full load

$$x + y = 54\,420 \quad \text{watts,}$$

and

$$x + \frac{y}{16} = 27\,770 \quad \text{watts,}$$

from which

$$x = 26\,000 \quad \text{watts,}$$

and

$$y = 28\,420 \quad \text{watts.}$$

The current at 17 320 volts to neutral is 154 amperes, so that the equivalent resistance of the transformer referred to the high-voltage circuit is

$$R_t = \frac{28\,420}{154 \times 154}$$
$$= 1{\cdot}198 \quad \text{ohms (to neutral).}$$

The equivalent reactance can be obtained from formula (33) as applied to the regulation of a transformer

$$\text{Regulation} = IR_t \cos\phi + IX_t \sin\phi,$$

or

$$X_t = \frac{\text{Regulation} - IR_t \cos\phi}{I \sin\phi}$$
$$= \frac{\frac{4}{100} \times 17\,320 - 154 \times 1{\cdot}198 \times 0{\cdot}8}{154 \times 0{\cdot}6}$$
$$= 5{\cdot}900 \quad \text{ohms (to neutral).}$$

Supposing, now, this transformer to be installed at the receiving end of the line, the resistance and reactance can be added to the corresponding values for the line conductors, and the characteristics of the load at the high-voltage side of the sending-end transformer calculated as though there were no transformers in circuit. In a similar way the effect of the sending-end transformer can be taken

into account, and the electrical conditions determined at the terminals of the generator.

REFERENCES.

[1] 'British Standard Spécification for Hard-drawn Copper Solid and Stranded Circular Conductors for Overhead Power Transmission Purposes,' *B.S.S.* No. 125.—1930.

[2] P. de Bancarel, 'Note sur le calcul des lignes de transmission d'énergie,' *Rev. Gen. d'Él.*, Vol. 2, p. 48 (1917).

CHAPTER IV.

PERFORMANCE OF LONG TRANSMISSION LINES (LOCALISED-CAPACITANCE METHODS OF SOLUTION).

29. Representation of Line Capacitance by Condensers.—In the transmission of power by overhead lines having a length of over 50 miles (and in nearly all cases where high-pressure underground cables are employed) the capacitance current of the system is appreciable and must be reckoned with in all calculations. This capacitance current is always flowing in the line while the generating-station switches are closed even though the receiving end of the line may be open-circuited. The actual value of the capacitance current flowing at any point along the line is that required to charge the section of line between the given point and the receiving end, hence it has a maximum value at the sending end and diminishes at a practically uniform rate down to zero at the receiving end.

The effect of the capacitance current can be approximately taken into account by assuming that the capacitance of the system is divided up and 'lumped' in the form of condensers shunted across the line at one or more points.

The commonest localised-capacitance methods of representation are:—

1. The middle-condenser, or nominal-T method in which the capacitance of each conductor is represented by a condenser of equal capacitance shunted between conductor and neutral midway along the line.

2. The split-condenser, or nominal-π method in which one-half the total capacitance of each conductor is assumed to be shunted between conductor and neutral at each end of the circuit.

30. Nominal-T Method.—This method assumes that the total capacitance of the line may be concentrated at its middle point and so the full charging current flows over half the line. The arrangement of the circuit is then as shown schematically in

the upper part of Fig. 21. At the terminals of the condenser the voltage has a value E' intermediate between the sending-end voltage E_s and the receiving-end voltage E_r. A capacitance current I_c, leading E' by 90 degrees, accordingly flows along the sending-end half of the line to charge the condenser. The current in the receiving-end half of the line is the load current I_r, and in the sending-end half of the line it is I_s the vector sum of the capacitance current I_c and the load current I_r.

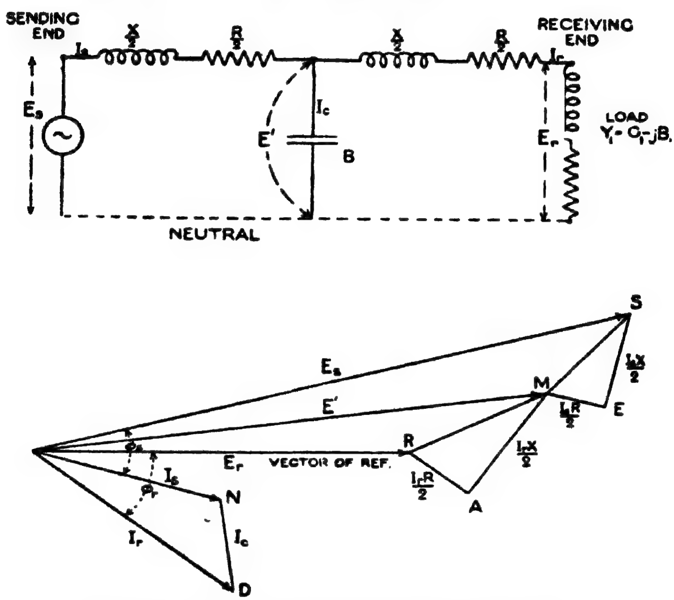

Fig. 21.—Middle-condenser or nominal-T method.

In the vector diagram the various quantities are represented as follows:—

OD is the current in the receiving circuit.

OR is the voltage to neutral at the receiving end drawn ϕ_r degrees in advance of *OD*: cos ϕ_r being the load power factor.

RA drawn parallel to *OD* is the ohmic pressure drop in the receiving-end half of the line.

AM drawn 90 degrees ahead of *OD* is the reactive pressure drop in the receiving-end half of the line.

OM is the voltage at the centre of the line.

DN drawn 90 degrees ahead of *OM* is the charging current of the line.

ON which is the vector sum of *OD* and *DN* is the current at the sending end.

ME drawn parallel to *ON* is the ohmic pressure drop in the sending-end half of the line.

ES drawn 90 degrees ahead of *ON* is the reactive pressure drop in the sending-end half of the line.

OS is the voltage at the sending end.

ϕ_s is the phase-angle at the sending end: $\cos \phi_s$ being the power factor at this end.

In order to determine the performance of the line by means of complex quantities let

$$Y_1 = G_1 - jB_1 = \text{admittance of receiving circuit,}$$
$$Z = R + jX = \text{impedance of line,}$$
and
$$B = \text{condensive susceptance of line.}$$

Then referring to the schematic diagram in the upper part of Fig. 21 we have

$$I_r = E_r(G_1 - jB_1).$$

$$E' = E_r + \frac{R + jX}{2} I_r$$
$$= E_r\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}$$

$$I_c = E' jB$$
$$= jBE_r\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}$$

$$I_s = I_r + I_c$$
$$= E_r\left[G_1 - jB_1 + jB\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}\right] \tag{46}$$

$$E_s = E' + \frac{R + jX}{2} I_s$$
$$= E_r\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2} + \frac{(R + jX)(G_1 - jB_1)}{2} + jB\left(\frac{R + jX}{2}\right) + jB\frac{(R + jX)^2(G_1 - jB_1)}{4}\right\}$$
$$= E_r\left\{1 + (R + jX)\left(G_1 - jB_1 + \frac{jB}{2}\right) + \frac{jB}{4}(R + jX)^2(G_1 - jB_1)\right\} \tag{47}$$

31. Nominal-π Method.—This method assumes that the total capacitance of the line is split up, one-half being placed across the line at each end. In this case one-half of the total charging current flows over the entire line.

The schematic diagram for this arrangement of the circuit is given in the upper part of Fig. 22. The current flowing in the line is I_r the vector sum of the load current I and the current I_{cr} taken by the receiving-end condenser. Knowing this resultant current, the voltage at the sending end can easily be found by the

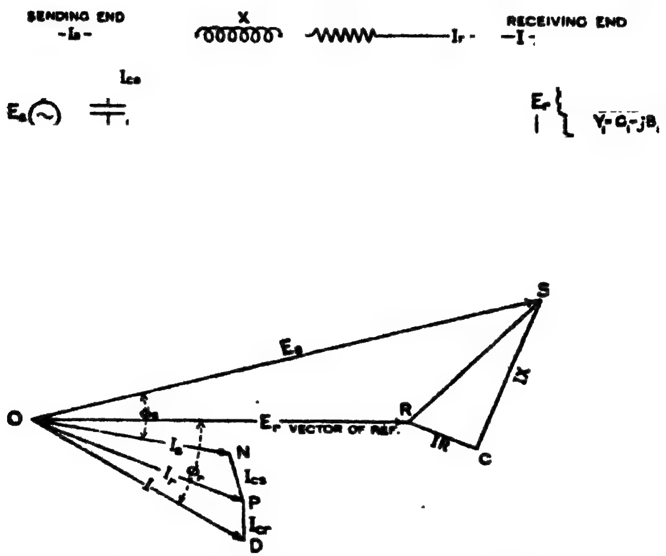

FIG. 22.—Split-condenser or nominal-π method.

fundamental impedance method used for short lines.
end voltage is therefore more readily calculated by this method than by the middle-condenser method which requires the calculation of the two separate halves of the circuit. If, however, the sending-end current and power factor are also required, a second calculation must be made to determine them. In such a case the current I_{cs} consumed by the condenser at the sending end must be added vectorially to that in the line.

In the vector diagram the various quantities are represented as follows:—

OR is the voltage to neutral at the receiving end.

OD is the load current drawn ϕ_r degrees behind OR: $\cos \phi_r$ being the load power factor.

DP drawn 90 degrees ahead of OR is the charging current consumed by the receiving-end condenser.

OP which is the vector sum of OD and DP is the resultant current in the line.

RC drawn parallel to OP is the ohmic pressure drop.

CS drawn 90 degrees ahead of OP is the reactive pressure drop.

OS is the voltage to neutral at the sending end.

PN drawn 90 degrees ahead of OS is the charging current consumed by the sending-end condenser.

ON which is the vector sum of OP and PN is the resultant sending-end current.

ϕ_s is the phase angle at the sending end: $\cos \phi_s$ being the power factor at this end.

Applying complex quantities to the solution of the same circuit we have the following relations:—

$$
\begin{aligned}
\dot{I}_r &= \dot{I} + \dot{I}_{cr} \\
&= \dot{E}_r(G_1 - jB_1) + \frac{jB}{2}\dot{E}_r \\
&= \dot{E}_r\left(G_1 - jB_1 + \frac{jB}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
\dot{E}_s &= \dot{E}_r + \dot{I}_r(R + jX) \\
&= \dot{E}_r + \dot{E}_r\left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX) \\
&= \dot{E}_r\left\{1 + \left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\}. \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
\dot{I}_{cs} &= \frac{jB}{2}\dot{E}_s \\
&= \frac{jB}{2}\dot{E}_r\left\{1 + \left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\}.
\end{aligned}
$$

$$
\begin{aligned}
\dot{I}_s &= \dot{I} + \dot{I}_{cs} \\
&= \dot{E}_r\left(G_1 - jB_1 + \frac{jB}{2}\right) + \frac{jB}{2}\dot{E}\left\{1 + \left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\} \\
&= \dot{E}_r\left\{G_1 - jB_1 + jB + \frac{jB}{2}\left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\}. \qquad (49)
\end{aligned}
$$

32. Numerical Example.—Transmission line problems may be solved graphically by drawing the vector diagram to scale, but it is difficult to obtain accurate results by this method owing to the varying magnitude of the quantities represented, and it is advisable to calculate all quantities. These calculations may be made trigonometrically, using the appropriate vector diagram as a guide, or the method of solution by means of complex quantities may be employed.

In order to show the nature of the agreement between the results obtained by the two localised-capacitance methods, the same problem will be solved by both methods.

Problem 3.—Find the characteristics of the load at the sending end of a three-phase line 120 miles long, consisting of three No. 3/0 S.W.G., hard-drawn copper conductors, spaced in a nine-foot delta arrangement. Temperature taken as 20° C. Load conditions at receiving end assumed as 15 000 kVA. (13 500 kW. at 90 °/ₒ power factor lagging), 88 000 volts, 50 cycles.

From wire tables the resistance per mile of wire is found to be 0· 406 ohms, and the diameter of the wire is 0· 372 inch.

The resistance of each conductor is therefore

$$R = 0{\cdot}\,406 \times 120 = 48{\cdot}7 \quad \text{ohms.}$$

The inductance of each conductor is

$$L = \left(0{\cdot}\,080 + 0{\cdot}\,741 \log_{10} \frac{108}{0{\cdot}\,186}\right) 10^{-3} \times 120 \quad . \quad (9)$$
$$= 0{\cdot}\,255 \quad \text{henries,}$$

and the reactance is

$$X = 2\pi \times 50 \times 0{\cdot}\,255$$
$$= 80{\cdot}2 \quad \text{ohms.}$$

The capacitance of each conductor from formula (15) is

$$C = \frac{0{\cdot}\,0388}{\log_{10} \dfrac{108}{0{\cdot}\,186}} 10^{-6} \times 120$$
$$= 1{\cdot}68 \times 10^{-6} \quad \text{farads,}$$

and the condensive susceptance is

$$B = 2\pi \times 50 \times 1{\cdot}68 \times 10^{-6}$$
$$= 0{\cdot}\,000529 \quad \text{mhos.}$$

At the receiving end of the line

$$kVA_r = \frac{15\ 000}{3} = 5\ 000 \quad \text{kVA. per phase.}$$

$$kW_r = \frac{13\ 500}{3} = 4\ 500 \quad \text{kW. per phase.}$$

$$E_r = \frac{88\ 000}{\sqrt{3}} = 50\ 810 \quad \text{volts to neutral.}$$

$$\cos\phi_r = 90\ \% \quad \text{lagging.}$$

The load current is

$$I = \frac{5\ 000 \times 1\ 000}{50\ 810} = 98{\cdot}5 \quad \text{amperes.}$$

The load admittance is

$$Y_1 = \frac{I}{E_r}$$

$$= \frac{98{\cdot}5}{50\ 810} = 0{\cdot}\ 001939 \quad \text{mhos per phase.}$$

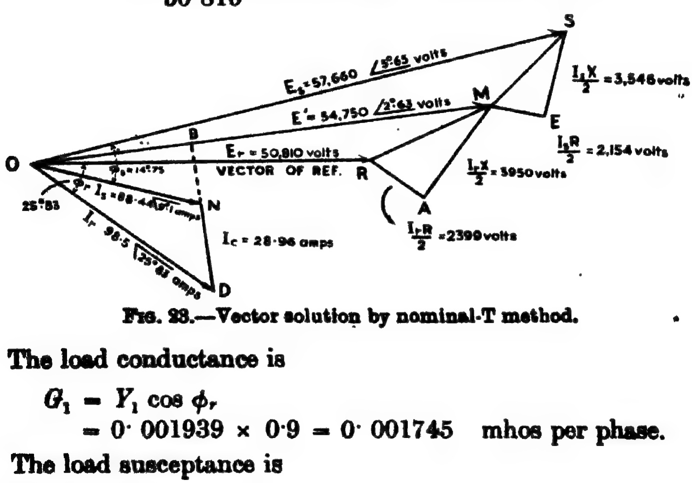

FIG. 23.—Vector solution by nominal-T method.

The load conductance is

$$G_1 = Y_1 \cos\phi_r$$
$$= 0{\cdot}\ 001939 \times 0{\cdot}9 = 0{\cdot}\ 001745 \quad \text{mhos per phase.}$$

The load susceptance is

$$B_1 = Y_1 \sin\phi_r$$
$$= 0{\cdot}\ 001939 \times 0{\cdot}\ 4359 = 0{\cdot}\ 0008450 \quad \text{mhos per phase.}$$

33. Nominal-T Solution.—The graphical vector solution for the above problem by the nominal-T method is indicated in Fig. 23, although, for the sake of clearness, this is not drawn strictly to scale.

The electrical conditions at the middle of the line are first of all determined as follows :—

$$\frac{I_r R}{2} = 98{\cdot}5 \times 24{\cdot}35 = 2\,399 \quad \text{volts (resistance drop).}$$

$$\frac{I_r X}{2} = 98{\cdot}5 \times 40{\cdot}1 = 3\,950 \quad \text{volts (reactance drop).}$$

$$E' = 50\,810\sqrt{\left(0{\cdot}9 + \frac{2\,399}{50\,810}\right)^2 + \left(0{\cdot}4359 + \frac{3\,950}{50\,810}\right)^2}$$

from formula (31)

$$= 50\,810\sqrt{(0{\cdot}9472)^2 + (0{\cdot}5135)^2}$$
$$= 54\,750 \quad \text{volts to neutral.}$$

E' leads the current vector OD by $\tan^{-1}\dfrac{0{\cdot}5135}{0{\cdot}9472} = 28{\cdot}47°$.

So

$E' = 54\,750 \,\underline{/28{\cdot}47°}$ volts to neutral with reference to vector OD.
$= 54\,750 \,\underline{/2{\cdot}63°}$ volts to neutral with reference to vector OR.

The current consumed by the condenser leads the voltage E' by 90 degrees and is

$$I_c = 0{\cdot}000529 \times 54\,750 = 28{\cdot}96 \quad \text{amperes.}$$

The current in the sending-end half of the line is determined as follows:—

$OB = 98{\cdot}5 \cos 28{\cdot}47° = 86{\cdot}59$ amperes.
$BD = 98{\cdot}5 \sin 28{\cdot}47° = 46{\cdot}95$ amperes.
$BN = 46{\cdot}95 - 28{\cdot}96 = 17{\cdot}99$ amperes.
$I_s = ON = \sqrt{86{\cdot}59^2 + 17{\cdot}99^2}$
$= 88{\cdot}44 \,\overline{\diagdown 11{\cdot}73°}$ amperes with reference to vector OB.
$= 88{\cdot}44 \,\overline{\diagdown 9{\cdot}10°}$ amperes with reference to vector OR.

The sending-end voltage is then found as follows:—

$$\frac{I_s R}{2} = 88{\cdot}44 \times 24{\cdot}35 = 2\,154 \quad \text{volts (resistance drop).}$$

$$\frac{I_s X}{2} = 88{\cdot}44 \times 40{\cdot}1 = 3\,546 \quad \text{volts (reactance drop).}$$

$$E_s = 54\,750\sqrt{\left(\cos 11{\cdot}73° + \frac{2\,154}{54\,750}\right)^2 + \left(\sin 11{\cdot}73° + \frac{3\,546}{54\,750}\right)^2} \qquad (31)$$

$$= 54\,750\sqrt{\left(0{\cdot}9791 + \frac{2\,154}{54\,750}\right)^2 + \left(0{\cdot}2034 + \frac{3\,546}{54\,750}\right)^2}$$
$$= 54\,750\sqrt{(1{\cdot}0184)^2 + (0{\cdot}2682)^2}.$$

$= 57\,860 \,\underline{/14{\cdot}75°}$ volts to neutral with reference to vector ON
$= 57\,860 \,\underline{/5{\cdot}65°}$ volts to neutral with reference to vector OR.

The other electrical conditions at the sending end are

$$\cos\phi_s = \cos 14{\cdot}75° = 96{\cdot}70\ \%\quad \text{lagging.}$$
$$\text{kVA}_s = 57\ 660 \times 88{\cdot}44 = 5\ 099\quad \text{kVA. per phase.}$$
$$\text{kW}_s = 5\ 099 \times 0{\cdot}9670 = 4\ 931\quad \text{kW. per phase.}$$
$$\text{Loss} = 4\ 931 - 4\ 500 = 431\quad \text{kW. per phase.}$$
$$\text{Efficiency} = \frac{4\ 500 \times 100}{4\ 931} = 91{\cdot}26\ \%$$

In order to solve the same problem by complex quantities, it is only necessary to substitute numerical values in the analytical expressions for voltage and current derived in Art. 30.

The sending-end voltage is

$$\underset{\bullet}{E}_s = \underset{\bullet}{E}_r\left\{1 + (R + jX)\left(G_1 - jB_1 + \frac{jB}{2}\right) + \frac{jB}{4}(R + jX)^2(G_1 - jB_1)\right\} \tag{47}$$

$$= 50\ 810\{1 + (48{\cdot}7 + 80{\cdot}2j)(0{\cdot}001745 - 0{\cdot}0008450j + 0{\cdot}0002645j) + 0{\cdot}0001322j(48{\cdot}7 + 80{\cdot}2j)^2(0{\cdot}001745 - 0{\cdot}0008450j)\}$$
$$= 50\ 810(1{\cdot}1293 + 0{\cdot}1116j),$$

and its absolute value is

$$E_s = 50\ 810\sqrt{(1{\cdot}1293)^2 + (0{\cdot}1116)^2}$$
$$= 57\ 660\ \angle 5{\cdot}65° \quad \text{volts to neutral.}$$

The sending-end current is

$$\underset{\bullet}{I}_s = \underset{\bullet}{E}_r\left[G_1 - jB_1 + jB\left\{1 + \frac{(R + jX)(G_1 - jB_1)}{2}\right\}\right] \tag{46}$$

$$= 50\ 810[0{\cdot}001745 - 0{\cdot}0008450j + 0{\cdot}000529j\{1 + (24{\cdot}35 + 40{\cdot}1j)(0{\cdot}001745 - 0{\cdot}0008450j)\}]$$
$$= 50\ 810(0{\cdot}001719 - 0{\cdot}0002753j),$$

and its absolute value is

$$I_s = 50\ 810\sqrt{(0{\cdot}001719)^2 + (0{\cdot}0002753)^2}$$
$$= 88{\cdot}44\ \overline{\diagdown 9{\cdot}10°} \quad \text{amperes.}$$

These results check exactly with those previously obtained by trigonometrical calculation.

34. Nominal-π Solution.—The graphical vector solution for Problem 3 by the nominal-π method is indicated in Fig. 24, although, like Fig. 23, this diagram is not drawn strictly to scale.

The charging current consumed by the receiving-end condenser leads the voltage at this end by 90 degrees and is

$$I_{cr} = 0{\cdot}0002645 \times 50\ 810 = 13{\cdot}44 \quad \text{amperes.}$$

The resultant current in each conductor is the vector sum of the load and condenser currents, and is determined as follows:—

$$OB = 98{\cdot}5 \times 0{\cdot}9 = 88{\cdot}65 \quad \text{amperes.}$$
$$BD = 98{\cdot}5 \times 0{\cdot}4359 = 42{\cdot}92 \quad \text{amperes.}$$
$$BP = 42{\cdot}92 - 13{\cdot}44 = 29{\cdot}48 \quad \text{amperes.}$$
$$I_r = OP = \sqrt{88{\cdot}65^2 + 29{\cdot}48^2}$$
$$= 93{\cdot}42 \;\overline{\diagdown 18{\cdot}40^\circ} \quad \text{amperes with reference to vector } OR.$$

The voltage consumed by the resistance and reactance of each conductor is

$$I_rR = 93{\cdot}42 \times 48{\cdot}7 = 4\,550 \quad \text{volts (resistance drop).}$$
$$I_rX = 93{\cdot}42 \times 80{\cdot}2 = 7\,492 \quad \text{volts (reactance drop).}$$

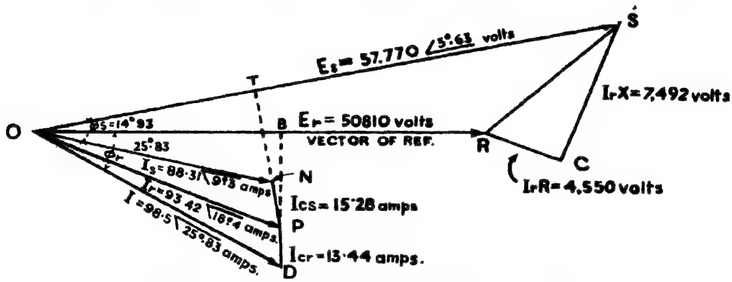

FIG. 24.—Vector solution by nominal-π method.

The sending-end voltage is

$$E_s = 50\,810\sqrt{\left(\cos 18{\cdot}40^\circ + \frac{4\,550}{50\,810}\right)^2 + \left(\sin 18{\cdot}40^\circ + \frac{7\,492}{50\,810}\right)^2}$$
$$= 50\,810\sqrt{(1{\cdot}0384)^2 + (0{\cdot}4631)^2}$$
$$= 57\,770 \;\underline{/24{\cdot}03^\circ} \quad \text{volts to neutral with reference to vector } OP.$$
$$= 57\,770 \;\underline{/5{\cdot}63^\circ} \quad \text{volts to neutral with reference to vector } OR.$$

The charging current consumed by the condenser at the sending end leads the voltage at this end by 90 degrees and is

$$I_{cs} = 0{\cdot}0002645 \times 57\,770 = 15{\cdot}28 \quad \text{amperes.}$$

The sending-end current is the vector sum of the current flowing in the conductor and the current consumed by the condenser at the sending end. It is found as follows:—

$$OT = 93{\cdot}42 \cos 24{\cdot}03^\circ = 85{\cdot}33 \quad \text{amperes.}$$
$$TP = 93{\cdot}42 \sin 24{\cdot}03^\circ = 38{\cdot}05 \quad \text{amperes.}$$
$$TN = 38{\cdot}05 - 15{\cdot}28 = 22{\cdot}77 \quad \text{amperes.}$$

$$I_s = ON = \sqrt{85{\cdot}33^2 + 22{\cdot}77^2}$$
$$= 88{\cdot}31 \overline{\diagdown 14{\cdot}93^\circ} \text{ amperes with reference to vector } OS$$
$$= 88{\cdot}31 \overline{\diagdown 9{\cdot}30^\circ} \text{ amperes with reference to vector } OR.$$

The electrical conditions at the sending end are therefore

$$\cos \phi_s = \cos 14{\cdot}93^\circ = 96{\cdot}62\ \% \text{ lagging.}$$
$$\text{kVA}_s = 57\ 770 \times 88{\cdot}31 = 5\ 102 \text{ kVA. per phase.}$$
$$\text{kW}_s = 5\ 102 \times 0{\cdot}\ 9662 = 4\ 929 \text{ kW. per phase.}$$
$$\text{Loss} = 4\ 929 - 4\ 500 = 429 \text{ kW. per phase.}$$
$$\text{Efficiency} = \frac{4\ 500 \times 100}{4\ 929} = 91{\cdot}29\ \%.$$

Applying complex quantities to the solution of the problem the sending-end voltage is

$$\dot{E}_s = \dot{E}_r\left\{1 + \left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\} \quad . \quad . \quad . \quad (48)$$
$$= 50\ 810\{1 + (0{\cdot}\ 001745 - 0{\cdot}\ 0008450j + 0{\cdot}\ 0002645j)(48{\cdot}7 + 80{\cdot}2j)\}$$
$$= 50\ 810(1{\cdot}\ 1315 + 0{\cdot}\ 1117j)$$
$$= 57\ 770 \underline{/5{\cdot}63^\circ} \text{ volts to neutral.}$$

The sending-end current is

$$\dot{I}_s = \dot{E}_r\left\{G_1 - jB_1 + jB + \frac{jB}{2}\left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\} \quad (49)$$
$$= 50\ 810\{0{\cdot}\ 001745 - 0{\cdot}\ 0008450j + 0{\cdot}\ 000529j + 0{\cdot}\ 0002645j(0{\cdot}\ 001745 - 0{\cdot}\ 0008450j + 0{\cdot}\ 0002645j)(48{\cdot}7 + 80{\cdot}2j)\}$$
$$= 50\ 810(0{\cdot}\ 001715 - 0{\cdot}\ 0002809j)$$
$$= 88{\cdot}31 \overline{\diagdown 9{\cdot}30^\circ} \text{ amperes.}$$

The above results correspond exactly with those previously obtained by the trigonometrical method.

35. Ferranti Effect.—On a long transmission line, open-circuited, or only very lightly loaded, a rise of pressure occurs at the receiving end. This is generally known as the 'Ferranti effect' owing to its first being observed on the Deptford mains laid down by S. Z. de Ferranti in 1890.[1]

This pressure rise is due to the e.m.f. of self-inductance of the charging current being in phase with the impressed voltage at the sending end of the line, and thus both capacitance and inductance are necessary to produce the phenomenon.

In order to determine the magnitude of the pressure rise one-half of the total line capacitance will be assumed to be concentrated at the receiving end. In Fig. 25 the receiving-end voltage to neutral is represented by *OR*, while *OD* drawn 90 degrees ahead of *OR* is the charging current consumed by the receiving-end condenser. When the load is entirely disconnected *OD* is the total current in the conductor. The voltage consumed by the resistance of each conductor is *RC* drawn in phase with *OD*, and the voltage consumed by the reactance of each conductor is *CS* drawn 90 degrees ahead of *OD*. Thus the voltage at the sending end of the line is *OS*, which is smaller than that at the receiving end.

In the case of short lines the magnitude of the effect is negligible,

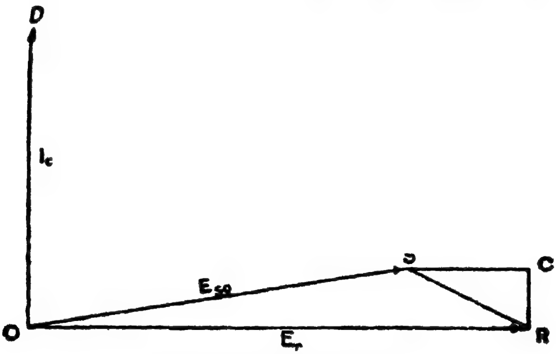

FIG. 25.—Ferranti effect.

but it increases rapidly as the length of the line increases, and on a 300-mile 50-cycle overhead transmission the receiving-end voltage on open-circuit would be 10 to 15 °/ₒ higher than the sending-end voltage.

Overhead lines having a sufficient length to bring the effect into prominence usually have a fairly large reactance compared with resistance, and by taking advantage of this fact a simple approximate formula to calculate the magnitude of the effect can be found. Thus, referring again to Fig. 25, if *RC* can be neglected in comparison with *CS*, the pressure rise *OR* – *OS* can be taken as equal to the induced volts without introducing any serious error.

The charging current flowing in the conductors is

$$I_c \quad 2\pi f \frac{C_0}{2} El, \tag{50}$$

where C_0 = the capacitance per mile of conductor in farads
and l = length of line in miles.

The induced volts are

$$I_c X = 2\pi f L_0 I_c l, \quad . \quad . \quad . \quad . \quad (51)$$

where L_0 = the inductance per mile of conductor in henries.

Substituting for I_c in the above,

$$I_c X = (2\pi f)^2 \frac{L_0 C_0}{2} l^2 E \quad \text{volts}, \quad . \quad . \quad . \quad (52)$$

but from formulæ (9) and (15) it can be seen that the product $L_0 C_0$ for all overhead lines has a practically constant value of 3×10^{-11}, hence the pressure rise at the end of the line is

$$I_c X = (2\pi f)^2 \frac{3 \times 10^{-11}}{2} l^2 E$$
$$= 5{\cdot}9 E f^2 l^2 10^{-10} \quad \text{volts.} \quad . \quad . \quad . \quad (53)$$

For 50-cycle lines this gives an approximate value of

$$1{\cdot}5 E \left(\frac{l}{1\,000}\right)^2 \quad \text{volts.} \quad . \quad . \quad . \quad (54)$$

In the case of the particular line having the constants given in Problem 3, the pressure rise at zero load is

$$1{\cdot}5 \times 50\,810 \left(\frac{120}{1\,000}\right)^2$$
$$= 1\,100 \quad \text{volts to neutral,}$$

or assuming that the receiving-end voltage was held constant at 50 810 volts to neutral, the corresponding sending-end voltage would be

$$E_{so} = 50\,810 - 1\,100$$
$$= 49\,710 \quad \text{volts to neutral.}$$

36. Charging Current and Losses in Open-circuited Line.—In transmission line operation the voltage at the receiving end is usually maintained at a constant value although the actual load on the system may vary considerably at different times.

The charging current of an open-circuited line then refers to the amount of current flowing into the line at the sending end with normal voltage and zero load at the receiving end.

In many cases the total charging current of the system is determined by multiplying the total susceptance of the line by the receiving-end voltage. This would be correct if the voltage

throughout the entire length of the line had the same value as at the receiving end. As shown above, this condition is only approximately complied with, but this method of finding the total charging current is sufficiently accurate for most commercial lines.

Making then the assumption of voltage equality along the line, the actual value of the charging current will decrease uniformly from its maximum value at the sending end to zero at the receiving end.

The value of the current at a point x miles from the sending end of a line l miles in length is

$$\frac{I_c}{l}(l - x), \qquad (55)$$

where I_c is the total charging current as found by the above method.

The average value of the square of this quantity is

$$\begin{aligned} &\frac{1}{l}\int_0^l \frac{I_c^2}{l^2}(l - x)^2 dx \\ &= \frac{I_c^2}{l^3}\int_0^l (l - x)^2 dx \\ &= \frac{I_c^2}{3}. \qquad (56) \end{aligned}$$

Hence at open-circuit, the power losses in each conductor are

$$\frac{I_c^2}{3}R. \qquad (57)$$

Applying these results to the line described in Problem 3, the charging current is

$$\begin{aligned} I_c &= 50\,810 \times 0{\cdot}000529 \\ &= 26{\cdot}87 \quad \text{amperes,} \end{aligned}$$

the kilovolt-amperes put into the line at the sending end are

$$\begin{aligned} \text{kVA}_{so} &= 50\,810 \times 26{\cdot}87 \times 10^{-3} \\ &= 1\,365 \quad \text{kVA. per phase,} \end{aligned}$$

and the open-circuit power losses are

$$\begin{aligned} &\frac{26{\cdot}87^2}{3} \times 48{\cdot}7 \times 10^{-3} \\ &= 11{\cdot}72 \quad \text{kW. per phase.} \end{aligned}$$

37. Effect of Capacitance on Performance of Loaded Line.—When, as usually obtains in practice, the transmission line

supplies an inductive load, the effect of capacitance is to reduce the pressure drop along the line and also to decrease the copper losses of the system. The magnitude of this effect depends on the phase-angle at the receiving end.

In any line, the current flowing along the conductors can be split up into two components, the load current and the charging current. The effect of the load current with lagging or unity power factor in the receiving circuit is to produce a voltage at the receiving end lower than that impressed on the line at the sending end. The charging current, however, tends to raise the receiving-end voltage, consequently the pressure drop along the line is diminished by the amount indicated by formula (53).

In this connection the distinction between the following terms should be noted: 1. Impedance drop, 2. Pressure drop, and 3. Regulation.

1. The impedance drop in the line is the voltage consumed in the line impedance. It is the vector difference between the sending-end voltage and the receiving-end voltage.

2. The pressure drop is the arithmetical difference between the sending- and receiving-end voltages.

3. The regulation is the change of pressure at the receiving end when full load is thrown off, the sending-end voltage being held constant. In the case of short lines the regulation and pressure drop are numerically equal. In lines where capacitance effects have to be reckoned with, the regulation is greater than the pressure drop by the pressure rise due to the Ferranti effect.

Thus, referring again to Problem 3, and using the results of the split-condenser method of solution,

$$\begin{aligned} \text{Regulation} &= \text{Pressure drop} + 1\,100 \\ &= 57\,770 - 50\,810 + 1\,100 \\ &= 8\,060 \quad \text{volts to neutral,} \end{aligned}$$

and expressed in terms of the receiving-end voltage

$$\begin{aligned} \text{Percentage regulation} &= \frac{8\,060 \times 100}{50\,810} \\ &= 15{\cdot}86\ \%. \end{aligned}$$

The presence of capacitance also influences the power losses of the system by an amount depending on the relative values of the charging current and the quadrature component of the load current.

For instance, referring to Fig. 22, the load current I can be

resolved into the two components $I \cos \phi_r$ in phase with E_r, and $I \sin \phi_r$ in quadrature behind E_r. The charging current I_{cr} is 90 degrees ahead of E_r and hence reduces the value of the quadrature component of the resultant line current from $I \sin \phi_r$ to $I \sin \phi_r - I_{cr}$ The resultant line current is thus

$$I_r = \sqrt{(I \cos \phi_r)^2 + (I \sin \phi_r - I_{cr})^2},$$

and the copper losses are reduced in the ratio

$$k = \frac{(I \cos \phi_r)^2 + (I \sin \phi_r - I_{cr})^2}{I^2}. \qquad (58)$$

It is easily seen from this formula that a reduction in the losses takes place so long as I_{cr} is less than $2I \sin \phi_r$.

Assuming now that the power factor of the load is unity there will be no quadrature component of the load current. Thus the effect of the charging current in a line supplying a non-inductive load is always to increase the copper losses.

Also if the power factor is lagging, but the actual load on the system is small, I_{cr} may be greater than $2I \sin \phi_r$, giving again an increased line loss.

In this connection Table 2 has been calculated from the data given in Problem 3, and shows the actual copper losses expressed as a percentage of what they would have been if there had been no charging current present in the line.

TABLE 2.—*Effect of Capacitance on Line Losses of a Particular Transmission.*

Load Power Factor.	Actual I^2R Losses as a Percentage of those which would Exist if Charging Current were Zero.
80 % lagging	85·5 %
85 % "	87·5 %
90 % "	90·0 %
95 % "	93·3 %
Unity	101·9 %

With the normal type of load-factor curve the actual effect of capacitance on the line losses during a 24-hour period is often negligible.

38. Limitations of Localised-capacitance Methods of Solution.—The localised-capacitance methods of solution are

capable of giving fairly accurate results for 50-cycle overhead lines up to about 150 miles in length. That is to say, the percentage error in the sending-end voltage, as calculated from the receiving-end data, by either of the methods described will be less than 1 °/ₒ for lines of this length. It is interesting to note that the nominal-T method over-compensates (that is, gives sending-end voltages which are too low), while the nominal-π method under-compensates.

A still higher degree of accuracy can be obtained by the three-condenser method suggested by Steinmetz,[2] in which the capacitance of the line is assumed to be concentrated across the line at three points, one-sixth being localised at each end and two-thirds midway along the line.

As a matter of fact, however, localised-capacitance methods are now chiefly of academic interest where very long lines are concerned, as formulæ have been developed which take into exact account the distributed character of the capacitance and other line constants, and at the same time do not involve much more labour in computation.

REFERENCES.

J. A. Fleming, 'On Some Effects of Alternating-Current Flow in Circuits having Capacity and Self-Induction,' *Jour. I.E.E.*, Vol. 20, p. 362 (1891).

[2] C. P. Steinmetz, *Theory and Calculation of Alternating Current Phenomena*, p. 169 (5th Ed., 1916).

CHAPTER V.

PERFORMANCE OF LONG TRANSMISSION LINES (RIGOROUS METHODS OF SOLUTION).

39. Alternating-current Phenomena in a Line having Distributed Electrical Properties.—In order to obtain a physical conception of the voltage and current variation from point to point on a long transmission line it may be imagined that the line is divided into n sections each having a resistance, inductance and capacitance $\frac{1}{n}$th of those for the whole line. Also to take account of the energy losses occurring through leakage over the insulators, or due to corona effect between conductors, a conductance of value $\frac{G}{n}$ may be assumed to be connected across the line at each section. The line is then represented schematically by the upper part of Fig. 26.

Starting from one end of the line, the voltage and current at any point can be found by taking each section in turn and constructing a diagram showing the voltages and currents consumed by the components of each section.

For example, in Fig. 26, $OR(E_r)$ represents the voltage to neutral at the receiving end of the line, and $OD(I_r)$ the current in each conductor lagging ϕ_r degrees behind E_r.

Consider first the section next to the receiving circuit.

A voltage RE_1, proportional to and in phase with OD, is consumed by the resistance, and a voltage E_1E_1', proportional to and 90 degrees ahead of OD, is consumed by the reactance. In the same section there is a current DI_1, proportional to and in phase with the voltage OR, representing the loss of current by leakage, and a current I_1I_1', proportional to and 90 degrees ahead of OR, being the charging current of the section.

Thus at the sending end of this section, the voltage and current are OE_1' and OI_1' respectively.

Passing now to the next section, voltages $E_1'E_2$ and E_2E_2' are consumed in phase and 90 degrees ahead of OI_1' respectively, and both proportional thereto, and also currents $I_1'I_2$ and I_2I_2' in phase and 90 degrees ahead of OE_1' respectively, and both proportional thereto.

In a similar manner, passing along from section to section, curves $RE_1'E_2'$ . . . and $DI_1'I_2'$ . . . are obtained which represent

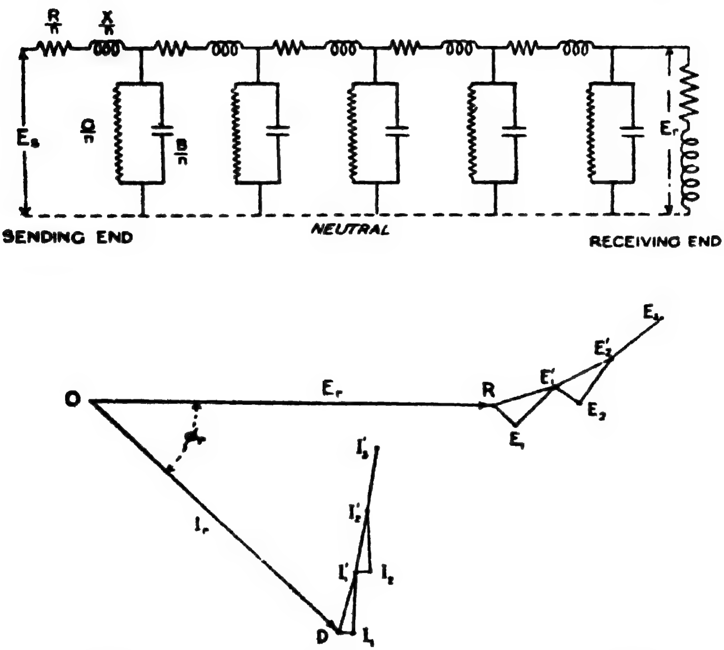

FIG. 26.—Transmission line with distributed electrical properties.

the loci of the voltage and current vectors of the line, and these can be traced out until the sending end of the line is reached.

When the number of line sections is infinite, the voltage and current loci can be readily calculated by means of hyperbolic functions, and then appear as smooth curves which have been called by Steinmetz 'Topographical circuit characteristics.' [1]

Fig. 27 shows the topographical circuit characteristics of a transmission line having the following constants per mile of conductor :—

Resistance	= 0·275 ohms.
Reactance	= 0·769 ohms.
Conductance	= $0{\cdot}15 \times 10^{-6}$ mhos.
Susceptance	= $5{\cdot}52 \times 10^{-6}$ mhos.
Frequency	= 60 cycles per second.

The receiving-end voltage to neutral is 50 000 volts, and the load current is 25 amperes, lagging 25 degrees, behind the voltage.

In order to show clearly the nature of the phenomena which occurs, a line length of 2 000 miles is taken, the vectors for the end of each 400-mile section being drawn.

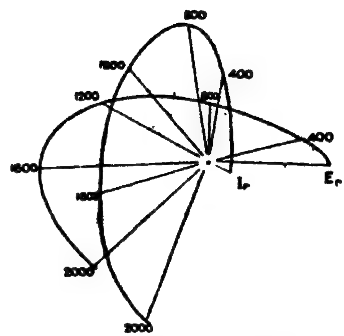

FIG. 27.—Topographical circuit characteristics for a long line.

It will be observed that passing from the receiving end towards the sending end of the line, the voltage and current alternately rise and fall, while the angle of phase difference changes periodically between lag and lead. This is shown more clearly in Fig. 28 where the

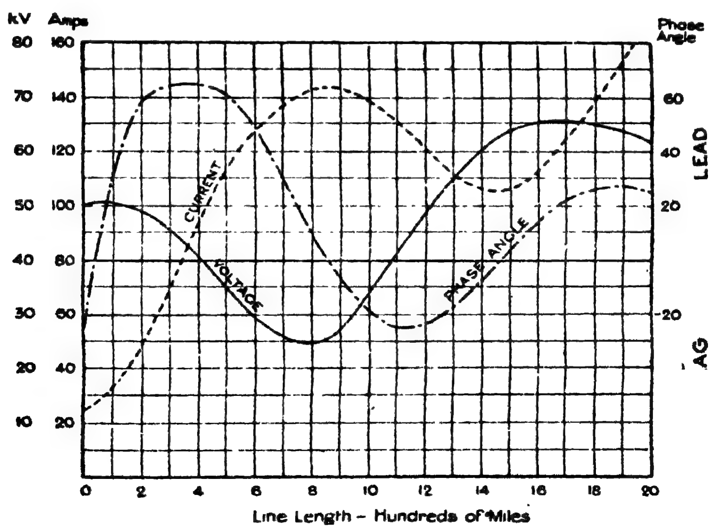

FIG. 28.—Voltage, current, and phase-angle in a long line.

values of voltage and current are plotted in rectangular co-ordinates with the distances from the receiving circuit as abscissæ.

40. Line Constants.—When the number of sections into which the line is supposed to be divided becomes infinite, the four electrical properties, *viz.* resistance, inductance, capacitance, and conductance, are evenly distributed throughout the length of the line, and thus conform to the conditions actually obtaining in practice. For this case rigorous mathematical formulæ can be applied to calculate the performance of the line.

Let

r = resistance per unit lengtn of line,

x = reactance per unit length of line,

b = susceptance per unit length of line,

and g = conductance per unit length of line.

These constants must be regarded as effective values, since the apparent properties of a transmission system carrying alternating currents are dependent, to some extent, on the frequency of the supply.

Then

$z = r + jx$ = impedance per unit length,

and $y = g + jb$ = admittance per unit length,

or in absolute values

$$z = \sqrt{r^2 + x^2}.$$

$$y = \sqrt{g^2 + b^2}.$$

41. Fundamental Differential Equations and Solutions.—The impedance of an element dl of the line is

$$zdl,$$

and the voltage $d\underset{\cdot}{E}$ consumed by the current $\underset{\cdot}{I}$ in this element

$$d\underset{\cdot}{E} = z\underset{\cdot}{I}dl.$$

The admittance of the line element dl is

$$ydl,$$

and the current $d\underset{\cdot}{I}$ consumed by the voltage $d\underset{\cdot}{E}$ of this element

$$d\underset{\cdot}{I} = y\underset{\cdot}{E}dl.$$

This gives the two equations

$$\frac{d\underset{\cdot}{E}}{dl} = z\underset{\cdot}{I}; \qquad (59)$$

$$\frac{d\underset{\cdot}{I}}{dl} = y\underset{\cdot}{E}. \qquad (60)$$

Differentiating the first equation and substituting therein the second gives

$$\frac{d^2\dot{E}}{dl^2} = zy\dot{E}, \quad . \quad . \quad . \quad . \quad . \tag{61}$$

and from the first equation follows

$$\dot{I} = \frac{1}{z}\frac{d\dot{E}}{dl}. \tag{62}$$

Equation (61) is integrated by

$$\dot{E} = A\epsilon^{Vl}, \quad . \quad . \quad . \quad . \tag{63}$$

and substituting (63) in (61) gives

$$V^2 = zy,$$

hence

$$V = \pm\sqrt{zy}.$$

Thus there exist two values of V which make (63) a solution of (61), and the most general solution therefore is

$$\dot{E} = A'\epsilon^{+\sqrt{zy}\,l} + A''\epsilon^{-\sqrt{zy}\,l}. \quad . \quad . \quad . \quad . \tag{64}$$

Substituting (64) in (62) gives

$$\dot{I} = \sqrt{\frac{y}{z}}\{A'\epsilon^{+\sqrt{zy}\,l} - A''\epsilon^{-\sqrt{zy}\,l}\}, \tag{65}$$

where l is counted from some point of the line as starting-point, as for instance from the receiving end.

If then, $\dot{E}_r$ and $\dot{I}_r$ are respectively the voltage and current at the receiving end of the line where $l = 0$ we have

$$\dot{E}_r = A' + A'',$$

$$\dot{I}_r = \sqrt{\frac{y}{z}}\{A' - A''\},$$

hence

$$A' = \tfrac{1}{2}\left\{\dot{E}_r + \sqrt{\frac{z}{y}}\dot{I}_r\right\}, \quad . \quad . \quad . \tag{66}$$

$$A'' = \tfrac{1}{2}\left\{\dot{E}_r - \sqrt{\frac{z}{y}}\dot{I}_r\right\}, \quad . \quad . \quad . \tag{67}$$

and substituting (66) and (67) into (64) and (65) respectively

$$\dot{E} = \dot{E}_r\frac{\epsilon^{+\sqrt{zy}\,l} + \epsilon^{-\sqrt{zy}\,l}}{2} + \sqrt{\frac{z}{y}}\,\dot{I}_r\,\frac{\epsilon^{+\sqrt{zy}\,l} - \epsilon^{-\sqrt{zy}\,l}}{2}. \tag{68}$$

$$\dot{I} = \dot{I}_r\frac{\epsilon^{+\sqrt{zy}\,l} + \epsilon^{-\sqrt{zy}\,l}}{2} + \sqrt{\frac{y}{z}}\,\dot{E}_r\,\frac{\epsilon^{+\sqrt{zy}\,l} - \epsilon^{-\sqrt{zy}\,l}}{2}. \tag{69}$$

Replacing the quantities in brackets by their equivalent hyperbolic functions and noting that

$$\pm\sqrt{zy}\,l = \pm\sqrt{ZY},$$

where

$$Z = zl = \text{total line impedance},$$

and

$$Y = yl = \text{total line admittance},$$

gives the sending-end voltage and current as

$$E_s = E_r \cosh\sqrt{ZY} + I_r\sqrt{\frac{Z}{Y}}\sinh\sqrt{ZY}, \quad . \quad (70)$$

$$I_s = I_r \cosh\sqrt{ZY} + E_r\sqrt{\frac{Y}{Z}}\sinh\sqrt{ZY}, \quad . \quad (71)$$

which are the fundamental equations of voltage and current for a transmission line.

42. Introduction of Convergent Series.—Provided that tables or charts of complex hyperbolic functions are available, the above equations may be applied directly for the calculation of the line performance. If, however, this material is not available, the equations can be written in a form suitable for computation by using an infinite series the successive terms in which decrease rapidly.

The series for the hyperbolic cosine and sine are

$$\cosh\theta = 1 + \frac{\theta^2}{\underline{|2}} + \frac{\theta^4}{\underline{|4}} + \frac{\theta^6}{\underline{|6}} + \cdots,$$

$$\sinh\theta = \theta + \frac{\theta^3}{\underline{|3}} + \frac{\theta^5}{\underline{|5}} + \frac{\theta^7}{\underline{|7}} + \cdots.$$

Introducing these series in (70) and (71) with

$$\theta = \sqrt{ZY},$$

gives

$$E_s = E_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \ldots\right) + I_r\sqrt{\frac{Z}{Y}}\left(\sqrt{ZY} + \frac{(ZY)^{\frac{3}{2}}}{6} + \ldots\right), \quad (72)$$

$$I_s = I_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \ldots\right) + E_r\sqrt{\frac{Y}{Z}}\left(\sqrt{ZY} + \frac{(ZY)^{\frac{3}{2}}}{6} + \ldots\right), \quad (73)$$

and by combining factors in the second portion of each

$$\dot{E}_s = \dot{E}_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \cdots\right) + \dot{I}_rZ\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \cdots\right), \quad (74)$$

$$= \dot{I}_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \cdots\right) + \dot{E}_rY\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \cdots\right). \quad (75)$$

43. Auxiliary Constants of Circuit.—In the above equations, the three auxiliary constants, which take into account the distributed nature of the line properties, are represented by the quantities

$$A = \cosh\sqrt{ZY} = \left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \cdots\right);$$

$$B = \sqrt{\frac{Z}{Y}}\sinh\sqrt{ZY} = Z\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \cdots\right);$$

$$C = \sqrt{\frac{Y}{Z}}\sinh\sqrt{ZY} = Y\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \cdots\right).$$

Equations (74) and (75) may therefore be expressed in terms of A, B, and C as follows :—

$$\dot{E}_s = \dot{E}_rA + \dot{I}_rB, \quad (76)$$
$$\dot{I}_s = \dot{I}_rA + \dot{E}_rC. \quad (77)$$

If the voltage and current values at the sending end are given, the corresponding values at the receiving end may be found by substituting $-l$ for l in equations (68) and (69), which gives

$$\dot{E}_r = \dot{E}_sA - \dot{I}_sB, \quad (78)$$
$$\dot{I}_r = \dot{I}_sA - \dot{E}_sC. \quad (79)$$

It should be noted that these auxiliary constants A, B, and C are functions of the physical properties of the line and of the frequency only, and not of the voltages and currents. In effect, all the various methods for transmission line solution are only schemes for determining the values of these constants. Having

these numerical values, they may then be applied directly to any numerical values of E and I for which a solution is desired. From this point on, the performance of the line may be determined either by mathematical or graphical methods.

44. Illustration of Convergence of Series.—When using the convergent series expressions for determining the auxiliary constants of the line, any desired degree of accuracy may be obtained by simply using a sufficient number of terms in the series.

To illustrate the manner of convergence of the series, Table 3 has been prepared, and shows the polar values of the auxiliary constants for a 300-mile, 50-cycle transmission having the line constants

$$R = 52{\cdot}5 \quad \text{ohms.}$$
$$X = 188{\cdot}4 \quad \text{ohms.}$$
$$G = 0$$
$$B = 0{\cdot}\,001411 \quad \text{mhos.}$$

TABLE 3.—*Convergence of Series Expressions for the Auxiliary Constants of 300-Mile Transmission Line.*

$R = 52{\cdot}5$ ohms. $X = 188{\cdot}4$ ohms. $G = 0.$ $B = 0{\cdot}\,001411$ mhos.

No. of Terms.	A.	B.	C.
1	$1{\cdot}00000 \angle 0{\cdot}000^\circ$	$195{\cdot}58 \angle 74{\cdot}429^\circ$	$0{\cdot}001411 \mid 90{\cdot}000^\circ$
2	$0{\cdot}86787 \angle 2{\cdot}446^\circ$	$186{\cdot}93 \angle 75{\cdot}169^\circ$	$0{\cdot}0013486 \diagdown 90{\cdot}740^\circ$
3	$0{\cdot}87052 \angle 2{\cdot}330^\circ$	$187{\cdot}03 \angle 75{\cdot}149^\circ$	$0{\cdot}0013494 \diagdown 90{\cdot}720^\circ$
4	$0{\cdot}87050 \angle 2{\cdot}332^\circ$	$187{\cdot}03 \angle 75{\cdot}150^\circ$	$0{\cdot}0013494 \diagdown 90{\cdot}721^\circ$
Inf.	$0{\cdot}87050 \angle 2{\cdot}332^\circ$	$187{\cdot}03 \angle 75{\cdot}150^\circ$	$0{\cdot}0013494 \diagdown 90{\cdot}721^\circ$

An inspection of the table shows that the use of three terms in the series expressions yields results which are sufficiently close to the exact values as obtained by the use of hyperbolic functions (infinite number of terms). In most cases occurring in practice, two terms will give a high degree of accuracy.

45. Solution of Problems by Convergent Series Method.—When the physical characteristics of the line and the frequency are stated, the form of solution and procedure will be on the following lines:—

The total resistance, reactance, and susceptance for one of the

line conductors is first calculated. The conductance, which represents the energy loss in the form of leakage over insulators and of corona loss between conductors, is usually negligible under normal operating conditions, and it is common practice to ignore its effect in calculations.

The values of Z and Y can now be set down in the form of complex quantities, and by successive multiplication the second, third and fourth power of ZY can be found, according to the degree of accuracy required.

After the values of ZY, Z^2Y^2, Z^3Y^3, etc., have been calculated they are divided by 2, 24, 720, etc., respectively, set down and added to unity. This gives the value of the auxiliary constant A, which is a complex quantity and may be referred to as $A_1 + jA_2$. The absolute value of constant A is of course $\sqrt{A_1^2 + A_2^2}$, and its polar angle is $\tan^{-1} \frac{A_2}{A_1}$.

The solution for B is of the same form as that for A except that the values of ZY, Z^2Y^2, Z^3Y^3, etc., are divided by 6, 120, and 5 040, etc., respectively. After these results are added to unity they are multiplied by Z, the product being the value of the auxiliary constant B, or $B_1 + jB_2$. The absolute value of B is obtained in the same manner as that of A.

The solution for C is the same as that for B except that in place of the series terms being multiplied by Z, they are multiplied by Y, and the value of C, or $C_1 + jC_2$ obtained.

Supposing now that the voltage $\dot{E}_r$ and current $\dot{I}_r$ at the receiving end of the line are specified, the conditions at the sending end can be found directly by applying the above constants. It should be noted that, in general, both $\dot{E}_r$ and $\dot{I}_r$ are complex quantities, but since the receiving-end voltage is kept constant $\dot{E}_r$ is taken as a vector of reference, and $\dot{I}_r$ must be specified in relation thereto. Hence the factor $(\cos\phi_r \pm j\sin\phi_r)$ is introduced, the minus sign being used when the current is lagging and the plus sign when the current is leading.

The equations for the voltage and current at the sending end of the line may thus be expressed as

$$\dot{E}_s = E_r(A_1 + jA_2) + I_r(\cos\phi_r \pm j\sin\phi_r)(B_1 + jB_2), \quad (80)$$

$$\dot{I}_s = I_r(\cos\phi_r \pm j\sin\phi_r)(A_1 + jA_2) + E_r(C_1 + jC_2). \quad (81)$$

Similarly, if the voltage and current at the sending end are given, the corresponding quantities at the receiving end are

$$\dot{E}_r = E_s(A_1 + jA_2) - I_s(\cos\phi_s \pm j\sin\phi_s)(B_1 + jB_2), \quad (82)$$

$$\dot{I}_r = I_s(\cos\phi_s \pm j\sin\phi_s)(A_1 + jA_2) - E_s(C_1 + jC_2). \quad (83)$$

With zero load, the line being open-circuited at the receiving end, the term containing $\dot{I}_r$ disappears, and the conditions at the sending end are

$$\dot{E}_{so} = \dot{E}_{ro}(A_1 + jA_2), \quad . \quad . \quad . \quad (84)$$

$$\dot{I}_{so} = \dot{E}_{ro}(C_1 + jC_2). \quad . \quad . \quad . \quad (85)$$

At zero load sending end, the conditions are really not specified but what is usually meant is that the line is open-circuited at the receiving end. Of course the capacitance current will still have to be supplied to the line. If E_{so}, then, is the sending-end voltage when all the switches are open at the other end of the line, the other quantities can be obtained as follows:—

From equation (84) we have

$$\dot{E}_{ro} = \frac{\dot{E}_{so}}{A_1 + jA_2}, \quad . \quad . \quad . \quad . \quad (86)$$

and substituting for $\dot{E}_{ro}$ in (85) gives

$$\dot{I}_{so} = \dot{E}_{so}\frac{C_1 + jC_2}{A_1 + jA_2}. \quad . \quad . \quad . \quad (87)$$

46. Numerical Example.—In order to illustrate the convergent series method of solution the following problem will be taken:—

Problem 4.—Find the characteristics of the load at the sending end of a three-phase line 300 miles long, consisting of three 0·418 sq. in. steel-cored aluminium conductors, spaced in a 12-foot flat arrangement. Temperature taken as 20° C. Load conditions at receiving end assumed as 40 000 kVA. (36 000 kW. at 90 % power factor lagging), 154 000 volts, 50 cycles.

From wire tables the resistance per mile of conductor at 20° C. is found to be 0·175 ohms, and the overall diameter of the conductor is 0·931 inch.

The inductance and capacitance as calculated by the usual formulæ are respectively

$$L_0 = 0\cdot 00200 \quad \text{henries per mile;}$$

$$C_0 = 0\cdot 01498 \times 10^{-6} \quad \text{farads per mile;}$$

and the leakage losses are taken as negligible.

Hence the line constants (total value per conductor) are

$$R = 52{\cdot}5 \quad \text{ohms.}$$
$$X = 188{\cdot}4 \quad \text{ohms.}$$
$$G = 0,$$
$$B = 0{\cdot}\,001411 \quad \text{mhos.}$$

The method of procedure is then as indicated in the previous article :—

Multiplication of ZY.

$$\begin{array}{rrr}
Y = & 0 & +\,0{\cdot}\,001411j \\
Z = & 52{\cdot}5 & +\,188{\cdot}4j \\
\hline
 & 0 & +\,0{\cdot}\,074077j \\
 & -\,0{\cdot}\,265832 & +\,0 \\
\hline
ZY = & -\,0{\cdot}\,265832 & +\,0{\cdot}\,074077j \\
 & -\,0{\cdot}\,265832 & +\,0{\cdot}\,074077j \\
\hline
 & +\,0{\cdot}\,070667 & -\,0{\cdot}\,019692j \\
 & -\,0{\cdot}\,005487 & -\,0{\cdot}\,019692j \\
\hline
Z^2Y^2 = & +\,0{\cdot}\,065180 & -\,0{\cdot}\,039384j \\
 & -\,0{\cdot}\,265832 & +\,0{\cdot}\,074077j \\
\hline
 & -\,0{\cdot}\,017327 & +\,0{\cdot}\,010470j \\
 & +\,0{\cdot}\,002917 & +\,0{\cdot}\,004828j \\
\hline
Z^3Y^3 = & -\,0{\cdot}\,014410 & +\,0{\cdot}\,015298j
\end{array}$$

Solution for A.

$$A = \left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \quad .\right).$$

$$\begin{array}{rrr}
 & 1{\cdot}\,000000 & \\
\dfrac{ZY}{2} = & -\,0{\cdot}\,132916 & +\,0{\cdot}\,037038j \\
\dfrac{Z^2Y^2}{24} = & +\,0{\cdot}\,002716 & -\,0{\cdot}\,001641j \\
\dfrac{Z^3Y^3}{720} = & -\,0{\cdot}\,000020 & +\,0{\cdot}\,000021j \\
\hline
A = & +\,0{\cdot}\,869780 & +\,0{\cdot}\,035418j \\
= & 0{\cdot}\,87050 & \angle 2{\cdot}\,332^\circ.
\end{array}$$

Solution for B.

$$B = Z\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \quad .\quad\right).$$

$$
\begin{aligned}
& \; 1\cdot 000000 \\
\frac{ZY}{6} &= -\,0\cdot 044305 + 0\cdot 012349j \\
\frac{Z^2Y^2}{120} &\doteq +\,0\cdot 000543 - 0\cdot 000328j \\
\frac{Z^3Y_r^3}{5\,040} &= -\,0\cdot 000003 + 0\cdot 000003j \\
& \; +\,0\cdot 956235 + 0\cdot 012024j \\
Z &= \,52\cdot 5 \qquad + 188\cdot 4j \\
& \; +\,50\cdot 2023 + 0\cdot 631j \\
& \; -\,2\cdot 2653 + 180\cdot 155j \\
B &= +\,47\cdot 9370 + 180\cdot 786j \\
&= 187\cdot 03 \,\angle 75\cdot 150^\circ.
\end{aligned}
$$

Solution for C.

$$C = Y\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5\,040} + \,.\,.\right)$$

$$
\begin{aligned}
+ & \quad 0\cdot 956235 + 0\cdot 012024j \\
Y = & \quad 0 \qquad\qquad + 0\cdot 001411j \\
C = & - 0\cdot 000017 + 0\cdot 0013490j \\
= & \quad 0\cdot 0013494 \,\diagdown 90\cdot 721^\circ.
\end{aligned}
$$

Solution for E_s.

$$E_s = E_r(A_1 + jA_2) + I_r(\cos\phi_r - j\sin\phi_r)(B_1 + jB_2). \quad (80)$$

$$
\begin{aligned}
A_1 + jA_2 &= 0\cdot 869780 + 0\cdot 035418j \\
E_r &= 88\,912 \\
E_r(A_1 + jA_2) &= 77\,333\cdot 8 \;+\; 3\,149\cdot 08j \\
(\cos\phi_r - j\sin\phi_r) &= 0\cdot 9 \;-\; 0\cdot 4359j \\
I_r &= 149\cdot 96 \\
I_r(\cos\phi_r - j\sin\phi_r) &= 134\cdot 965 \;-\; 65\cdot 3680j \\
(B_1 + jB_2) &= 47\cdot 9370 \;+\; 180\cdot 786j \\
& \; 6\,469\cdot 8 \;-\; 3\,133\cdot 5j \\
& \; 11\,817\cdot 6 \;+\; 24\,399\cdot 8j \\
I_r(\cos\phi_r - j\sin\phi_r)(B_1 + jB_2) &= 18\,287\cdot 4 \;+\; 21\,266\cdot 3j \\
E_r(A_1 + jA_2) &= 77\,333\cdot 8 \;+\; 3\,149\cdot 08j \\
E_s = (E_s' + jE_s'') &= 95\,621\cdot 2 \;+\; 24\,415\cdot 4j
\end{aligned}
$$

$E_s = 98\,689 \,\angle 14\cdot 324^\circ$ volts to neutral.

Solution for I_s.

$$I_s = I_r(\cos\phi_r - j\sin\phi_r)(A_1 + jA_2) + E_r(C_1 + jC_2). \quad . \quad (81)$$

$$\begin{array}{rrcr}
I_r(\cos\phi_r - j\sin\phi_r) & 134\cdot 965 & - & 65\cdot 3680j \\
(A_1 + jA_2) & 0\cdot 869780 & + & 0\cdot 035418j \\
\hline
 & 117\cdot 390 & - & 56\cdot 8558j \\
 & 2\cdot 315 & + & 4\cdot 7802j \\
\hline
I_r(\cos\phi_r - j\sin\phi_r)(A_1 + jA_2) = & 119\cdot 705 & - & 52\cdot 0756j \\
(C_1 + jC_2) = & -\ 0\cdot 000017 & + & 0\cdot 001349j \\
E_r = & 88\ 912 & & \\
\hline
 & -\ 1\cdot 512 & + & 119\cdot 942j \\
I_r(\cos\phi_r - j\sin\phi_r)(A_1 + jA_2) & 119\cdot 705 & - & 52\cdot 076j \\
\hline
I_s = (I_s' + jI_s'') & 118\cdot 193 & + & 67\cdot 866j \\
I_s & 136\cdot 29\ \underline{/29\cdot 864^\circ} & & \text{amperes.}
\end{array}$$

Having found the values of sending-end voltage and current, all the other electrical conditions at this end are readily determined as follows:—

$$\text{kVA}_s = \frac{98\ 689 \times 136\cdot 29}{1\ 000}$$

$$= 13\ 450 \quad \text{kVA. per phase.}$$

$$\text{kW}_s \quad \frac{E_s'I_s' + E_s''I_s''}{1\ 000}$$

$$= \frac{(95\ 621\cdot 2 \times 118\cdot 193) + (24\ 415\cdot 4 \times 67\cdot 866)}{1\ 000}$$

$$= 12\ 959 \quad \text{kW. per phase.}$$

$$\text{Loss} = 12\ 959 - 12\ 000$$

$$= 959 \quad \text{kW. per phase.}$$

$$\text{Efficiency} = \frac{12\ 000 \times 100}{12\ 959}$$

$$= 92\cdot 60\ \%$$

$$\text{Cos}\ \phi_s = \frac{12\ 959 \times 100}{13\ 450}$$

$$= 96\cdot 35\ \% \quad \text{leading.}$$

$$\phi_s = 29\cdot 864^\circ - 14\ 324^\circ.$$

$$= 15\cdot 54^\circ \quad \text{leading.}$$

Similarly, if the line is open-circuited at the receiving end the electrical conditions at the sending end can be found as follows:—

$$E_{so} = E_{ro}(A_1 + jA_2) \quad . \quad . \quad . \quad . \quad . \quad (84)$$
$$= 77\,333{\cdot}8 + 3\,149{\cdot}08j$$
$$= 77\,398\ \underline{/2{\cdot}\,332^\circ} \quad \text{volts to neutral.}$$

$$I_{so} = E_{ro}(C_1 + jC_2) \quad . \quad . \quad . \quad . \quad . \quad (85)$$
$$= -1{\cdot}\,512 + 119{\cdot}\,942j$$
$$= 119{\cdot}95\ \underline{\backslash 90{\cdot}\,722^\circ} \quad \text{amperes.}$$

$$\text{kVA}_{so} = \frac{77\,398 \times 119{\cdot}95}{1\,000}$$
$$= 9\,284{\cdot}0 \quad \text{kVA. per phase.}$$

$$\text{kW}_{so} = \frac{(77\,333{\cdot}8 \times -1{\cdot}\,512) + (3\,149{\cdot}08 \times 119{\cdot}\,942)}{1\,000}$$
$$= 260{\cdot}8 \quad \text{kW. per phase.}$$

$$\cos\phi_{so} = \frac{260{\cdot}8 \times 100}{9\,284{\cdot}0}$$
$$= 2{\cdot}81\ \% \quad \text{leading.}$$
$$\phi_{so} = 90{\cdot}\,722^\circ - 2{\cdot}\,332^\circ$$
$$= 88{\cdot}\,390^\circ \quad \text{leading.}$$

All the above results are collected together in Table 4, and for purposes of comparison the numerical results obtained by using the two localised-capacitance methods of solution are also given.

47. Semi-graphical Solution for Various Receiving-end Conditions.—It is often preferable to adopt a semi-graphical method of solution, and this method is particularly advantageous in cases where the sending-end conditions are required for many different receiving-end currents and power factors. In order to show clearly the graphical treatment for various load conditions the numerical example already solved mathematically will be taken.

The first step in the work is to obtain the numerical values of the three auxiliary constants by calculation, and it will be found more convenient to have these expressed in polar rather than rectangular co-ordinates.

Referring to Table 3, these values are

$$A = 0{\cdot}\,87050\ \underline{/2{\cdot}\,332^\circ}.$$
$$B = 187{\cdot}03\ \underline{/75{\cdot}\,150^\circ}.$$
$$C = 0{\cdot}\,0013494\ \underline{\backslash 90{\cdot}\,721^\circ}.$$

TABLE 4.—*Performance of 300-Mile Transmission Line Calculated by Various Methods.*

$R = 52{\cdot}5$ ohms. $X = 188{\cdot}4$ ohms. $G = 0$. $B = 0{\cdot}001411$ mhos.

Quantity.	Calculated Values Determined by		
	Rigorous Solution.	Nominal-T Method.	Nominal-π Method.
Load Conditions.			
E_s	98 689 volts	97 836	99 754
I_s	136·29 amps.	140·39	134·16
kVA_s	13 450 kVA.	13 736	13 383
kW_s	12 959 kW.	13 108	12 956
Loss	959 kW.	1 108	956
Efficiency	92·60 %	91·55 %	92·62 %
Cos ϕ_s	96·35 %	95·43 %	96·81 %
ϕ_s	15·54°	17·39°	14·51°
Zero Load Conditions.			
E_{so}	77 398 volts	77 164	77 164
I_{so}	119·95 amps.	125·45	117·14
kVA_{so}	9 284 kVA.	9 681	9 039
kW_{so} = Loss on open-circuit	260·8 kW.	413	207
Cos ϕ_{so} = ...	2·81 %	4·27 %	2·28 %
ϕ_{so}	88·39°	87·55°	88·69°

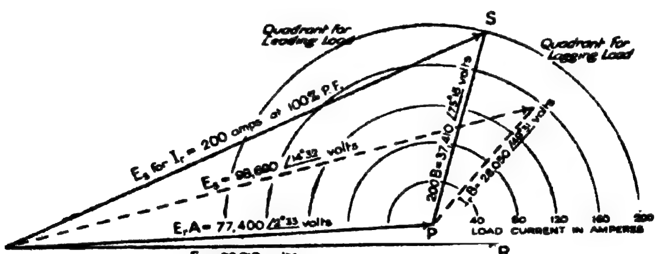

FIG. 29.—Voltage solution for various loads.

Hence

$$E_s = E_r A + I_r B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$
$$= E_r(0{\cdot}87050 \ \angle 2{\cdot}332^\circ) + I_r(187{\cdot}03 \ \angle 75{\cdot}150^\circ),$$

and

$$I_s = I_r A + E_r C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$
$$= I_r(0{\cdot}87050 \ \angle 2{\cdot}332^\circ) + E_r(0{\cdot}0013494 \ \diagdown 90{\cdot}721^\circ).$$

Fig. 29 is the solution for the sending-end voltage. Since the receiving-end voltage is supposed to be kept constant at 88 910

volts, the diagram is constructed on OR, which is drawn horizontally to represent $E_r = 88\ 910$ volts. The resultant at the sending end is made up of two parts, one of which is proportional to the receiving-end voltage, and the other to the receiving-end current. The vector OP representing $E_rA = 77\ 400\ \underline{/2{\cdot}33^\circ}$ is drawn at an angle of 2·33° from OR, and with a length to the same scale as OR, representing 77 400 volts. This vector, which is one of the two which go to make up the resultant vector for E_s, remains fixed regardless of the current in the line.

To complete the diagram, and make it applicable to any and all values of receiving-end current at any power factor, select a base value of current of, say, 200 amperes at unity power factor.

Then

$$\dot{I}_r' = 200\ \underline{/0^\circ},$$

and

$$\dot{I}_r'B = (200\ \underline{/0^\circ})(187{\cdot}03\ \underline{/75{\cdot}15^\circ})$$
$$= 37\ 410\ \underline{/75{\cdot}15^\circ} \quad \text{volts.}$$

The resultant sending-end voltage is then obtained by drawing PS at an angle of 75·15° with the vector of reference OR, and to a scale length of 37 410 volts.

This gives the solution for only one value of the receiving-end current, but it is readily seen that for any other value of current delivered at unity power factor the solution is obtained by taking the proportional part of the vector PS. For example, the line PS may be divided into five equal parts, giving thereby the values of sending-end voltage for $I_r = 0$, 40, 80, 120, 160, and 200 amperes. By continuing this subdivision on PS produced, solutions are found for receiving-end currents greater than 200 amperes.

The length of the vector PS depends only on the absolute value of I_r, while the angle it makes with the reference vector OR depends only upon the power factor of the load, being in fact equal to the algebraic sum of the receiving-end phase-angle and the angle of the auxiliary constant B. Thus for any given receiving-end power factor it is only necessary to turn the vector PS as drawn in the diagram for unity power factor through an angle equal to the phase-angle at the receiving end. By drawing arcs of circles through the points subdividing the line PS, solutions are provided for the intermediate values of current at any power factor.

The dotted lines in the figure indicate the voltage solution for

the load conditions of Problem 4, *i.e.* a total load of 40 000 kVA. delivered at 90 % power factor lagging, the current in this case being 150·0 amperes.

Fig. 30 is the corresponding solution for the sending-end current with $OR(E_r)$ again used as vector of reference. The fixed component of the sending-end current is

$$\dot{E}_r C = 88\,910(0\cdot 0013494 \diagdown \underline{90\cdot 721^\circ})$$
$$= 120\cdot 0 \quad \text{amperes,}$$

and is represented in length and direction by the vector OM.

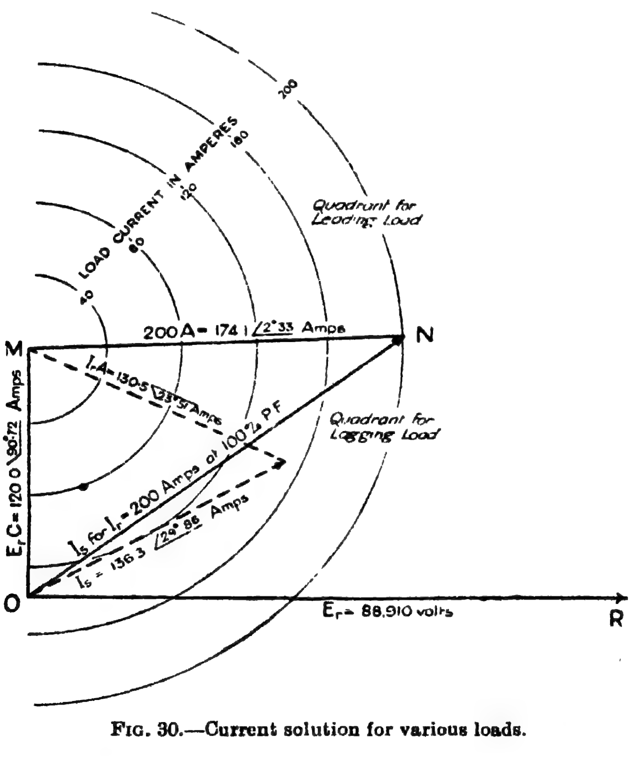

FIG. 30.—Current solution for various loads.

The other component I_rA has a variable value depending on the receiving-end current and power factor. Selecting then a base value of current of 200 amperes at unity power factor gives

$$\dot{I}_r A = (200 \angle 0^\circ)(0\cdot 87050 \angle 2\cdot 332^\circ)$$
$$= 174\cdot 1 \quad \text{amperes.}$$

The vector MN representing this quantity is drawn to scale in the diagram making an angle of 2·33° with OR, and the sending-end current is found as ON.

In an analogous method to that used for the voltage diagram the sending-end current for any value of receiving-end current at unity power factor is obtained by taking the proportionate part of the vector MN. Circular arcs are drawn through the points of subdivision of MN and for other power factors than unity the vector MN is rotated through an angle equal to the receiving-end phase-angle.

The dotted lines in the figure indicate the current solution for Problem 4.

By drawing both the voltage and current diagrams on the same sheet of paper the sending-end phase-angle may be found by direct measurement, and thus the power input to the line, and efficiency of transmission may be readily determined.

48. Hyperbolic Method of Solution.—The fundamental equations of voltage and current for a long transmission line are generally expressed by means of hyperbolic functions, and providing that suitable tables of these functions are available their use leads to the most direct method of solution.

Such tables were, until recently, incomplete, but numerous tables and charts of complex hyperbolic functions have now been calculated and published by Kennelly.[2] Even now, with these latest tables, a considerable amount of interpolation is required, and thus the same refinement of calculation cannot be obtained as with the convergent series method using three or four terms. Nevertheless the hyperbolic method yields results with an accuracy which is amply sufficient for all calculations on ordinary power lines. The charts are particularly useful for rapid work as they enable the values of the functions to be read off directly and with an accuracy of at least three significant figures.

A discussion of the theory of real and complex hyperbolic functions is outside the scope of the present work, and for a detailed exposition of their derivation and employment in transmission line calculations reference should be made to the works of Dr. Kennelly. Fortunately, although a knowledge of hyperbolic trigonometry is desirable, it is by no means essential, and it is hoped that the information given in the following pages will enable the reader to apply the hyperbolic method successfully to any particular problem.

49. Real and Complex Hyperbolic Angles.—Real hyperbolic angles are derived from an equilateral hyperbola in a similar manner to the way circular angles are derived from a circle. Circular angles are generally expressed in radians, and hyperbolic angles in hyperbolic angles or hyps.

The hyperbolic functions sinh x, cosh x, tanh x, etc., also are analogous to the circular functions sin x, cos x, tan x, etc., and many of the trigonometrical formulæ used in connection with circular angles can be applied with very slight modification to the hyperbolic case. One important difference, however, is that circular functions have a period of 2π radians so that

$$\cos x = \cos (x + 2\pi n),$$

where n is an integer, while hyperbolic functions, having no true period, continually increase.

Mathematically the following relations exist and may be employed for purposes of calculation:—

$$\sinh x \quad \frac{\epsilon^{x} - \epsilon^{-x}}{2}$$

$$x + \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} + \frac{x^7}{\underline{|7}} +$$

$$\cosh x \quad \frac{\epsilon^{x} + \epsilon^{-x}}{2}$$

$$1 + \frac{x^2}{\underline{|2}} + \frac{x^4}{\underline{|4}} + \frac{x^6}{\underline{|6}} +$$

$$\tanh x - \frac{\sinh x}{\cosh x}.$$

In alternating-current work, however, the quantities used for purposes of calculation are complex, and hence hyperbolic functions of *complex* variables are required.

A complex hyperbolic function is of the three-dimensional order and has two components, a real component relating to a hyperbolic angle, and an imaginary component relating to a circular angle. The angle may be expressed either in polar co-ordinates as $v\underline{/\theta_v}$, or in rectangular co-ordinates as $a + j\beta$, where a is the real part of the angle expressed in hyps, and β is the imaginary part expressed in circular radians.

The published tables of complex hyperbolic functions, previously referred to, give functions of complex angles for polar values up to 3·0 by steps of 0·1 and for angles from 45° to 90° by steps of one

degree; also functions in terms of rectangular co-ordinates $\alpha + j\beta$ to $\alpha = 10$ by steps of 0·05 and of β virtually to infinity by steps of 0·05. Functions of angles lying between the values for angles given in the tables are obtained by simple proportion, the values thus obtained being sufficiently accurate for all practical calculations.

The charts cover exactly the same ground as the tables but are quicker in use as they reduce the work of interpolation to a minimum.

The following formulæ giving the relationship between the functions of the complex angle and those of its component parts may be employed if exact values are required in any particular case:—

$$\sinh(\alpha + j\beta) = (\sinh\alpha\cos\beta + j\cosh\alpha\sin\beta).$$
$$\cosh(\alpha + j\beta) = (\cosh\alpha\cos\beta + j\sinh\alpha\sin\beta).$$

50. Line Angle.—In the hyperbolic theory the part of the complete circuit formed by the line is said to subtend a certain complex angle $\sqrt{ZY} = \theta$. This quantity represents in a sense the electrical length of the line, and is an indication of how much distortion in the distribution of voltage and current is to be expected owing to the line impedance and admittance being distributed, instead of being localised at one point.

In the case of long-distance overhead transmission systems the conductors usually have a fairly high ratio of reactance to resistance, and the conductance representing the power loss by leakage is negligible at normal operation, hence the line angle is approximately proportional to $\sqrt{XB}$. In other words, the line angle is proportional to the physical length of the line and approximately proportional to the frequency of the supply.

As an indication of the values of line angle to be expected in practice, it may be stated that for 50-cycle overhead systems the modulus, or size of the angle, will vary from about 0·17 for a 100-mile line to about 0·50 for a 300-mile line. The argument, or slope of the angle, will have a value of about 80 degrees.

51. Determination of Auxiliary Constants.—As shown in Art. 43 the three auxiliary constants of the line may be expressed in terms of the complex angle θ as follows:—

$$A = \cosh\theta.$$
$$B = \sqrt{\frac{Z}{Y}}\sinh\theta.$$
$$\sqrt{\frac{Y}{Z}}\sinh\theta.$$

To determine their value by the hyperbolic method, the line angle θ is first calculated from the known values for the impedance and admittance of the line. The complex hyperbolic functions $\cosh\theta$ and $\sinh\theta$ are then obtained from the tables or charts, and substituted in the above formulæ, together with the calculated value of $\sqrt{\frac{Z}{Y}}$ or its reciprocal.

The operations of multiplication, division, and evolution of complex quantities are most readily carried out when the terms are expressed in polar co-ordinates, and hence in the hyperbolic method it is preferable to work throughout with polar values.

Thus to obtain the auxiliary constants for Problem 4:—

$$
\begin{aligned}
Z = R + jX &= 52{\cdot}5 + 188{\cdot}4j \\
&= 195{\cdot}58\ \angle 74{\cdot}429^\circ. \\
Y = G + jB &= 0 + 0{\cdot}001411j \\
&= 0{\cdot}001411\ |\underline{90^\circ}. \\
\theta = \sqrt{ZY} &= \sqrt{195{\cdot}58\ \angle 74{\cdot}429^\circ \times 0{\cdot}001411\ |\underline{90^\circ}} \\
&= \sqrt{0{\cdot}27596\ \diagdown 164{\cdot}429^\circ} \\
&= 0{\cdot}52532\ \angle 82{\cdot}214^\circ \\
\sqrt{\frac{Z}{Y}} &= \sqrt{\frac{195{\cdot}58\ \angle 74{\cdot}429^\circ}{0{\cdot}001411\ |\underline{90^\circ}}} \\
&= 372{\cdot}30\ \diagdown 7{\cdot}786^\circ \\
\sqrt{\frac{Y}{Z}} &= \frac{1}{372{\cdot}30\ \diagdown 7{\cdot}786^\circ} \\
&= 0{\cdot}0026860\ \angle 7{\cdot}786^\circ.
\end{aligned}
$$

By interpolation from the tables is found

$$
\begin{aligned}
\cosh\theta &= \cosh 0{\cdot}52532\ \angle 82{\cdot}214^\circ \\
&= 0{\cdot}8698\ \angle 2{\cdot}36^\circ, \\
\sinh\theta &= \sinh 0{\cdot}52532\ \angle 82{\cdot}214^\circ \\
&= 0{\cdot}5019\ \angle 82{\cdot}94^\circ
\end{aligned}
$$

Hence

$$
\begin{aligned}
A &= \cosh\theta \\
&= 0{\cdot}8698\ \angle 2{\cdot}36^\circ, \\
B &= \sqrt{\frac{Z}{Y}}\sinh\theta \\
&= 372{\cdot}30\ \diagdown 7{\cdot}786^\circ \times 0{\cdot}5019\ \angle 82{\cdot}94^\circ \\
&= 186{\cdot}9\ \angle 75{\cdot}15^\circ,
\end{aligned}
$$

$$C = \sqrt{\frac{Y}{Z}} \sinh \theta,$$
$$= 0{\cdot}0026860 \,\angle 7{\cdot}786^\circ \times 0{\cdot}5019 \,\angle 82{\cdot}94^\circ$$
$$= 0{\cdot}001348 \,\diagdown 90{\cdot}73^\circ.$$

Having found the auxiliary constants the remainder of the solution proceeds along the lines of the calculation given in Art. 46, or the graphical method of treatment may be employed.

52. Comparison of Methods.—Now that the chief methods of transmission line solution have been described it is interesting to evaluate the three auxiliary constants corresponding to the approximate localised-capacitance or localised-admittance methods, and see how they compare with their true value. Loss of power by leakage was not mentioned in connection with the approximate methods, but it can be allowed for if necessary by supposing a high resistance shunting each condenser.

In the nominal-T method

$$E_s = E_r\left\{1 + (R+jX)\left(G_1 - jB_1 + j\frac{B}{2}\right) + j\frac{B}{4}(R+jX)^2(G_1 - jB_1)\right\}, \quad (47)$$

$$I_s = E_r\left[G_1 - jB_1 + jB\left\{1 + \frac{(R+jX)(G_1 - jB_1)}{2}\right\}\right]. \quad (46)$$

Substituting I_r for $E_r(G_1 - jB_1)$, Z for $R + jX$ and Y for jB gives

$$E_s = E_r + Z\left(I_r + \frac{E_rY}{2}\right) + \frac{Y}{4}Z^2I_r$$
$$= E_r\left(1 + \frac{ZY}{2}\right) + I_rZ\left(1 + \frac{ZY}{4}\right). \quad (88)$$

Also

$$I_s = I_r + E_rY + \frac{ZYI_r}{2}$$
$$= I_r\left(1 + \frac{ZY}{2}\right) + E_rY. \quad (89)$$

Hence the auxiliary constants corresponding to the nominal-T method are

$$A = 1 + \frac{ZY}{2}.$$
$$B = Z\left(1 + \frac{ZY}{4}\right).$$
$$C = Y.$$

In the nominal-π method

$$E_s = E_r\left\{1 + \left(G_1 - jB_1 + j\frac{B}{2}\right)(R + jX)\right\}, \quad . \quad (48)$$

$$I_s = E_r\left\{G_1 - jB_1 + jB + \frac{jB}{2}\left(G_1 - jB_1 + \frac{jB}{2}\right)(R + jX)\right\}. \quad (49)$$

Substituting I for $E_r(G_1 - jB_1)$ and making the other substitutions as before gives

$$\begin{aligned} E_s &= E_r + \left(I + \frac{E_rY}{2}\right)Z \\ &= E_r\left(1 + \frac{ZY}{2}\right) + IZ, \quad . \quad . \end{aligned} \quad (90)$$

and

$$\begin{aligned} I_s &= I + E_rY + \frac{Y}{2}\left(I + \frac{E_rY}{2}\right)Z \\ &= I\left(1 + \frac{ZY}{2}\right) + E_rY\left(1 + \frac{ZY}{4}\right). \end{aligned} \quad (91)$$

Hence the auxiliary constants corresponding to the nominal-π method are

$$A = 1 + \frac{ZY}{2}$$

$$B = Z$$

$$C = Y\left(1 + \frac{ZY}{4}\right).$$

The numerical values of these constants for the 300-mile line of Problem 4 have been calculated, and are given in Table 5 for purposes of comparison with the values obtained by the more rigorous methods of solution.

TABLE 5.—*Comparison of Auxiliary Constants for 300-Mile Transmission Line, as Determined by Various Methods.*

R = 52·5 ohms. X = 188·4 ohms. G = 0. B = 0·001411 mhos.

Method of Calculation.	A.		C.
Convergent series method (Exact Values) . .	$0·87050 \angle 2·332°$	$187·03 \angle 75·150°$	$0·0013494 \angle 90·721°$
Hyperbolic method (values obtained from Tables for cosh θ and sinh θ)		$186·9 \angle 75·15°$	$0·001348 \angle 90·73°$
Nominal-T method . .	$0·8679 \angle 2·45°$	$182·6 \angle 75·57°$	$0·001411 \angle 90·00°$
Nominal-π method . .	$0·8679 \angle 2·45°$	$195·6 \angle 74·43°$	$0·001317 \angle 91·14°$

53. Electrical Conditions at Intermediate Points.—So far, only the electrical conditions at the termini of the line have been considered. In many cases, however, and particularly with long lines, it is necessary to determine the voltage and current at points along the line between the termini. One way of doing this would be by the step-by-step method.

In this method (supposing the receiving-end conditions to be specified) the values of voltage and current are calculated as though the sending end were successively at, say, 30, 60, 90, . . . 300 miles from the receiving end. The results obtained are then plotted against distance from one of the line termini and a smooth curve drawn through the various points. It is readily seen that this method involves a large number of tedious calculations, and a much shorter solution is provided by the employment of hyperbolic functions.

In using hyperbolic functions for this purpose advantage is taken of the fact that the potentials of any and all points on a smooth transmission line vary as the sines, and the currents as the cosines, of the corresponding position angles. This relationship is expressed in equation form as

$$\frac{E_\rho}{E_c} = \frac{\sinh \delta_\rho}{\sinh \delta_c}, \qquad . \quad . \quad . \quad . \quad . \quad (92)$$

$$\frac{I_\rho}{I_c} = \frac{\cosh \delta_\rho}{\cosh \delta_c}, \qquad . \quad . \quad . \quad . \quad . \quad (93)$$

where the subscripts c and ρ relate to points along the line, c being some point where the electrical conditions are known, and ρ the point for which they are to be computed.[3]

54. Position Angles.—Reference has been made to the line as subtending a certain complex hyperbolic angle θ. The remaining part of the circuit formed by the impedance which represents the load may be said to subtend another complex hyperbolic angle θ_r, so that the receiving end of the line occupies an angular position $\delta_r = \theta_r$. The total angle of the complete circuit (line and load) is thus $\delta_s = \theta_r + \theta$, as shown in Fig. 31.

By similar reasoning, all points lying between the receiving and sending ends of the line will occupy, or assume, an angular position intermediate between δ_r and δ_s. Thus at the point P distant $\frac{1}{n}$th of the total line length from the receiving end, the position angle is $\delta_p = \delta_r + \frac{\theta}{n}$.

The following cases will be considered :—

1. *Line Earthed.*—If the line is short-circuited or earthed at the receiving end, the load angle vanishes, and the distribution of position angles along the line is purely a linear function of the total line angle. In such a case $\delta_s = \theta$.

2. *Line Supplying Load.*—Before the position angle of any intermediate point on the line can be found, it is first necessary to determine the angle subtended by the terminal load. This is obtained from the following relationship:—

$$\theta_r = \tanh^{-1}\frac{\sigma}{Z_0}, \qquad (94)$$

where σ is the impedance load to earth, or neutral, at the receiving

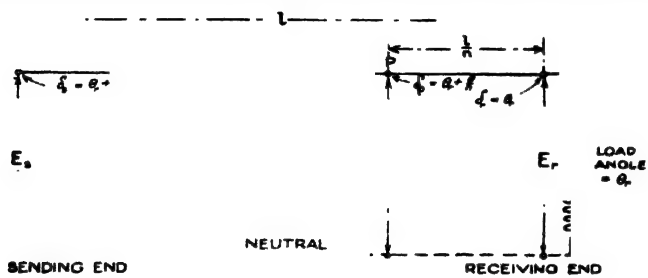

FIG. 31.—Position angles.

end of the line, and Z_o is the surge impedance of the line to neutral. The two latter quantities are both complex and are given by

$$\sigma = \frac{E_r}{I_r}, \qquad (95)$$

and

$$Z_o = \sqrt{\frac{Z}{Y}}. \qquad (96)$$

After $\frac{\sigma}{Z_o}$ has been calculated, the corresponding angle may be obtained with sufficient accuracy from the chart of complex hyperbolic tangents.

The change in the position angle from point to point along the line due to the line impedance and admittance is purely a linear function of the line angle θ. This is the case whether the receiving end is earthed, loaded, or open-circuited. Thus to obtain the

position angle of an intermediate point, it is necessary to add to the load angle a part of the line angle proportional to the distance of the point from the receiving end. The point P, distant $\frac{1}{n}$th of the line length from the receiving end, has thus a position angle of $\delta_p = \delta_r + \frac{\theta}{n}$.

3. *Line Open-circuited.*—In this case there is no load, and consequently no real part to the load angle. The impedance of the load may be regarded as infinite, that is $\sigma = \infty$, and substituting in formula (94),

$$\theta_{ro} = \tanh^{-1}\frac{\infty}{Z_o}.$$

Thus, the effect of the infinite impedance existing at the receiving end of an open-circuited line is to cause a phase rotation of 90 degrees, or $j\frac{\pi}{2} = 1\cdot 5708$ circular radians.

Consequently at zero load $\delta_{ro} = (0 + 1\cdot 5708j)$, and to find the position angle of a point P on the line δ_{ro} must be added to the part of the line angle proportional to the distance of the point from the receiving end. In equation form $\delta_{po} = \delta_{ro} + \frac{\theta}{n}$.

The method of procedure will be seen clearly from a numerical example. Referring to Problem 4,

$$\sigma = \frac{E_r}{I_r} = \frac{88\,912}{149{\cdot}96 \diagdown 25{\cdot}84^\circ}$$
$$= 592{\cdot}90 \diagup 25{\cdot}84^\circ,$$

and from Art. 51,

$$Z_o = \sqrt{\frac{Z}{Y}} = 372{\cdot}30^\circ \diagdown 7{\cdot}79^\circ.$$

Hence

$$\tan\delta_r = \frac{\sigma}{Z_o} = \frac{592{\cdot}90 \diagup 25{\cdot}84^\circ}{372{\cdot}30 \diagdown 7{\cdot}79^\circ}$$
$$= 1\cdot 5925 \diagup 33{\cdot}63^\circ,$$

and by interpolation from the table of complex hyperbolic tangents the angle subtended by the load is found as

$$\delta_r = 0\cdot 4855 + 1\cdot 144j.$$

The angle subtended by the line

$$\theta = \sqrt{ZY} = 0\cdot 0712 + 0\cdot 520j.$$

Hence the position angle of the point P, distant, say, 60 miles from the receiving end of the line is

$$\delta_p = \delta_r + \frac{\theta}{5}$$

$$= 0\cdot 4855 + 1\cdot 144j + \frac{1}{5}(0\cdot 0712 + 0\cdot 520j)$$

$$= 0\cdot 4997 + 1\cdot 248j,$$

and at zero load the position angle of the same point is

$$\delta_{po} = \delta_{ro} + \frac{\theta}{5}$$

$$= 0 + 1\cdot 571j + \frac{1}{5}(0\cdot 0712 + 0\cdot 520j)$$

$$= 0\cdot 0142 + 1\cdot 675j.$$

55. Voltage and Current Distribution along Line.—Having found the position angles and corresponding values of the sines and cosines for various points along the line, and knowing the absolute electrical conditions at one of the points, the voltage and current distribution is easily determined from the relationship given in equations (92) and (93).

For example, under the load conditions of Problem 4

$$E_r = 88\ 910 \quad \text{volts to neutral,}$$
$$I_r = 150{\cdot}0 \,\overline{\diagdown 25{\cdot}84^\circ} \quad \text{amperes,}$$

and the functions of the load angle are

$$\sinh \delta_r = \sinh (0\cdot 4855 + 1\cdot 144j)$$
$$= 1\cdot 0405 \,\underline{/78{\cdot}39^\circ},$$
$$\cosh \delta_r = \cosh (0\cdot 4855 + 1\cdot 144j)$$
$$= 0\cdot 6538 \,\underline{/44{\cdot}77^\circ}.$$

At the point P, distant 60 miles from the receiving end,

$$\sinh \delta_p = \sinh (0\cdot 4997 + 1\cdot 248j)$$
$$= 1\cdot 081 \,\underline{/81{\cdot}19^\circ},$$
$$\cosh \delta_p = \cosh (0\cdot 4997 + 1\cdot 248j)$$
$$= 0\cdot 6104 \,\underline{/54{\cdot}06^\circ}.$$

Hence the voltage and current existing at this point are respectively

$$E_s = \frac{E_r \sinh \delta_s}{\sinh \delta_r}$$

$$= \frac{88\,910(1\cdot 081 \angle 81\cdot 19^\circ)}{1\cdot 0405 \angle 78\cdot 39^\circ}$$

$$= 92\,370 \angle 2\cdot 80^\circ \quad \text{volts to neutral,}$$

and

$$I_s = \frac{I_r \cosh \delta_s}{\cosh \delta_r}$$

$$\frac{(150\cdot 0 \overline{\diagdown 25\cdot 84^\circ})(0\cdot 6104 \angle 54\cdot 06^\circ)}{0\cdot 6538 \angle 44\cdot 77^\circ}$$

$$= 140\cdot 0 \overline{\diagdown 16\cdot 55^\circ} \quad \text{amperes.}$$

The power factor is the cosine of the angular difference between these two vectors or

$$\cos \phi_s = \cos (2\cdot 80^\circ + 16\cdot 55^\circ)$$
$$= \cos 19\cdot 35^\circ$$
$$= 94\cdot 35\ \%\ \text{lagging.}$$

The intermediate electrical conditions at zero load are obtained, of course, in an exactly analogous manner.

The voltage and current distribution for Problem 4 has been determined, using functions of position angles as described above, and values of voltage, current and power factor for points 30 miles apart are given in Tables 6 and 7. The graphs shown in Fig. 32 have been plotted from these results.

It will be noted that when the specified load is being delivered the dotted graph representing the current at intermediate points along the line has a minimum value about 100 miles from the sending end. This is due to the fact of the charging current of the system being greater than the lagging quadrature component of the load current. Starting at the receiving end the power factor is 90 % lagging. As the centre of the line is approached, the increasing charging current neutralises an increasing portion of the lagging component of the load current. At a point about 100 miles from the sending end of the line this lagging component is entirely neutralised, and the actual line current has a minimum value, the power factor being unity. Passing the centre, and approaching the sending end, the increasing charging current now causes a decreasing leading power factor which, when the sending end is reached, becomes 96·35 %.

Table 6.—*Voltage and Current Distribution (Load Conditions) on 300-Mile Line.*

R = 52·5 ohms. X = 188·4 ohms. G = 0. B = 0·001411 mhos.

Distance from Sending End.	Position Angle, $\delta_{p'}$	Sinh $\delta_{p'}$	$E_{p'}$	Cosh $\delta_{p'}$	$I_{p'}$	Power Factor Cos $\phi_{p'}$
300 miles	0·4855 + 1·144j	1·0405 $\underline{/78{\cdot}39^\circ}$	88 910 $\underline{/0{\cdot}00^\circ}$	0·6538 $\underline{/44{\cdot}77^\circ}$	150·0 $\overline{\backslash 25{\cdot}84^\circ}$	90·00 % lagging
270 „	0·4926 + 1·196j	1·062 $\underline{/79{\cdot}80^\circ}$	90 750 $\underline{/1{\cdot}41^\circ}$	0·6309 $\underline{/49{\cdot}28^\circ}$	144·7 $\overline{\backslash 21{\cdot}38^\circ}$	92·19 „
240 „	0·4997 + 1·248j	1·081 $\underline{/81{\cdot}19^\circ}$	92 370 $\underline{/2{\cdot}80^\circ}$	0·6104 $\underline{/54{\cdot}06^\circ}$	140·0 $\overline{\backslash 16{\cdot}55^\circ}$	94·35 „
210 „	0·5069 + 1·300j	1·098 $\underline{/82{\cdot}57^\circ}$	93 820 $\underline{/4{\cdot}18^\circ}$	0·5941 $\underline{/59{\cdot}29^\circ}$	136·3 $\overline{\backslash 11{\cdot}32^\circ}$	96·36 „
180 „	0·5140 + 1·352j	1·114 $\underline{/83{\cdot}96^\circ}$	95 190 $\underline{/5{\cdot}57^\circ}$	0·5804 $\underline{/64{\cdot}74^\circ}$	133·1 $\overline{\backslash 5{\cdot}87^\circ}$	98·02 „
150 „	0·5211 + 1·404j	1·127 $\underline{/85{\cdot}40^\circ}$	96 300 $\underline{/7{\cdot}01^\circ}$	0·5703 $\underline{/70{\cdot}65^\circ}$	130·8 $\underline{/0{\cdot}04^\circ}$	99·26 „
120 „	0·5282 + 1·456j	1·137 $\underline{/86{\cdot}81^\circ}$	97 160 $\underline{/8{\cdot}42^\circ}$	0·5662 $\underline{/76{\cdot}65^\circ}$	129·9 $\underline{/6{\cdot}04^\circ}$	99·91 „
90 „	0·5353 + 1·508j	1·145 $\underline{/88{\cdot}24^\circ}$	97 940 $\underline{/9{\cdot}85^\circ}$	0·5657 $\underline{/82{\cdot}68^\circ}$	129·8 $\underline{/12{\cdot}07^\circ}$	99·93 leading
60 „	0·5425 + 1·560j	1·151 $\underline{/89{\cdot}69^\circ}$	98 350 $\underline{/11{\cdot}30^\circ}$	0·5703 $\underline{/88{\cdot}73^\circ}$	130·8 $\underline{/18{\cdot}12^\circ}$	99·29 „
30 „	0·5496 + 1·612j	1·153 $\underline{\backslash 91{\cdot}17^\circ}$	98 520 $\underline{/12{\cdot}78^\circ}$	0·5804 $\underline{\backslash 94{\cdot}65^\circ}$	133·1 $\underline{/24{\cdot}04^\circ}$	98·08 „
0 „	0·5567 + 1·664j	1·155 $\underline{\backslash 92{\cdot}74^\circ}$	98 690 $\underline{/14{\cdot}35^\circ}$	0·5943 $\underline{\backslash 100{\cdot}57^\circ}$	136·3 $\underline{/29{\cdot}96^\circ}$	96·31 „

TABLE 7.—*Voltage and Current Distribution (Zero-load Conditions) on 300-Mile Line.*

R = 52·5 ohms. X = 188·4 ohms. G = 0. B = 0·001411 mhos.

Distance from Sending End.	Position Angle $\delta_{po'}$	Sinh $\delta_{po'}$	$E_{po'}$	Cosh $\delta_{po'}$	$I_{po'}$	Power Factor Cos $\phi_{po'}$
300 miles	0·0000 + 1·571j	1·000 | 90·00°	88 910 / 0·00°	0·0000 | 90·00°	0·0 \ 7·79°	—
270 ,,	0·0071 + 1·623j	0·998 \ 90·02°	88 730 / 0·02°	0·0525 \ 172·21°	12·6 | 90·00°	0·03 °/₀ leading
240 ,,	0·0142 + 1·675j	0·996 \ 90·08°	88 550 / 0·08°	0·1048 \ 172·23°	25·1 \ 90·02°	0·10 ,,
210 ,,	0·0214 + 1·727j	0·988 \ 90·19°	87 840 / 0·19°	0·1568 \ 172·27°	37·5 \ 90·06°	0·23 ,,
180 ,,	0·0285 + 1·779j	0·979 \ 90·34°	87 040 / 0·34°	0·2084 \ 172·32°	49·8 \ 90·11°	0·40 ,,
150 ,,	0·0356 + 1·831j	0·967 \ 90·54°	85 980 / 0·54°	0·2595 \ 172·38°	62·0 \ 90·17°	0·65 ,,
120 ,,	0·0427 + 1·883j	0·953 \ 90·79°	84 730 / 0·79°	0·3099 \ 172·46°	74·1 \ 90·25°	0·94 ,,
90 ,,	0·0498 + 1·935j	0·935 \ 91·09°	83 130 / 1·09°	0·3595 \ 172·55°	86·0 \ 90·34°	1·31 ,,
60 ,,	0·0570 + 1·987j	0·916 \ 91·44°	81 440 / 1·44°	0·4081 \ 172·66°	97·6 \ 90·45°	1·73 ,,
30 ,,	0·0641 + 2·039j	0·895 \ 91·85°	79 570 / 1·85°	0·4556 \ 172·78°	108·9 \ 90·57°	2·23 ,,
0 ,,	0·0712 + 2·091j	0·8705 \ 92·33°	77 400 / 2·33°	0·5019 \ 172·92°	120·0 \ 90·71°	2·83 ,,

Taking now the zero-load graphs, and assuming that the normal voltage of 88 910 is maintained constant at the receiving end of the line, the full curve showing the voltage at intermediate points gradually falls due to the Ferranti effect to a sending-end value of 77 400 volts. The rate of change of voltage is greater as the sending end is approached, owing to the larger charging current at this end.

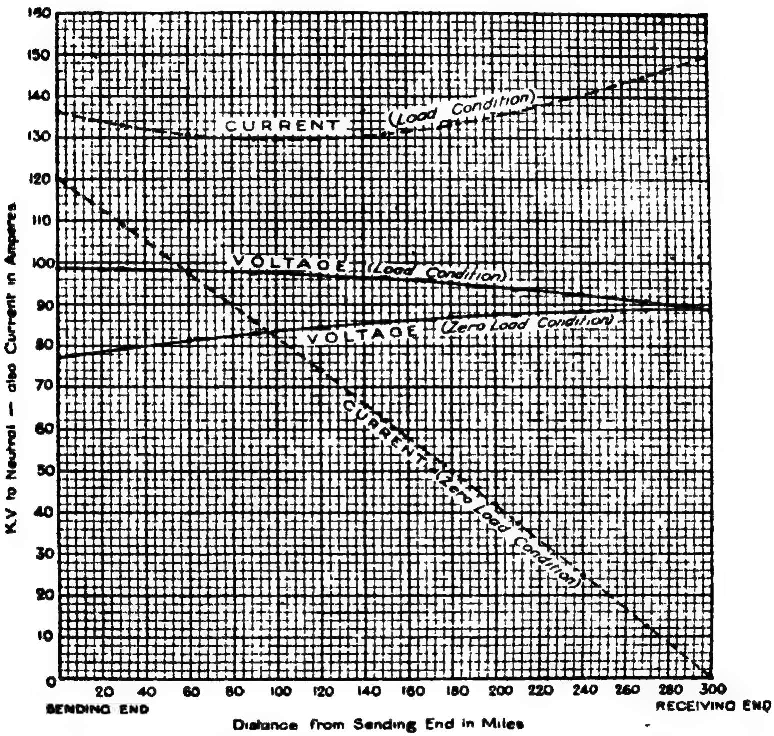

FIG. 32.—Voltage and current distribution on a 300-mile line.

From the dotted current graph under zero-load conditions, it will be seen that the charging current at the sending end has a value not much below that established when the line is supplying full load of 12 000 kW. per phase. As the receiving end of the line is approached, the current falls lower and lower until at the terminus of the line its value is, of course, zero.

56. Effect of Non-sinusoidal Sending-end Voltage.—So far in the treatment of the subject, all equations and data have been derived on the assumption that the voltage wave impressed on the line was sinusoidal in shape. If this is not the case, a rigid line solution involves the separation of the distorted wave into its constituent terms, consisting of a fundamental wave and its odd harmonics. Calculations of the effect due to each separate frequency must then be made as though the others did not exist, and the final result obtained by the method of superposition. Where only the R.M.S. value of the resultant wave is required this may be obtained by perpendicular summation. Thus if E_1, E_3, E_5, etc., are the R.M.S. values of the various harmonics of the receiving-end voltage as obtained by calculation, the R.M.S. value of the receiving-end voltage wave is

$$E_r = \sqrt{E_1^2 + E_3^2 + E_5^2 + \ . \ . \ .}$$

A similar method of treatment must be applied in the case of plural-frequency currents.

The effect of a distorted sending-end wave is brought most into prominence when the line is open-circuited at the far end. Both the susceptance ωC and reactance ωL of the line increase in proportion to the frequency of the various harmonics. Hence a high-voltage harmonic of quite small value may produce charging currents, of the same order of magnitude as those produced by the fundamental voltage wave. Furthermore, these high charging currents flowing through the correspondingly increased reactance of the line, magnify the distortion of the voltage wave at points along the line and at the receiving end.

For purposes of illustration, a problem worked out in detail by Pernot will be taken.[4] In this example, a voltage having a wave shape whose equation as a function of $\theta = 2\pi ft$ is

$$\begin{aligned} E_s = 100\,000 \sin \theta \\ + 8\,000 \sin (3\theta - 40°) \\ + 6\,000 \sin (5\theta - 295°) \\ + 5\,000 \sin (7\theta - 110°) \\ + 2\,000 \sin (9\theta - 50°) \\ + 500 \sin (11\theta - 170°) \end{aligned}$$

is assumed to be impressed on a 100-mile open-circuited line having the following constants:—

$R = 27{\cdot}5$ ohms.
$X = 76{\cdot}9$ ohms.
$G = 0{\cdot}15 \times 10^{-4}$ mhos.
$B = 5{\cdot}52 \times 10^{-4}$ mhos.
$f = 60$ cycles.

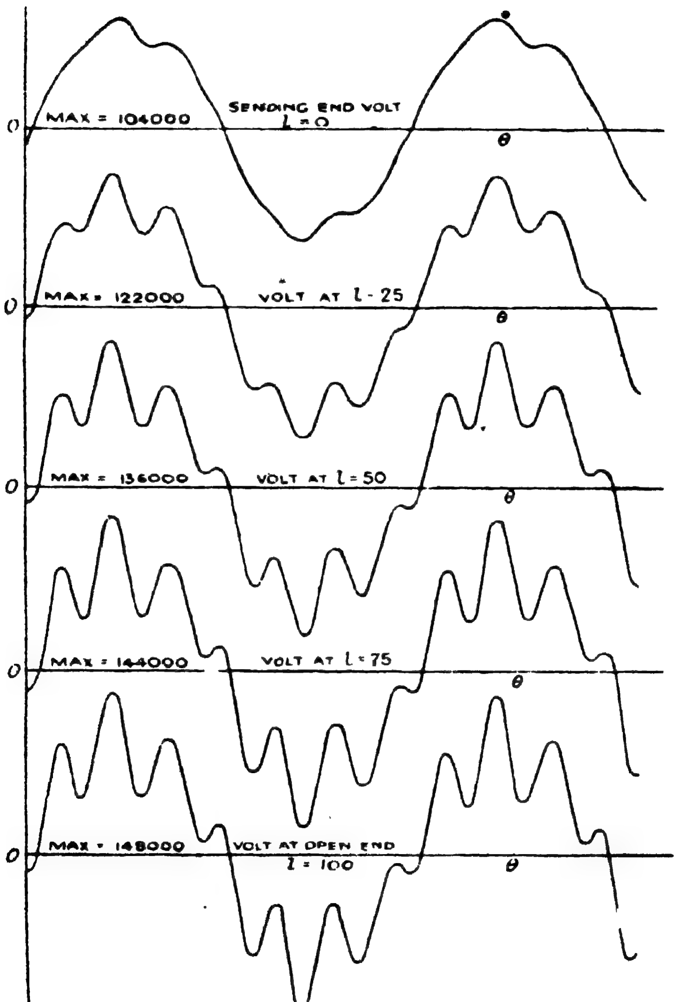

Fig. 33.—Voltage wave shapes along a 100-mile open-circuited line.

The resultant voltage at the receiving end is

$$\begin{aligned} E_r = {} & 102\,100 \sin(\theta - 0{\cdot}47^\circ) \\ & + 9\,808 \sin(3\theta - 41{\cdot}61^\circ) \\ & + 11\,604 \sin(5\theta - 298{\cdot}76^\circ) \\ & + 36\,835 \sin(7\theta - 126{\cdot}81^\circ) \\ & + 7\,134 \sin(9\theta - 222{\cdot}20^\circ) \\ & + 782 \sin(11\theta - 347{\cdot}27^\circ), \end{aligned}$$

and the intensification of the various harmonics should be noted.

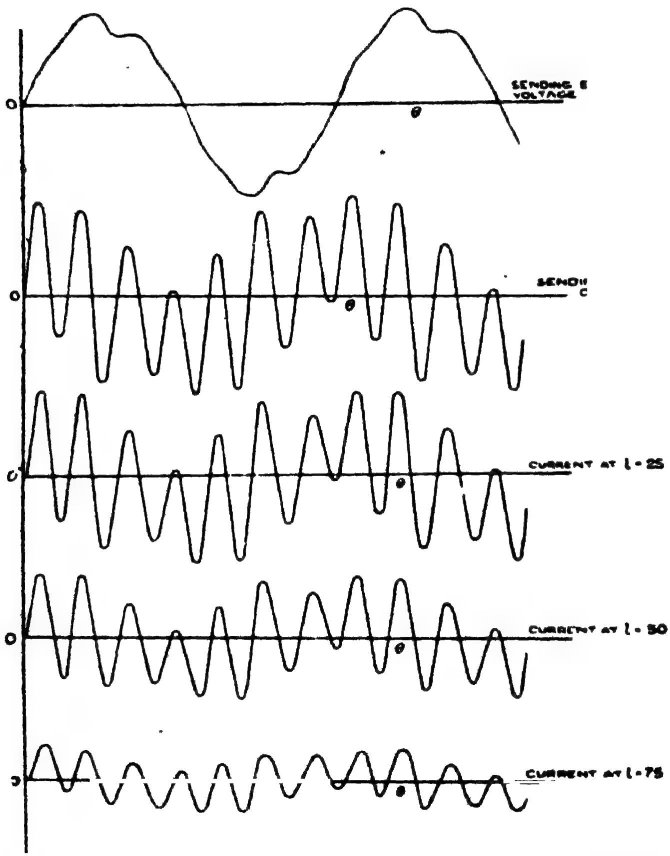

Fig. 34.—Current wave shapes along a 100-mile open-circuited line.

The line is approximately in resonance for the seventh harmonic, so that although the seventh harmonic in the impressed voltage is only 5 °/₀ of the fundamental, the seventh harmonic in the voltage at the receiving end of the line is 36 °/₀ of the fundamental.

The charging current at the sending end of the line is

$$\begin{aligned} I_c = {} & 55\,800 \sin (\theta + 88{\cdot}12^\circ) \\ & + 15\,184 \sin (3\theta + 48{\cdot}35^\circ) \\ & + 26\,604 \sin (5\theta - 208{\cdot}23^\circ) \\ & + 97\,725 \sin (7\theta - 35{\cdot}76^\circ) \\ & + 18\,346 \sin (9\theta - 130{\cdot}50^\circ) \\ & + 1\,611 \sin (11\theta - 254{\cdot}53^\circ). \end{aligned}$$

Here again, the seventh harmonic component is magnified the most, its value being approximately twice as great as the fundamental.

The wave shapes of voltage and current at various points along the line are shown in Figs. 33 and 34. Special attention should be given to the former curves as on these have been indicated the maximum instantaneous values of the voltage wave. The phase relations of the several component harmonics is such that the maximum value of the impressed voltage wave is only a little greater than the maximum value of the fundamental—104 000 maximum, with 100 000 volts as maximum value of the fundamental. At the open end of the line, the maximum value of the voltage wave is 148 000 volts, or an increase of nearly 50 °/₀ over the impressed voltage, which might be serious in its effect upon the line insulation. Furthermore, in a line designed to operate normally without corona formation at the voltage assigned to the fundamental frequency in this illustrative example, the presence of such large superimposed harmonics, which increase the maximum voltage by nearly 50 °/₀, might be sufficient to start corona formation, and thus introduce an additional power loss in the system.

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[2] A. E. Kennelly, *Tables of Complex Hyperbolic and Circular Functions* (Harvard University Press). *Chart Atlas of Complex Hyperbolic and Circular Functions* (Harvard University Press).

[3] A. E. Kennelly, *Artificial Electric Lines*, pp. 38-40 (1917).

[4] F. E. Pernot, *Electrical Phenomena in Parallel Conductors*, p. 181 (1918).

CHAPTER VI.

LINE CONDUCTORS AND SUPPORTING STRUCTURES.

57. Properties of Stranded Conductors.—All conductors employed on overhead lines are preferably stranded, on account of the increased flexibility thereby obtained. Solid wires, except in the smaller sizes, are difficult to handle, and when used for long spans tend to crystallise at the points of support due to swinging in the wind.

In stranded conductors there is generally one central wire, and round this, successive layers of wires containing 6, 12, 18, 24 . . . wires. Thus if there are n layers, the total number of individual wires employed is

$$N = 3n(n + 1) + 1. \qquad (97)$$

In the process of manufacture, the consecutive layers of wires are twisted or spiralled in opposite directions, the effect being to bind all the layers together. This method of construction is known as 'concentric lay.'

With very large sections of conductor, however, another method of stranding called 'rope lay' is sometimes used as it gives a more flexible conductor.

When a current enters a stranded conductor it divides among the wires, and each separate current, for all practical purposes, remains in its own wire throughout the length of the conductor. This is because the individual wires being circular, touch only along lines, and the surface resistance, due to dirt and the formation of oxide or sulphide, has a fairly high value. The result is that each current, in general, pursues a spiral path of greater length than the length of the conductor as a whole, and this effective increase of the path length correspondingly increases the resistance. The precise magnitude of this effect depends on the *lay* adopted for the conductor, meaning by this term the axial length of one complete turn of any wire. The lay is usually

expressed numerically in terms of the mean diameter of the layer containing the wire.

There is no fixed lay used by all manufacturers, but in wire tables the assumption is usually made that the length, and corresponding resistance, of all wires except the straight central one, is increased by 2 °/₀ above the values for the central one. This is equivalent to assuming that every twisted wire has a lay ratio of about 15·6.

Another effect of stranding is to modify slightly the fundamental formula for inductance which is based on a solid round conductor. According to Dwight,[1] the inductance per mile of concentric-lay conductor is as follows:—

$$\text{3-strand conductor, } L_0 = \left(0\cdot 125 + 0\cdot 741 \log_{10}\frac{d}{r}\right)10^{-3} \text{ henries,} \tag{98}$$

$$\text{7-strand conductor, } L_0 = \left(0\cdot 103 + 0\cdot 741 \log_{10}\frac{d}{r}\right)10^{-3} \text{ henries,} \tag{99}$$

$$\text{19-strand conductor, } L_0 = \left(0\cdot 089 + 0\cdot 741 \log_{10}\frac{d}{r}\right)10^{-3} \text{ henries,} \tag{100}$$

$$\text{37-strand conductor, } L_0 = \left(0\cdot 085 + 0\cdot 741 \log_{10}\frac{d}{r}\right)10^{-3} \text{ henries,} \tag{101}$$

$$\text{61-strand conductor, } L_0 = \left(0\cdot 083 + 0\cdot 741 \log_{10}\frac{d}{r}\right)10^{-3} \text{ henries,} \tag{102}$$

where d is the interaxial distance between conductors, and r is the overall radius of the conductor, both measured in the same units.

For conductors having more than sixty-one strands, the formula for solid conductors

$$L_0 = \left(0\cdot 080 + 0\cdot 741 \log_{10}\frac{d}{r}\right)10^{-3} \text{ henries per mile,} \tag{9}$$

is used.

The above formulæ do not take into account the effect of *spiralling* which tends to increase the inductance of stranded conductors, but the magnitude of this effect is negligible in the case of non-magnetic conductors (see Art. 59).

58. Conductivity.—The resistivity of all conductor materials is usually referred to the standard for annealed copper as adopted

by the International Electrotechnical Commission in 1913 (corresponding for the first four significant figures with the older Matthiessen Standard). The International Annealed Copper Standard is expressed in terms of mass resistivity as 0·15328 ohms, this being the resistance of a uniform round wire one metre long, weighing one gramme, at the temperature of 20° C.

Since the density of copper at this temperature is taken as 8·89 grammes per cubic centimetre, the standard may be alternatively expressed in terms of volume resistivity as $1{\cdot}7241 \times 10^{-6}$ ohms representing the resistance between opposite faces of a centimetre cube of the metal at 20° C.

The most convenient method of comparing different materials is then to express their conductivity (reciprocal of volume resistivity) at 20° C. as a percentage of the conductivity of the International Standard taken as 100 per cent.

The resistance of all pure metals increases with increase of temperature, and the variation can be expressed by a linear relation for the temperature changes occurring in transmission line operation. Hence

$$R_t = R_0(1 + at), \qquad (103)$$

where R_0 is the resistance at 0° C., R_t is the resistance at a temperature of t° C., and a is the temperature coefficient of resistance. For example, the value of a for copper of 100 °/₀ conductivity is 0·00427.

If the temperature t_1° C. is chosen as a standard of reference, the formula may be written

$$R_t = R_{t_1}\{1 + a(t - t_1)\}, \qquad (104)$$

where a is not the same as before but is smaller, because the increase of resistance is expressed as a fraction of the resistance at t_1° C. which is greater than the resistance at 0° C. Hence

$$a_{t_1} = a_0\frac{R_0}{R_{t_1}} = \frac{a_0 R_0}{R_0(1 + a_0 t_1)}$$

$$= \frac{1}{\frac{1}{a_0} + t_1}$$

$$= \frac{1}{234{\cdot}5 + t_1}. \qquad (105)$$

At a temperature of 20° C. the temperature coefficient for copper of 100 °/ₒ conductivity is therefore

$$a_{20} = \frac{\quad}{234\cdot5 + 20} \quad 0\cdot00393.$$

59. Current Distortion Effects.—In previous discussions it has been taken for granted that an alternating current distributes itself uniformly over the conductor cross-section. In actual practice, however, the distribution of current is more or less distorted due to: (1) Skin effect, (2) Proximity effect, and (3) Spirality effect. Although the sum total of these effects is quite small at normal supply frequencies, they are the chief factor in determining the conductor resistance under high-frequency conditions.

(1) *Skin Effect.*—A solid round conductor may be thought of as composed of a large number of separate filaments or strands, each carrying a small finite part of the total current. The inductance of the individual filaments will vary according to their position, due to the fact that those filaments near the centre of the conductor are surrounded by a greater magnetic flux than the filaments near the outer surface. The self-induced back e.m.f. is therefore greater in the filaments located near the centre of the conductor. The higher reactance of the inner filaments causes the current to distribute in such a manner that the current density is less in the interior of the conductor than at the surface. This crowding of the current towards the surface of the conductor is known as skin effect, and its result is to apparently increase the resistance, and to a smaller extent decrease the reactance, of the conductor. The same phenomena occurs also in the case of the spirally stranded conductors used in practice.

Maxwell[2] and Rayleigh[3] have shown that for solid cylindrical non-magnetic conductors the effective values of resistance and internal reactance are given by

$$R' = R\left\{1 + \frac{1}{12}\left(\frac{m^2}{4}\right)^2 - \frac{1}{180}\left(\frac{m^2}{4}\right)^4 + \dots\right\}, \qquad (106)$$

and $$X'_2 = X_2\left\{1 - \frac{1}{24}\left(\frac{m^2}{4}\right)^2 + \frac{13}{4320}\left(\frac{m^2}{4}\right)^4 - \quad \right\} \qquad (107)$$

where

R = resistance of conductor to a steady current,

R = effective resistance of conductor to an alternating current,

$m = \sqrt{\dfrac{8\pi fl \times 10^{-9}}{R}}$,

f = frequency of alternation in cycles per second,

$X_2 = \pi fl$ = internal reactance of conductor with a steady current (see formula 7).

X_2' = effective internal reactance of conductor with an alternating current,

and l = length of conductor in centimetres.

The magnitude of the effect depends largely on the frequency of alternation. As this is gradually raised the resistance steadily rises, and the internal reactance (instead of increasing directly with f like the external reactance), increases also but at a slower rate than f. When the frequency is very great, it is interesting to note that the effective resistance and the effective internal reactance of the conductor are equal, being given by

$$R' = X'_2 = \frac{m}{2\sqrt{2}R}, \quad . \quad . \quad . \quad (108)$$

which steadily increases without limit with f.

In the case of overhead conductors, the internal reactance is always small compared with the external reactance produced by the necessary separation of the conductors, and since skin effect can only alter the value of the former component its influence on the total reactance of the conductor is negligible.

On the other hand, the increase of resistance due to skin effect is noticeable at normal supply frequencies with large sections of conductor, and assumes overwhelming importance in the case of high-frequency currents, such as are induced in overhead lines by lightning disturbances, etc.

Unfortunately both the above formulæ have a limited range of utility. Formula (106) is too slowly convergent and should not be used for values of m above 2 (in which case the use of the first two terms is sufficient), and formula (108) should only be used for values of m above 60. In this connection it should be noted that for copper conductors under average temperature conditions the value of m is approximately as follows:—

$$m = 0{\cdot}3\sqrt{fA}, \quad . \quad . \quad . \quad (109)$$

so that under 50-cycle conditions

$$m = 2{\cdot}1\sqrt{A}, \quad . \quad . \quad . \quad . \qquad (110)$$

where A is the cross-sectional area of the conductor in square inches.

For practical calculations of skin effect, therefore, the following empirical formulæ are preferable :—

(1) *For values of* m *from* 0 *to* 3

$$R' = R\left\{\left(1 + \frac{m^4}{48}\right)^{\frac{1}{4}}\right. \qquad (111)$$

(2) *For values of* m *above* 3

$$R' \quad R\left(\frac{m}{2\sqrt{2}} + 0{\cdot}26\right), \quad . \qquad (112)$$

the maximum error of each of these formulæ over their respective ranges being less than 1 °/ₒ.

As a rough guide to the magnitude of skin effect at 50 cycles in practical sizes of conductors, the percentage increase of resistance can be taken as $9A^2$.

The sub-division of a solid conductor into straight filaments or strands does not alter the alternating-current resistance, and m has been defined in such a way that all the above formulæ for skin effect can be used for stranded, as well as solid, conductors. In order to give some idea of the magnitudes involved, values of $\frac{R'}{R}$ have been calculated for various frequencies and conductor sizes and are given in Table 8.

TABLE 8.—*Increase of Resistance of Copper Conductors Due to Skin Effect.*

[Annealed copper (100 °/ₒ conductivity). Temp. 40° C.
Hard-drawn copper (97 °/ₒ conductivity). Temp. 30° C.]

Cross-sectional Area in Sq. In.	Frequency.	m.	Method of Calculation.	$\frac{R'}{R}$.
0·25	50	1·05	Formula (111)	1·006
0·50	50	1·48	"	1·025
1·00	50	2·10	"	1·092
0·25	1 000	4·70	Formula (112)	1·92
0·25	10 000	14·8	"	5·49
0·25	100 000	47·0	"	16·9
0·25	1 000 000	148·0	"	52·6

It is interesting to note that when aluminium is used in place of copper the larger diameter necessary to give the same conductivity does not lead to a greater power loss through skin effect. In fact, equivalent sections of all non-magnetic conductors have exactly the same percentage increase of resistance; the increased cross-section being, exactly compensated for by the increased resistivity.

(2) *Proximity Effect.*—In the discussion of skin effect it is assumed that the return conductor or conductors are so remote, or arranged in such a manner, that the field due to the return current can be neglected. If this is not the case the current is crowded to one side of the conductor due to proximity effect. Thus considering two solid round conductors A and B, lying comparatively close together, the flux due to B cutting the near half of A is greater than the flux cutting the remote half of A, and consequently the current distribution over the cross-section of A is distorted. Assuming that A and B are carrying currents in opposite directions, the currents distribute themselves in such a way that their maximum density occurs at those portions of the conductors which are nearest to one another. Proximity effect, like skin effect, increases the resistance and decreases the internal inductance of a conductor. The phenomenon is more pronounced for large conductors, high frequencies and close proximity. The magnitude of the effect, at normal supply frequencies, in the case of the wide spacings of conductors required for overhead transmission lines, is so small that it can be ignored.

In the case of stranded overhead conductors, due to the spiralling of the wires, each wire traverses alternately weaker and stronger portions of the magnetic field produced by an external current-carrying conductor. Hence the average value of the field along the path of any wire will be the same, and assuming that the currents in the conductor follow to a large extent the paths of the individual wires, the proximity effect must be substantially eliminated.

(3) *Spirality Effect.*—The spirality effect, first mentioned by Stirnimann,[4] tends to increase both the resistance and internal reactance of a stranded conductor, and is a necessary corollary to the elimination of proximity effect. Due to the spiral path of the individual wires, the conductor may be regarded as built up of a series of coaxial solenoids. Each of these solenoids formed by consecutive layers produces a magnetic field whose direction

along the axis of the solenoid, *i.e.* the axis of the conductor. The outermost layer contains the greater number of wires so that its field will be stronger than that due to the other layers. The next layer is spiralled in the opposite direction and contains fewer wires than the outer layer, so that there is produced a weaker field in the opposite direction. Owing to the reversal of spirality, the layer below gives rise to a still weaker field in the original direction, and so on. The resultant effect is, that travelling in a radial direction from the surface of the conductor, the axial magnetic field alternates in magnitude although it does not actually change in direction on passing from one layer to another. As the potential difference applied to the ends of all the layers is the same, this variation in the axial magnetic flux leads to phase differences among the currents in consecutive layers. Hence the sum of the R.M.S. values of the currents in the various layers may be appreciably larger than the R.M.S. value of the total current passing along the conductor, or in other words, the resistance of the conductor has been apparently increased.

The magnitude of the effect depends on the size and method of construction of the conductor. At present there is a lack of numerical data relating to the phenomenon, but it appears to be very small at normal supply frequencies, and can quite safely be ignored in the case of non-magnetic conductors.

60. Conductor Materials.—The materials used at the present time for overhead transmission line conductors include copper, aluminium, steel-cored aluminium, copper-clad steel, and to some extent steel alone. It is very difficult to make hard-and-fast comparisons between these various metals, because not only the electrical, but also the mechanical, properties have to be taken into account, and the requirements in the latter respect depend to a large extent on local legislative restrictions affecting the design of the line. The question of first cost, also, except when based on current quotations, is of little value, and must be gone into for each particular case. In the following articles, however, leading technical particulars and data will be given regarding the materials mentioned above. Particulars of resistances, sizes and weights will not be given, as they are readily obtained from wire tables and other sources. In the case of copper, aluminium, steel-cored aluminium, and cadmium-copper conductors, standard specifications have been issued by the British Standards Institution.[5]

61. Copper.—Copper ranks first in importance among transmission line conductors, owing to its high electrical conductivity and great tensile strength. The conductivity of the metal depends to an enormous extent on its freedom from impurities. For instance, the presence of 0·02 °/₀ of phosphorus, or 0·07 °/₀ of arsenic, in the copper, lowers the conductivity by about 30 °/₀ Metallic impurities, even where these have good conductivity in themselves, also reduce the conductivity by a substantial amount, and to obtain a high-quality product most copper is electrolytically refined.

The conductivity also depends on the physical state of the metal, as the operations of cold-rolling and drawing when manufacturing the hard-drawn copper conductors used for overhead work lowers the conductivity by 2 to 3 °/₀. The tensile properties, on the other hand, are considerably improved by this cold working. Within the range of 20 to 30 tons per sq. in. there is a linear relation between tensile strength and conductivity, so that taking the conductivity of 20-ton quality copper as 98 °/₀, the corresponding value for 30-ton quality would be 97 °/₀.[6]

It is found, by experiment, that the temperature coefficient of commercial copper at a given temperature varies with the conductivity of the copper, being greater the purer the copper. Dellinger[7] has shown that if n be the fractional conductivity of the sample as compared with standard annealed copper, then its temperature coefficient is $n\alpha_t$ where α_t is the temperature coefficient for pure copper. If, for example, the percentage conductivity of a sample of copper is 96 °/₀, its temperature coefficient at 20° C. will be

$$n\alpha_t = 0{\cdot}96 \times 0{\cdot}00393 = 0{\cdot}00377.$$

As previously mentioned, line conductors should preferably be stranded, but solid copper wires may be used with care up to No. 3/0 S.W.G., which is about the practical limit.

62. Aluminium.—Aluminium possesses many advantages as a conductor material, and a considerable saving in the first cost of the line is sometimes possible by the employment of this metal in place of copper.

Cold rolling and drawing, as in the case of copper, slightly decreases the electrical conductivity of the metal and improves the tensile properties. At 20° C. the conductivity of the hard-drawn wire is about 60 °/₀, its temperature coefficient of resistance is 0.00390, and its density is 2·71 grammes per cubic centimetre.

The smaller conductivity of the metal necessitates that, for any particular efficiency of transmission, the sectional area of the conductor must be larger in aluminium than in copper. Comparing aluminium of 60 % with copper of 97 % conductivity, the diameter of the aluminium conductor would be 1·26 times the diameter of a copper conductor of equal resistance. In spite of the greater section of aluminium necessary, the density of the metal is so low, that an aluminium conductor has almost exactly one-half the weight of the equivalent copper conductor. The tensile strength is much lower than that of copper, but the larger sectional area of metal neutralises the difference to some extent, and an aluminium conductor has about 75 % of the ultimate strength of the equivalent copper conductor.

On the other hand, the increased cross-section of conductor exposes a greater surface to wind pressure, and so the supporting towers must be designed for a greater transverse strength. In many cases also, higher towers must be used with aluminium conductors than would be required with copper conductors having the same length of span. This is in consequence of the greater sag of aluminium conductors, due to the decreased working stress permissible, and also owing to the fact that the linear coefficient of expansion of aluminium is 1·4 times that of copper. When considering an aluminium installation, it is therefore necessary to set the additional cost of towers against any saving which may be effected on the conductors. Apparently the chief field for aluminium is for heavy-current transmissions where the conductor size is large and its cost forms a large proportion of the total cost of the complete installation.

63. Steel-cored Aluminium.—Steel-cored aluminium conductors have a central core of galvanised steel wires with one or more layers of aluminium wires outside. This type of conductor has been developed to permit of longer spans being employed with the ordinary height of tower. The breaking load of such a composite conductor is considerably larger than that of the equivalent copper conductor, and its weight is still about 25 % smaller. The importance of this lies in the fact that the sag in an overhead conductor is dependent upon the ratio of the loading in the span to the strength of the conductor. The loading factor is the resultant of the weight of the conductor and the wind pressure, ice deposits and other artificial loads, and though the diameter of a steel-cored

aluminium conductor is larger than that of any other conductor of equal resistance, and the artificial loads are therefore greater, the weight factor is so small that no other conductors can equal steel-cored aluminium in the smallness of the ratio of total loading to strength. The result is that the sag with steel-cored aluminium is much less than with copper, and the supporting towers may therefore be shorter, or alternatively the span length longer.

Since the composite type of conductor is more costly than the all-aluminium type, it is chiefly used in those cases where an all-aluminium conductor would necessitate a much higher and more costly support than copper. This occurs principally where spans of 300 feet and over are involved, steel structures generally being used to support the line conductors.

In many long-distance transmission systems, the use of copper conductors necessitates a lower line voltage than would be otherwise desirable, in order to avoid excessive corona losses. In these cases, the use of the equivalent steel-cored aluminium conductor with its greater diameter has the advantage of raising the critical voltage limits of the line. The magnitude of this increase may be of the order of 30 to 50 °/$_o$, and this point will be raised again in the course of the following chapter.

For the purpose of obtaining the resistance and reactance of a steel-cored conductor, Dwight[8] has shown that the resistance may be taken as equal to that of the core and the outer wires connected in parallel, and the reactance as equal to that of a non-magnetic conductor of the same outside diameter. Since the resistance of the steel core to alternating currents is very high compared to that of the aluminium wires, it may generally be neglected, and the resistance of the composite conductor taken as equal to that of the aluminium wires alone.

These remarks apply chiefly to conductors having two layers of aluminium wires. In cases where there are one or three layers of aluminium the axial magnetic field is much greater, and sets up hysteresis and eddy-current losses in the steel core so that at high-current densities the alternating-current resistance may be 20 °/$_o$ greater than at low values.[9]

64. Copper-clad Steel.—A composite wire known as copper-clad steel wire is obtained by welding a coating of copper on to a steel wire core. Line conductors made of this material are preferably stranded, and have a considerably greater tensile

strength than the equivalent all-copper conductors. The proportion of copper and steel is so chosen that the composite wire has a conductivity either 30 or 40 °/₀ of a copper conductor of equal diameter. This material appears to be very suitable for river crossings, or other places where an exceptionally long span is necessary.

65. Iron and Steel.—Galvanised iron and steel conductors have been used to advantage for extremely long spans, or for short line sections exposed to abnormally high stresses due to climatic conditions. They have also been employed for lines supplying rural areas and operating at pressures of about 11 kV. Generally speaking, the use of iron or steel is most advantageous for short-distance small-power transmissions, where the size of the copper conductor desirable from economic considerations would be smaller than No. 8 S.W.G., and would thus be unsuitable for use owing to its lack of mechanical strength.

With magnetic conductors, current distortion effects cause a material increase in resistance which is impossible to calculate, as it depends not only on the frequency, but also on the magnitude, of the currents flowing. The application of any skin effect formulæ, for instance, is impossible as the value to be assigned to μ, the permeability of the material, is unknown. The magnetic field strength varies during each cycle, and also at different points over the cross-section, so that the corresponding value of μ varies also between wide and unknown limits. The alternating-current resistance is further increased by hysteresis losses, and in the case of stranded conductors by spirality effect.

Similarly, the internal inductance (which is negligible in the case of non-magnetic conductors at normal spacings) is greatly increased, its value often exceeding that of the external inductance.

Furthermore, the various electrical and magnetic constants for any particular grade of iron or steel wire cannot be given with any degree of accuracy, because of the method of manufacture. A slight change in the composition of the wire, that does not affect the mechanical properties, often causes a material change in the electrical and magnetic characteristics.

For the above reasons, all calculations should be based on test data, obtained from sizes and grades of wire approximating to those of the conductor it is proposed to use.

Fig. 35, for example, which is due to Walton,[10] shows the

alternating-current resistance, and internal reactance for two different grades of 7/12 S.W.G. galvanised stranded steel conductors, the data being obtained from tests on experimental lines. The value of resistance extrapolated for zero current is, of course, the continuous-current resistance.

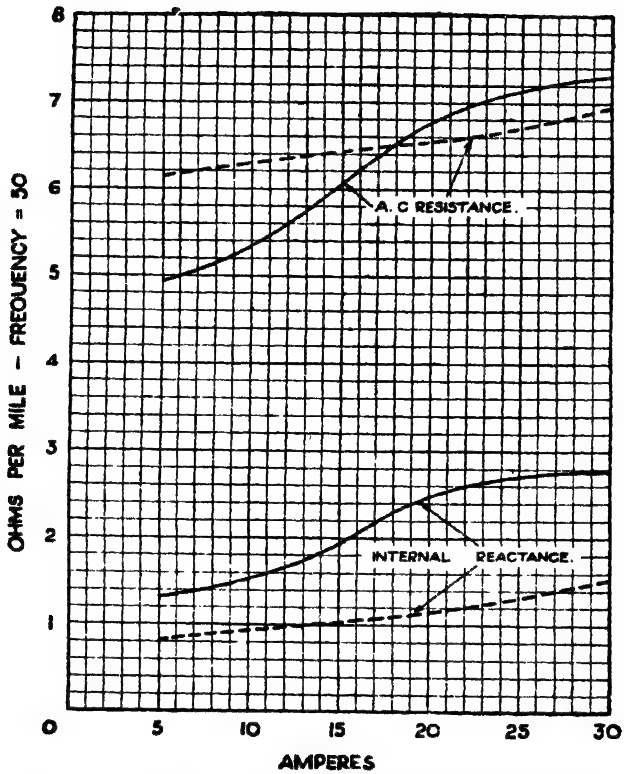

FIG. 35.—Alternating-current electrical properties of 7/12 S.W.G. galvanised steel conductors.

Owing to the high internal reactance of iron and steel conductors, the pressure drop per mile of line is far greater than would occur if the equivalent copper or aluminium conductors were used. To illustrate the method of calculation, assume a transmission line consisting of 7/12 S.W.G. conductors having the electrical characteristics shown by the dotted lines in Fig. 35, sr

30 inches apart (delta arrangement) and carrying a current of 30 amperes at a frequency of 50.

From Fig. 35 the alternating-current resistance per mile is 6·95 ohms, so the ohmic pressure drop in each wire is

$$6{\cdot}95 \times 30 = 208 \quad \text{volts per mile.}$$

Also the internal reactions per mile is 1·52 ohms, hence the total reactance of each wire is

$$\begin{aligned} 1{\cdot}52 + \left(0{\cdot}741 \log_{10} \frac{30}{0{\cdot}156}\right)10^{-3} \times 2\pi \times 50 \\ = 1{\cdot}52 + 0{\cdot}53 \\ = 2{\cdot}05 \quad \text{ohms per mile,} \end{aligned}$$

and the reactive pressure drop is

$$2{\cdot}05 \times 30 = 61 \quad \text{volts per mile.}$$

Given the receiving-end power factor as 80 °/ₒ lagging, the resultant fall of pressure in each conductor is

$$\begin{aligned} IR \cos \phi_r + IX \sin \phi_r \\ = 208 \times 0{\cdot}8 + 61 \times 0{\cdot}6 \\ = 203 \quad \text{volts per mile.} \end{aligned}$$

66. Cadmium Copper.—A material now being employed in certain cases is copper alloyed with 1 to 2 °/ₒ of cadmium. This increases the tensile strength by about 40 °/ₒ, and the conductivity is only reduced by 17 °/ₒ below that of pure copper. However, the cost is appreciably more than that of copper, so that the use of this cadmium alloy would probably only show an economy in lines of small cross-section, *i.e.* where the cost of conductor material is relatively small compared with the cost of supports.

67. Line Supporting Structures.—The supporting structures for overhead power conductors are of various kinds, including wood, steel, and reinforced-concrete poles, and steel towers either of the rigid or flexible type. A few typical designs from present-day practice are exhibited in Fig. 36, and the drawings also show the various methods of spacing the conductors.

The design of a transmission line support depends very materially upon whether the support is rigid, or has a certain amount of flexibility in the direction of the line. Wooden poles and certain special types of steel structures fall within the latter category, and in designing these it is usual to consider only the

transverse wind pressure which occurs upon the conductors and upon the support itself. The longitudinal pull of the conductors is normally balanced on either side of the support, but in the event of

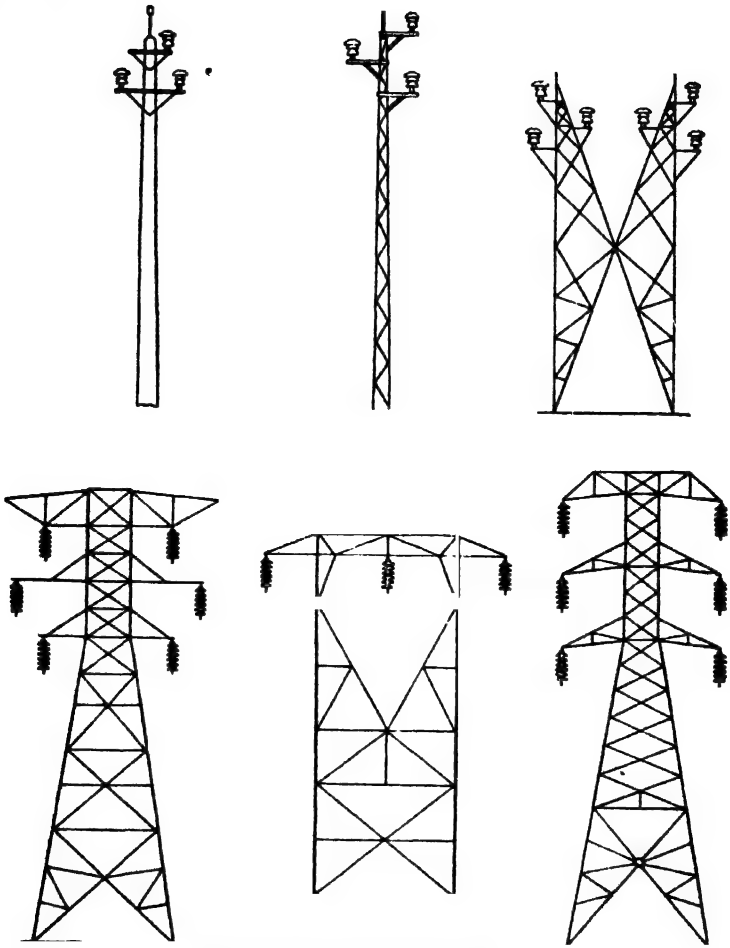

Fig. 36.—Typical line supports.

the breaking of one or more conductors on one side, there will be an unbalanced load which may be greatly in excess of the transverse wind pressure. With flexible supports this unbalanced load is quickly absorbed into the system. The supports on each side of

the wrecked span bend towards the adjoining spans, and this movement of the pole tops produces a reduction of tension in the wires of the adjoining span owing to the increased sag of these wires. There will be an appreciable deflection of the second and third poles beyond the break, but the magnitude of these successive deflections decreases rapidly, and is rarely noticeable beyond the fifth pole. The greater the flexibility of the supports in the direction of the line, the smaller will be the extra load which any one support may be called upon to withstand. In any case, a certain general rigidity in the longitudinal direction is provided for, by employing at intervals of a mile or so rigid anchoring towers, designed to withstand the failure of one conductor in three on one side, as well as the transverse load.

In the case of rigid supports, such as the lattice-steel broad-base structures which are typical of American and Canadian practice, it is usual to provide equal strength in both the longitudinal and transverse directions, and to design every tower to withstand the unbalanced load due to the breaking of one conductor in three, on the same side. In addition, anchor towers are often provided, in which the support is capable of withstanding the failure of two conductors out of three, or even of the whole of the conductors, on one side.

68. Wood Poles.—The use of wood poles is chiefly limited to comparatively low pressures, and spans having a maximum length of 250 or 300 feet. There are a few notable exceptions, however, existing in districts having a plentiful supply of suitable timber, and where the cost of transporting steel towers would be high. Under these special conditions, single and 'H' poles have been employed for lines working at voltages up to 130 000, and average span lengths of 500 feet.

Wood poles are very elastic, and lines using this type of support are often designed throughout for the transverse load; longitudinal strength at terminals and for anchor supports being provided by means of guys. Double-pole structures, of the 'A' or 'H' type, are used in many situations to obtain a higher transverse strength than could be economically provided by means of the single poles. Theoretically these structures are from three to five times as strong as one of the constituent poles, but under actual service conditions failure occurs due to yielding of the foundations long before these values are reached.

Latterly more efficient types of double-pole structures, such as the Rutter pole, have come into use, in which adequate resistance to overturning is provided, and which have a measured strength of about six times that of a single pole.[11]

The chief objection to wood supports is the tendency to rot at the ground level, but the life of a wood pole under normal conditions can be taken as from twenty-five to thirty years, provided that the wood has been well impregnated with creosote, or other preservative compound.

69. Steel Poles.—Steel poles of tubular construction, or narrow-base lattice-steel masts, are often used as a substitute for wood poles. They enable a rather longer span to be used, at the same time taking up little more base room than a wood pole, and may therefore be employed for crossing cultivated fields or for running along the roadside. In the flexible system of line construction, 'A' or 'H' supports are generally used.

All steel supports should be well galvanised, and a life of at least thirty years can be counted on. There is apparently no reason why the life should not be indefinitely prolonged, if the structure is regularly scraped and painted after the galvanising fails.

70. Reinforced-concrete Poles.—Reinforced-concrete poles have been employed as line supports in recent installations, particularly on the Continent. The best quality poles are factory made by a centrifugal or other mechanical process, but owing to their great weight the transport costs are usually high. Hence, where cheapness is essential, poles are often cast on site. The most attractive quality of reinforced concrete is its very long life, and the fact that maintenance charges are exceedingly low.[12]

71. Steel Towers.—Broad-base, lattice-steel towers are employed for the majority of long-distance transmission lines running across open country, although their first cost is from two to four times greater than that of wooden poles. Steel towers have the advantage of a long life, which may be extended almost indefinitely by a reasonable amount of attention to their maintenance. Furthermore, their reliability is much greater than that of wooden supports. The substantial construction of the towers renders them capable of withstanding the most severe climatic conditions, and immune from destruction by forest fires. The risk of interrupted service, due to broken or punctured insulators, is considerably reduced owing to the longer spans rendered possible by the stronger and taller steel

supports. Lightning troubles are also minimised as each tower is a lightning conductor, whereas on wood pole lines shattered poles and wrecked line sections are not infrequent. At a moderate additional cost double-circuit towers can be provided, thus giving a further insurance against discontinuity of supply. In case of break-down to one circuit it is then possible to carry out repairs while maintaining the supply on the other circuit.

72. Spacing and Arrangement of Conductors.—There is no general agreement among engineers as to the best arrangement of

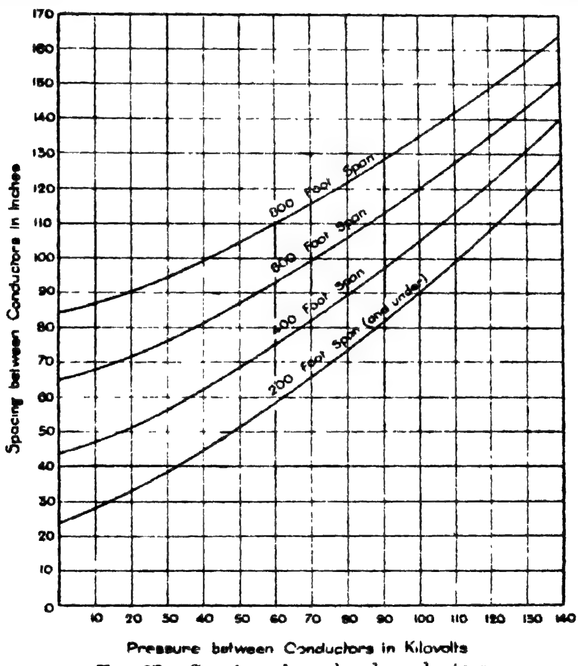

FIG. 37.—Spacing of overhead conductors.

the conductors on the support. Apparently there is no special advantage to be obtained by using the symmetrical-delta arrangement, and in most cases flat-horizontal or vertical spacing is adopted from mechanical considerations, particularly when suspension insulators are employed. In unsymmetrical arrangements, the conductors are usually transposed at regular intervals, in order to balance the electrical characteristics of the several phases, and prevent inductive interference with neighbouring telephone circuits

The spacing of the conductors is determined by considerations partly electrical and partly mechanical. Increased spacing increases the inductance of the line and hence the pressure drop, so that to keep the latter within a reasonable value the conductors should be as close together as is consistent with the prevention of corona. In many cases, however, the most powerful consideration which governs the spacing is the necessity for preventing the conductors from swinging together in the wind. Usually wires swing synchronously in the wind, but with the longer spans there is always a possibility of non-synchronous swinging taking place, and so the spacing should increase somewhat with the span length. According to Still,[13] present-day practice in this respect is given by Fig. 37.

73. Span Lengths.—Neglecting the deciding influence upon the span length of such local conditions as the necessity for following the configurity of roads, canals, or railways, it is interesting to note that there is one definite value for the span length which will give the minimum overall cost of the line. It will be obvious that this must be so, because, as the length of span increases, the total number of supports and insulators decreases, but the height of the support and the conductor spacing must be increased to allow for the greater sag. It can readily be seen that the higher the operating voltage of the system, the greater will be the economical length of span, owing to the higher relative cost of insulators to supports. Again, where line supports are placed on private property, the cost of pole or tower rights is generally reckoned as a fixed amount per support irrespective of the size of support, and this is another factor which operates in favour of longer spans and should receive consideration. It is not possible, therefore, to give any hard-and-fast rules as to the best span length to be employed, and the only way of determining this is to calculate the total line cost per mile for a number of different span lengths, and plot the results.

In this connection it should be noted that with small sizes of conductors the length of span is often unduly restricted, owing to the mechanical requirements of the line. For example, with No. 8 S.W.G. copper conductors it is not advisable to employ spans over 150 feet in length as the sag then becomes excessive. Thus, when the section of conductor as determined from electrical calculations comes out rather small, it is often possible to reduce the total line cost by using a larger and stronger conductor, and increasing the spacing of the line supports.

In the case of long-distance transmission systems employing the steel-tower form of construction, spans of 600 to 1 000 feet are commonly used, but there is a definite tendency towards longer span lengths. One reason for this is that the insulators constitute the weakest part of a transmission line, and a reduction in the number of towers per mile provides a fewer number of points of potential break-down. Modern views on this point are typified in a recently constructed line where steel-cored aluminium conductors supported on high tensile limit steel towers have been adopted, the average span length being 1 250 feet.

For river or ravine crossings exceptionally long single spans are sometimes necessary. Under these special circumstances spans of 4 000 to 6 000 feet have been satisfactorily employed, the conductor material being steel-cored aluminium, copper-clad steel, or simply galvanised steel.

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CHAPTER VII.

INSULATION OF OVERHEAD LINES AND CORONA EFFECT.

74. Insulator Materials.—The successful operation of a transmission system depends to a great extent on the proper maintenance of the line insulation, and hence the selection of suitable insulators should be made very carefully. The material most commonly used for insulators on overhead lines is porcelain, but glass, steatite, and special composition materials are also used to a limited extent.

Glass is cheaper than porcelain in the simpler shapes, and when properly annealed it has a high resistivity and dielectric strength. Owing to its transparency, flaws in the material can be readily detected by visual examination. On the other hand, moisture readily condenses on its surface and facilitates the accumulation of dirt deposits, thus giving a high surface leakage. Glass is also less reliable than porcelain in the irregular-shaped pieces forming the make-up of large high-voltage insulators, on account of internal strains resulting from irregular cooling after casting. Nevertheless, glass insulators are well adapted for use on installations employing pressures up to about 25 000 volts, and under specially favourable climatic conditions they have been used for very much higher voltages.

Steatite is a naturally occurring magnesium silicate usually found combined with oxides in varying proportions. It has a much higher tensile and bending stress than porcelain, and has been used extensively on the Continent for particular types of insulators in which the ceramic part is actually in pure tension.

Special composition insulators are available for the lower voltages, and have the advantage that they may be readily moulded to the most complicated shapes without any internal stresses being caused. Metallic fittings can also be firmly embedded in the material without any difficulty. Some of these materials, however, deteriorate fairly rapidly under bad climatic conditions, or may

yield by softening in the sun when subjected to high mechanical stresses. The insulator surface also is liable to be carbonised, and conducting paths formed, if subjected to flash-over.

Porcelain is produced by firing at a high temperature a mixture which consists essentially of kaolin, feldspar, and quartz. The resulting material is stronger mechanically than glass, gives less trouble from leakage, and is less affected by changes of temperature. On the other hand, it is not so homogeneous as glass, owing to the fact that each component shell of a porcelain insulator is glazed during the process of manufacture, and the satisfactory performance of the insulator in service depends to a considerable extent on the preservation of this glaze which is only of the order of one-thousandth of an inch in thickness. Also it is not possible to detect flaws in the material by simple inspection, and each separate insulator shell has to be subjected to pressure tests as part of the manufacturing routine.

In the manufacture of porcelain, under-firing produces good mechanical qualities but causes porosity in the material, which results in rapid deterioration and failure when put in service. Over-firing, on the other hand, minimises porosity, but makes the material brittle. The quality of electrical porcelain has been greatly improved in recent years due to new methods of manufacture, and particularly to close control of the kiln temperature.[1]

75. Factors Involved in Insulator Design.—Insulators are required to withstand both mechanical and electrical stresses, in addition to which the surface leakage path must have sufficiently high resistance to prevent any appreciable current flowing to earth. Electrical break-down may occur either by flash-over or puncture. In a flash-over, an arc occurs between the line conductor and earth (the latter being represented by the supporting pin of the insulator), and the discharge jumps across the air gaps in its path. In a puncture, the discharge occurs from conductor to pin through the body of the insulator. When a break-down of the former type is involved, the insulator will continue to act in its proper capacity after the event unless fractured by the heat of the arc, but after a puncture it is permanently ruined. It is thus of importance to provide sufficient thickness of porcelain in the insulator to resist puncture by the combined effect of the line voltage and any probable transient pressure rises on the line. The ratio of puncture strength to flash-over voltage may be termed the factor of safety of

the insulator against puncture, and this ratio should be high so that a good margin is obtained to protect the insulator from complete failure.

Fig. 38 shows the flash-over distances as usually measured on a pin-type insulator. The sum of $A + B + C$, or $D + B + C$ (depending on whether A or D is the shorter distance), gives the length of path on a dry flash-over. When the exposed upper surfaces of the insulator are wet with rain they are practically conducting, and thus the wet flash-over distance is the sum of $F + G + H$. The exact path of the arc will depend somewhat on the shape of the parts, and will tend to lie between 45 degrees and a normal from one shell to the outer surface of the next lower shell.

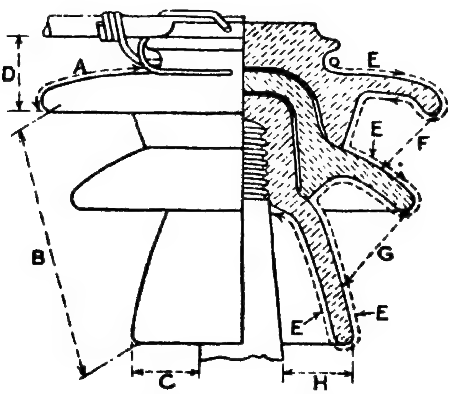

Fig. 38.—Flash-over and leakage distances of a pin-type insulator.

The leakage distance is generally measured from tie wire to pin taken radially along the surfaces as shown by the line E, but obviously this measurement only gives a rough idea of the leakage resistance as it neglects the varying width of the leakage path.

76. Pin-type Insulators.—One-piece, pin-type insulators can be employed satisfactorily for operating voltages up to about 25 000 volts. An adequate length of leakage path is obtained by providing the insulator with two or three petticoats or rain sheds. These are so designed, that even when the outer surface of the insulator is wet due to rain, sufficient leakage resistance is still given by the inner dry surfaces.

In the past, it has been found difficult to vitrify porcelain

properly if the thickness was considerable. Hence for higher voltages, where the total thickness of the material required would be greater than desirable from a manufacturing point of view, multi-part insulators are employed. These consist of shells of moderate thickness fastened firmly together by neat Portland cement, the cemented surfaces being unglazed and corrugated in order to obtain a good bond. Multi-part, pin-type insulators are

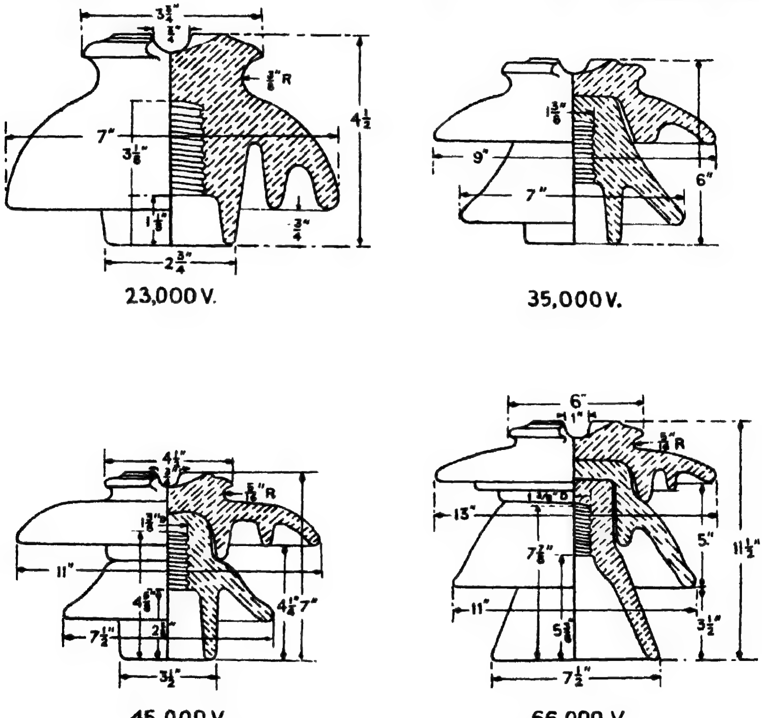

FIG. 39.—Typical designs of pin-type insulator.

supplied for all voltages up to 80 000 volts, but the suspension type of insulator is generally more satisfactory, and also more economical for pressures above 50 000 volts. Typical pin-type insulators are illustrated in Fig. 39.

For supporting the insulator, steel pins are generally used, these being screwed direct into the inner shell of the insulator, or fastened by means of a metal thimble. Wooden supporting pins

impregnated with paraffin are occasionally employed on lines working at the lower pressures. They have the advantage of cheapness, but are mechanically weak and liable to give trouble by burning due to leakage currents.

The commonest method of attaching the conductor to the insulator is to tie it down with soft copper, or soft aluminium, binding wire according to the conductor material. This holds the conductor firmly, but the restraining force is gradually applied, and the swinging of the conductor in the wind does not cause continuous bending backwards and forwards at one point. Mechanical clamps have been employed for such fastenings but suffer from lack of flexibility.

In their electrical behaviour, pin-type insulators may be compared to a complicated series of condensers with resistances in series and shunt. The petticoats, with the intervening air spaces, form the condenser system, and the leakage paths over the surface and through the body of the material are represented by the resistances. Now considering any two consecutive insulator shells, the two surfaces which are opposite to each other are at different potentials except where they are cemented together. Also the potential difference between corresponding points on opposite surfaces is greater, the further they are removed from the joint at the centre. Hence in designing the insulator the shells must be arranged so that they diverge, so as to increase the distance between the shells as the potential difference increases. It should also be noted that owing to the higher permittivity of porcelain (the value of which is 4—6) the greater part of the electrical stress occurs in the air spaces. If the air spaces are of insufficient length corona tends to form, reducing both the arcing distance and the leakage path.

As regards the leakage resistance, this quantity is directly proportional to the length and inversely proportional to the width of the leakage path. Hence after a certain point is reached, increasing the diameter of the petticoats does not appreciably increase the leakage resistance. Increasing the number of petticoats increases the length of path without necessarily increasing the width, and therefore forms the best means of securing a high leakage resistance.

During the last few years the shape of pin-type insulators has undergone a radical change, mainly on the lines suggested by

Gilchrist and Klinefelter.[2] This improved design comprises the following features:—

1. The surfaces of the petticoats or rain sheds conform to the equipotential surfaces of the electrostatic field between the pin and the tie wire or cap.

2. The body of the insulator conforms to the lines of the electrostatic field.

3. The leakage resistance for each shell is about the same value, and the capacitance per shell is also approximately constant.

These improvements have the effect of minimising the formation of corona, and increasing the flash-over voltage and also the mechanical strength of the insulator.

Fig. 40 shows the electrostatic flux distribution between pin

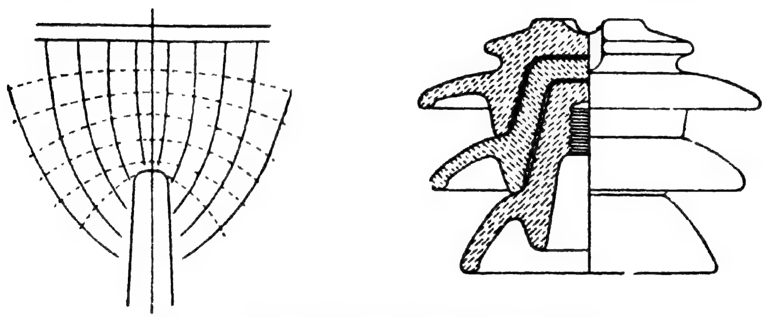

FIG. 40.—Improved design of pin-type insulator.

and cap of a pin-type insulator, and also illustrates the actual design of insulator used on 33 000-volt installations.

77. Suspension-type Insulators.—With the suspension type of insulator, the line conductor is hung below the point of support at the end of a string of insulator units, or discs, connected to each other by metal links. There are three main designs in use at the present time, the Hewlett or interlinking type, the cemented-cap type, and the core-and-tine type.

The Hewlett suspension insulator, Fig. 41 (*a*), was the first type to be successfully developed and is characterised by great simplicity of design. Each disc consists only of one piece of porcelain, the central bulbous portion of which is provided with two curved tunnels lying in planes at right angles to each other. The short steel strips forming the connection between individual discs are threaded through these tunnels and thus loop through

each other, being separated by a layer of porcelain which is wholly in compression. This method of construction gives a high mechanical strength and has the further advantage that should the porcelain between the links be accidentally broken, the links still hold the other units together and thus prevent any interruption to

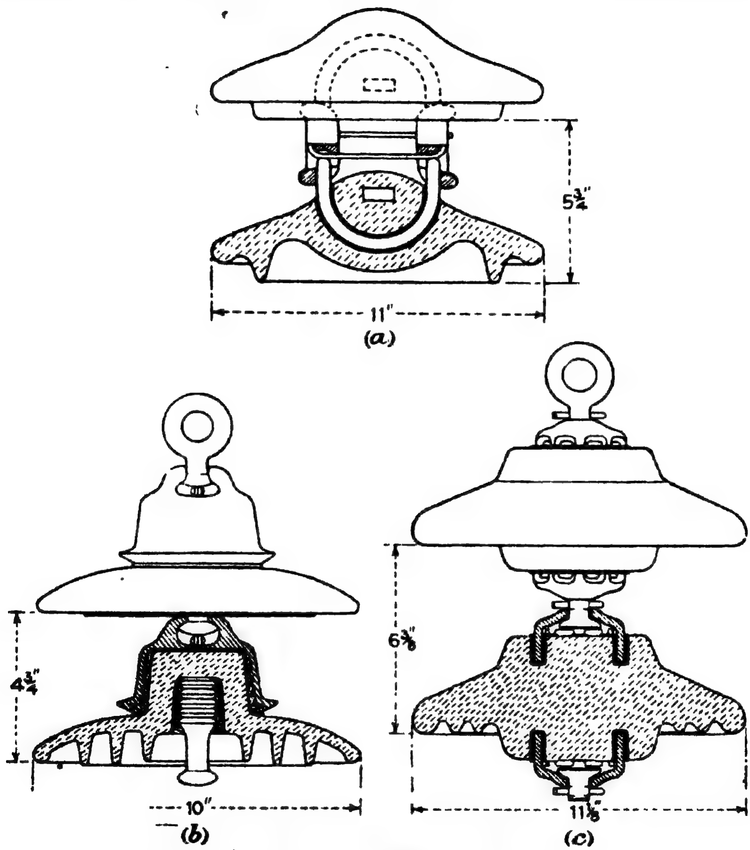

FIG. 41.—Types of suspension-insulator units.

the service. There is also no risk of breakage due to the difference in expansion or contraction of the connecting links and the insulating material. On the other hand, the Hewlett insulator appears to be rather more liable to puncture than other types of suspension insulator, owing to the high electrostatic stress in the material between the links.

In the cemented-cap type of insulator shown in Fig. 41 (*b*), each disc is covered by a metal cap cemented in place, the upper end of each cap terminating in a lug to which the pin of the unit above is fastened. The pins also are firmly cemented in place, and consecutive discs are joined together either by pin-and-clevis or ball-and-socket joints, the porcelain being partly in tension. A very uniform distribution of the electrostatic stress in the material between the connecting-links is obtained in this type of insulator. The disadvantage of the cemented-cap type hitherto has been the high rate of deterioration, insulators failing in service at rates varying up to 20 °/₀ per year. This appears to have been mainly due to the insulator being cemented together very rigidly and no provision being made for expansion. The three materials, porcelain, cement, and steel, have different coefficients of cubical expansion, and the sudden temperature changes occurring in service were sufficient to set up internal stresses which ultimately cracked the porcelain, leading to electrical failure. Furthermore, the cement itself, which is subject to volumetric changes depending on its moisture content, has no doubt often materially assisted in the process of fracturing the insulator. These causes of ultimate failure are now clearly recognised by insulator manufacturers, and modern designs show a great improvement so far as reliability of service is concerned.

One way of avoiding trouble is to substitute for the cementing-in of the pin a purely mechanical fixing, such as the "spring-ring" design. In this method of fixing a spiral spring ring of steel wire carried on the stem of the pin is forced into the interior of the insulator head which is of bulbous shape. The ring immediately expands and is locked in position, the interior being then filled with a lead alloy to prevent any movement of the various parts and protect the fittings from the weather.

In the core-and-tine type of insulator manufactured by the Jeffrey-Dewitt Insulator Co., and illustrated in Fig. 41 (*c*), it is claimed that several important advantages exist. Each insulator disc is symmetrical, and conforms as far as possible to the lines of the electrostatic field, thus avoiding placing materials of different permittivities in series. The shape is such that the equipotential planes between the electrodes of the insulator intersect, for equal increments of potential, fairly equal zone widths on the insulator surface. The metal work consists of pressed steel spiders, the legs

of which are fastened into the porcelain by an alloy having approximately the same coefficient of cubical expansion as the porcelain. By this method of construction high mechanical stresses on the porcelain, whether due to sudden temperature variations or to the employment of cement, are entirely avoided. The manufacturers state that the difficult problem of producing thick porcelain discs has been solved, and the discs actually used have a maximum thickness of 4·25 inches, with a correspondingly high puncture strength.

78. **Limits of Pin-type Insulators.**—Pin-type insulators are available for all pressures up to about 80 000 volts, but generally speaking, the suspension type is preferable on systems operating at above 50 000 volts. Some of the points in favour of the latter type are:—

1. Suspension insulators are usually found to be cheaper than pin-type insulators for pressures over 50 000 volts.

2. Pin-type insulators become very heavy and cumbersome when designed for the higher voltages, and owing to the great length of the supporting pin the bending moment near the point of attachment to the cross-arm tends to become excessive.

3. In cases where the load supplied by the transmission line develops rapidly, it is often found more satisfactory to supply the greater demand by raising the line voltage than to provide another set of conductors. With suspension insulators it is an easy matter to obtain the additional line insulation required, by adding one or more discs to the string.

4. From a mechanical point of view the suspension insulator greatly improves the flexibility of the line. The conductor is secured by a clamp, but the connection at the cross-arm is such that the insulator string is free to swing in any direction, and thus takes up a position where it experiences only a pure tensile stress.

5. The suspension arrangement, when used in conjunction with steel supporting structures, has the advantage of rendering the conductor less liable to be affected by lightning disturbances. At every point of support the wire is hung below the earthed cross-arm, thus enabling the tower to function as a lightning rod.

79. **Strain Insulators.**—Special mechanically-strong suspension insulators are used on all lines (irrespective of the voltage) where the conductors are dead ended, or at intermediate anchor

towers. They are also installed on sharp curves, where the conductors would tend to break the pin of a pin-type insulator, or pull a string of suspension insulators too far from the vertical. On extra long spans, as at river crossings, two or three strings of strain insulators arranged in parallel are often used.

80. Arcing Horns and Rings.—In case of insulator flash-over the porcelain is often cracked or broken up by the power arc which follows the initial potential discharge. To protect against this trouble, the arcing horns or rings originally introduced by Nicholson [3] are installed on some overhead systems. One of the horns or rings is earthed, and the other is in electrical connection with the line conductor, and the arrangement is such that the arc is taken by the electrodes and held at a sufficient distance from the porcelain to prevent damage by the heat of the arc. The advantage of the ring design lies in the fact that the arc can form at any point around the insulator, and in case of formation on the windward side, it may be blown round without injury to the insulator. With definite terminals, on the other hand, the arc may be blown under the insulator and injure it. In some recent designs the horns at the earth end are shaped in such a way as to lead this end of the arc upwards and outwards from the insulator string with a view to increasing the arc length as quickly as possible, and thereby making it self-extinguishing. It should be noted that the application of these devices lowers the flash-over voltage of the insulator, but this disadvantage is held to be more than counterbalanced by the protective value of the equipment. Fig. 42 shows the arrangement of the horns on a 4-unit string of suspension insulators.

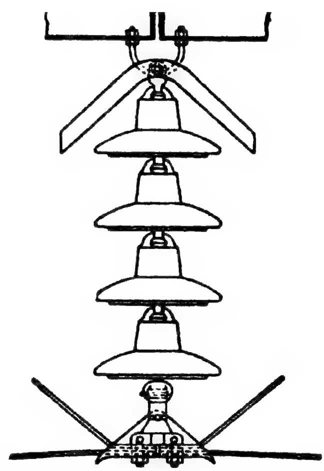

FIG. 42.—Arrangement of arcing horns on suspension insulator.

81. Distribution of Potential over a String of Suspension Insulators.—The voltage impressed on a string of suspension insulators does not distribute itself uniformly across the individual discs owing to the proximity of earth (represented by the supporting

tower). The metal-work (cap and connecting-link) of each disc has a capacitance c_2 relative to the metal-work of the next disc, and also a capacitance c_1 relative to earth. Hence the arrangement may be represented by a series of condensers as shown diagrammatically in Fig. 43 for the specific case of a 4-unit string. A greater capacitance current flows through the line disc 1 than through disc 2, hence the voltage across disc 1 is greater than the voltage across disc 2. Similarly the voltage across disc 2 is greater than that across disc 3, and so on, the least share of the total voltage being taken by the cross-arm unit. The inequality of the voltage distribution is accentuated both by an increase in the number of units in the string, and a decrease in the value of the ratio $\frac{c_2}{c_1}$.

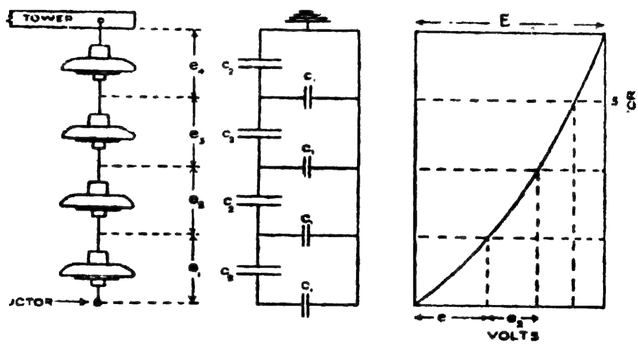

Fig. 43.—Potential distribution on string of suspension insulators.

Assuming that $\frac{c_2}{c_1}$ is known, Peek[4] has shown that the voltage stresses on the discs can be calculated by the following formulæ:—

Volts across 1st or line unit,

$$e_1 = \frac{E}{k}\left(\frac{C_n}{c_1} - 1\right).$$

Volts across 2nd unit,

$$e_2 = E - e_1$$

Volts across mth unit,

$$e_m = e_{m-1} - \frac{\{E - (e_1 + e_2 + e_3 + \ldots e_{m-1})\}}{k} \qquad (113)$$

where E total voltage across complete string of n units,

$$k \quad \frac{c_2}{c_1} = \frac{\text{mutual capacitance}}{\text{earth capacitance}},$$

C_n = total capacitance of complete string

$$= c_1 + c_2 - \cfrac{c^2_2}{2c_2 + c_1 - \cfrac{c^2_2}{2c_2 + c_1 - \cfrac{c^2_2}{2c_2 + c_1 - \cdots}}}$$

the function to be carried to $(n - 1)$ of the $2c_2 + c_1$ terms.

With long insulator strings the above formulæ are rather tedious to apply, and the voltage distribution can be obtained more conveniently by using hyperbolic functions. In this method, the potential at the cross-arm end of the mth unit is given by

$$v_m = E\frac{\sinh\left\{(n - m)\sqrt{\frac{1}{k}}\right\}}{\sinh\left(n\sqrt{\frac{1}{k}}\right)} \tag{114}$$

and after having calculated this value for the various units, the actual voltage across any particular unit can be seen at once.

Table 9 shows the voltage distribution over a 10-unit string as calculated by the latter formula, and assuming $k = 16$. It is seen that the line disc takes 22·5 °/。 of the applied pressure, while the disc at the cross-arm end of the string only takes about 4·2 °/。.

TABLE 9.—*Voltage Distribution over a 10-Unit Suspension Insulator.*

$k = 16.$ $E = 100.$

No. of Unit.	Potential at Line End of Unit.	Voltage across Unit.
1	100	22·5
2	77·5	17·6
3	59·9	13·8
4	46·1	10·9
5	35·2	8·7
6	26·5	7·1
7	19·4	5·8
8	13·6	5·0
9	8·6	4·4
10	4·2	4·2

Supposing now that the applied pressure is gradually increased, flash-over of the string will occur when the stress across the line disc exceeds the flash-over voltage of a single disc. When this happens, the flash-over voltage of the string

$$E_a = \frac{e_a}{e}E, \quad . \quad . \quad . \quad . \quad (115)$$

where e is the voltage taken by the line unit when a voltage E is applied to the insulator, and e_a is the flash-over voltage of a single unit.

The term 'string efficiency' has been applied by Peek to the ratio

$$\frac{\text{Flash-over voltage of string of } n \text{ units}}{n \text{ times flash-over voltage of one unit}} = \frac{E_a}{ne_a}. \quad . \quad (116)$$

In the particular example considered, the string efficiency would be about

$$\frac{100}{22 \cdot 5 \times 10} = 44 \%.$$

It is thus important when installing suspension-type insulators to consider carefully not only the electrical tests made on individual discs, but also the spacing between the discs, and between the insulator string and the tower. By attention to these details, a short insulator string will often give better results than a much longer series of discs installed with no special regard to the question of high string efficiency.

So far the presence of surface leakage has been ignored, but its effect is to equalise to some extent the voltage drops across the individual units. Fig. 44, taken from actual measurements,[5] exhibits this phenomenon clearly. When the insulator is dry the voltage distribution is unequal, and the string efficiency for varying numbers of discs is shown by the upper curve in the diagram. When the leakage surfaces of the discs are covered with moisture the flash-over voltage is lowered, but the distribution of voltage is equalised completely, the string efficiency under these conditions being unity. This is owing to the heavy leakage current swamping the effect of the capacitance current. For these wet tests the insulator was heavily sprayed with water for a considerable period and the mist formed damped the underside of the discs. When only the top surfaces of the discs are wet, the flash-over curve

occupies an intermediate position to the dry and wet curves shown, and the voltage distribution is unequal but not to the same extent as with a dry insulator.

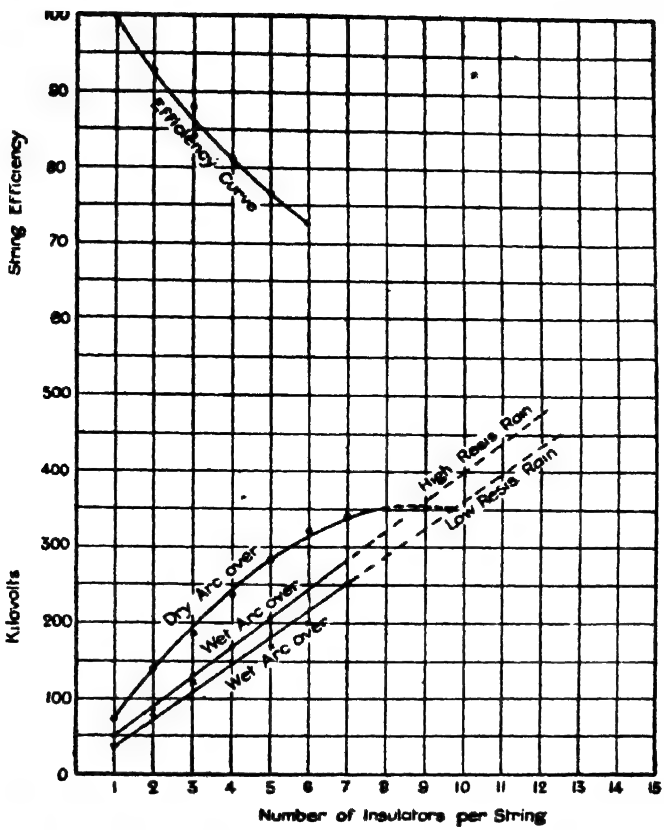

FIG. 44.—Effect of leakage in modifying potential distribution on suspension-insulator string.

82. Shielding and Grading of Suspension-type Insulators.—It will now be readily recognised that the voltage rating of an insulator string is determined by the electrical stresses on the line disc. With the usual ten-inch discs and spacings employed in practice it is found that for strings of five or more units the line unit takes about one-quarter of the total applied voltage. The voltage distribution is most unequal on insulators of the Hewlett

and Jeffrey-Dewitt type, in both of which the line disc takes about 25 to 30 % of the applied pressure. In the cemented-cap type of insulator the metal cap secures a higher value for k, the line disc taking approximately 20 to 25 % of the applied pressure.

In order to show clearly how this phenomenon limits the rating of the insulator, the voltage across the line unit has been calculated for strings of 5, 6, . . . 10 units, assuming as before that $k = 16$, and the results are given in Table 10.

TABLE 10.—*Voltage across Line Disc for Various Lengths of Suspension Insulator.*

$k = 16.$

Length of String.	Percentage Voltage across Line Disc.
5-unit	26·6 %
6-unit	24·8 %
7-unit	23·7 %
8-unit	23·1 %
9-unit	22·7 %
10-unit	22·5 %

Supposing now that the safe operating limit for a single disc is 20 000 volts, the voltage rating for a string of five units is

$$20\,000 \times \frac{100}{26 \cdot 6} = 75\,200 \quad \text{volts,}$$

and for a string of ten units is

$$20\,000 \times \frac{100}{22 \cdot 5} = 88\,900 \quad \text{volts.}$$

Hence, increasing the number of discs by 100 % only results in an 18·2 % increase of voltage rating. Furthermore, even with a 10-unit string the maximum line pressure is

$$88\,900 \times \sqrt{3} = 154\,000 \quad \text{volts.}$$

With the advent of very much higher line pressures the problem of equalising the electrical stresses has become of considerable importance. Various methods have been proposed for this purpose such as:—

1. Increasing the c_2 capacitances without increasing the c_1 capacitances so that the effect of the earth current is relatively less.

This may be done by fitting sheet-metal caps or rings in intimate contact with the porcelain.

2. Increasing the c_9 capacitances of the discs on the line half of the string by using larger units.

3. Increasing the c_2 capacitances of the insulator along the string in proportion to the current flowing through the unit. To effect this, metal caps or rings are employed, the size of these being varied for the different units so as to give the greatest c_2 capacitance for the line unit, a less c_2 capacitance for unit 2, and so on. This is generally known as grading.

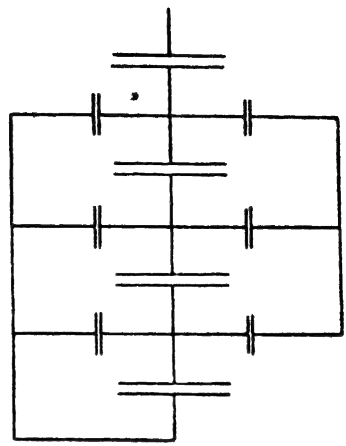

FIG. 45.—Principle of action of antenna shield.

4. Elimination of the earth capacitances by means of an antenna shield from the line conductor. This is called shielding, and Fig. 45 illustrates the principle of action of the device.

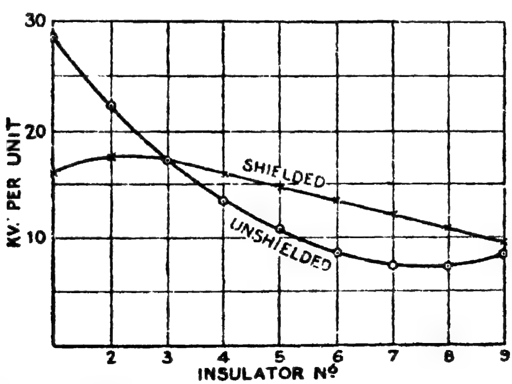

FIG. 46.—Effect of grading on a 9-unit suspension insulator.

Good results can be obtained by all these methods, but method 4 has the special advantage of not requiring different types or

kinds of insulator discs to be installed, and has been adopted on the 220 000-volt system of the Southern California Edison Co.[6] The antenna shield in this case consists of a metal ring 28 inches outside diameter, surrounding the line unit and electrically connected to the conductor. At the cross-arm end of the string, arcing horns or a light ring is fitted, so that in the event of flash-over the arc is held between the horn and the shield ring and prevented from injuring any of the discs. The curves in Fig. 46 show the effect of the shield ring in equalising the voltage distribution over a 9-unit string of cemented-cap type units. Contrary to what might be expected, the flash-over voltage of an insulator string instead of being increased by shielding is usually slightly decreased, but not to a serious extent.

83. Recent Developments in the Design of Suspension-type Insulators.—The advance in transmission line voltages during the last thirty years has been continuous, and there seems no reason to doubt that before long pressures of 300 000 or 400 000 volts will be used for the transmission of very large amounts of power. The length and cost of a conventional type suspension insulator designed for pressures of this magnitude would be prohibitive, and there is great need of an insulator which would be better adapted to increasing line voltages.

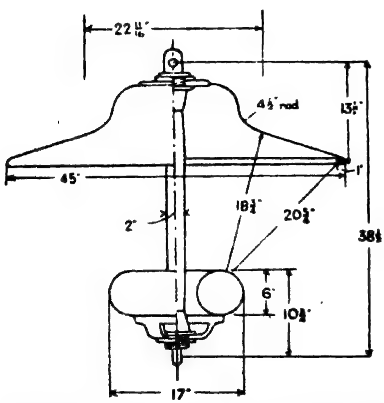

FIG. 47.—Experimental design for 110-kV. suspension insulator.

Research work by Smith[7] has resulted in the development of a suspension-type insulator of simple construction, and possessing greatly improved electrical and mechanical characteristics. The form finally adopted for a series of insulators which are being tried out under actual service conditions is shown in Fig. 47. The mechanical strain member consists of a special impregnated-wood spindle about 2 inches in diameter. Metallic flux-distributing terminals surround the spindle at each end so that the electrostatic field produced in the neighbourhood of the spindle conforms

closely to the surface of the latter. The upper flux terminal which is a ring of about 45 inches diameter runs round the edge of a hood or water shed.

The flash-over voltage of the insulator has a value of 280 kV. or over, dry, and 200 kV. or over, wet, under standard wet-spray test. It is designed for use on 110 000 volt lines, two units in series would be sufficient for 220 000 volt lines, and when necessary three units in series on 330 000 volt lines. No corona is visible on any point of the insulator until within about 5 °/ₒ below break-down voltage. Owing to the absence of arcing conditions on the surface of the insulating member the use of treated-wood spindles can be permitted, and it is claimed that the mechanical properties of the latter are far superior to any porcelain yet available.

84. High-frequency Effects.—When an insulator, either of the pin or suspension type, is subjected to the action of high-frequency potentials, the electrical stresses are usually of a different character than when the potential is applied at the ordinary line frequency. The distribution of potential among any arrangement of pure capacitances is independent of frequency, but an insulator besides consisting of many capacitances may be considered to have a certain amount of leakage path in series thereto. Even if this leakage path can be considered non-inductive, the distribution of potential across such an arrangement is not the same at high as at low frequencies. This is probably one of the chief reasons why insulators, that will flash-over rather than puncture on tests conducted at the line frequency, will sometimes fail through puncture during atmospheric electrical disturbances. Another reason for this phenomenon is to be found in the fact that at high frequencies corona has no time to form and relieve the stress by rupture of the air.

In considering high-frequency effects care should be taken to differentiate between :—

1. Continuous high-frequency oscillations.
2. Damped high-frequency oscillations.
3. Impulses with steep wave front.

1. Continuous or undamped high-frequency oscillations such as might be obtained from an Alexanderson alternator, when applied to an insulator cause high dielectric losses and heating,

resulting in failure at comparatively low voltages. This is a condition that does not occur in the case of transmission lines.

2. Damped high-frequency oscillations occur frequently on transmission lines due to switching operations or arcing grounds.

3. Impulses are often produced in overhead systems by a lightning discharge in the proximity of the line. The voltage induced thereby, travels over the line as a disturbance with a steep wave front. At one instant an insulator may be subjected to the normal voltage, and a millionth of a second later, to the normal voltage plus or minus the induced voltage. If the voltage impressed on the insulator due to the impulse rises from zero to a maximum value in t seconds, the impulse may be said to have an equivalent frequency of $\frac{1}{4t}$ cycles per second.

It is found that both with impulses and single trains of damped high-frequency oscillations, a much higher voltage can be impressed on the insulator than the 50-cycle flash-over voltage. This is owing to the fact that the disruptive discharge through a dielectric requires not only a sufficiently high voltage but also a definite minimum amount of energy. Thus, on applying voltage rapidly to an insulator as by an impulse, flash-over does not occur when the continuously-applied break-down voltage is reached. The voltage overshoots the mark for such a time as will enable this amount of energy to be supplied to the dielectric, and rupture then takes place. The excess voltage is greater, the greater the rate of application. Also for a given rate of increase of pressure, the rupturing voltage is greater, the greater the ununiformity of the dielectric field. With a parallel-plate condenser, or spheres of large diameter compared with their distance apart, the impulse-frequency flash-over voltage is practically equal to the 50-cycle value. For ununiform fields, such as exist between needle points, the impulse-frequency flash-over voltage may be two or three times the 50-cycle value. Peek[8] has applied the term 'impulse ratio' to the ratio of the flash-over voltage at impulse frequency to that at commercial frequency. For insulators the ratio is of the order of 1·2 to 1·6 for the suspension type, and 1·3 to 2·3 for the pin type.

85. Causes of Insulator Failure.—The chief causes of insulators failing in service are:—

1. Deterioration by cracking of the porcelain.
2. Porosity.
3. Puncture of weak porcelain.
4. Shattering by power arcs.
5. Flash-over caused by dust deposits.
6. Failure from mechanical stresses.
7. Short-circuits by birds and similar objects.

1. Failure by cracking of the insulator after a certain period of service has given great trouble both with pin-type insulators and the cemented-cap suspension type. The continued action of alternate and sometimes rapidly varying heat and cold, dryness and dampness, on the complete assembly of porcelain, steel, and cement, produces high stresses on the porcelain and ultimate fracture. This may be due to the differential expansion of the component parts which have different coefficients of cubical expansion, or to volume changes in the cement produced by hydration. Improvements have been made in later designs of insulators by providing for the relative expansion of the different materials. In some cases elastic cushions have been used between the shells, and the steel pins fixed into the insulator by means of metal thimbles or tarred-hemp wrappings.

2. Porosity in the porcelain, due to under-firing or other causes, always leads to failure after a comparatively short period of service. The porous material absorbs water, either from the atmosphere or the cement, thus decreasing its electrical resistance. The leakage current which then flows through the porcelain tends to heat up a localised portion to a high temperature. This local heat may then cause mechanical failure followed by the passage of a power arc through the insulator shell, or under certain circumstances the leakage current may gradually increase with increasing temperature until the porcelain is punctured. In this connection it should be noted that good glazing may postpone the deterioration of porous porcelain but it cannot eliminate it.

3. Puncture of insulators occurs occasionally due to improperly vitrified material. Severe routine electrical tests during the course of manufacture are the best form of insurance against this trouble.

4. The shattering of insulators by the intense heat of the power arc which usually follows a lightning flash-over is one of the most common interruptions to service. Arcing horns or rings are

satisfactorily employed on many systems to reduce this particular operating hazard.

5. Flash-over caused by salt, cement-dust, or soot deposits may give trouble on localised line sections. These deposits are partially conducting and produce a considerable reduction in the flash-over voltage of the insulator, particularly, when in addition to the dust deposits on the interior surfaces, the exterior surfaces of the shells are wet under conditions of mist or fog. It is often recommended to install larger insulators on line sections where the atmospheric conditions are bad, but this is only a palliative measure, and trouble due to this cause can only be eliminated by periodically cleaning the insulators.

Special suspension insulators for erection on coastal lines or in industrial areas are now being manufactured. Some of these rely on the possession of a very long leakage surface, and others of more unconventional design have the main corrugated leakage surface on the upper side of each unit where it can be washed by the combined action of wind and rain.[9]

6. Failure of insulators by mechanical stresses which cause actual pulling apart is very rare as defective material is usually weeded out by routine factory tests.

7. Short-circuiting of conductors by large birds or similar objects gives trouble in certain localities. This can often be prevented by increasing the spacing of conductors and using suspension, instead of pin-type, insulators.

86. Safety Factors.—Insulators are generally so designed that the wet and dry flash-over voltages are several times the operating voltage of the line, so as to make provision for abnormal pressure rises. The ratio between the flash-over voltage of the insulator at normal frequency, and the operating voltage is known as the safety factor. Average values of this factor for pin- and suspension-type insulators are exhibited by the curves of Fig. 48. It will be noticed that the safety factor is far higher on the low-voltage lines. This is as it should be, seeing that all overhead lines in a given locality are subject to virtually the same pressure rises due to lightning. The high-voltage lines are thus relatively less affected. In any case it does not appear desirable to use safety factors less than 2, as low-frequency surges of double voltage may appear in the line caused by normal switching operations.

The severest electrical stresses on insulators are caused by steep

wave front impulses and other high-frequency potentials produced by lightning disturbances. Fortunately, the impulse flash-over voltage of an insulator is always higher than the 50-cycle flash-over voltage, and has the important advantage of not being lowered by rain or moisture. Hence the protective value of an insulator against high-frequency disturbances is greater than indicated by

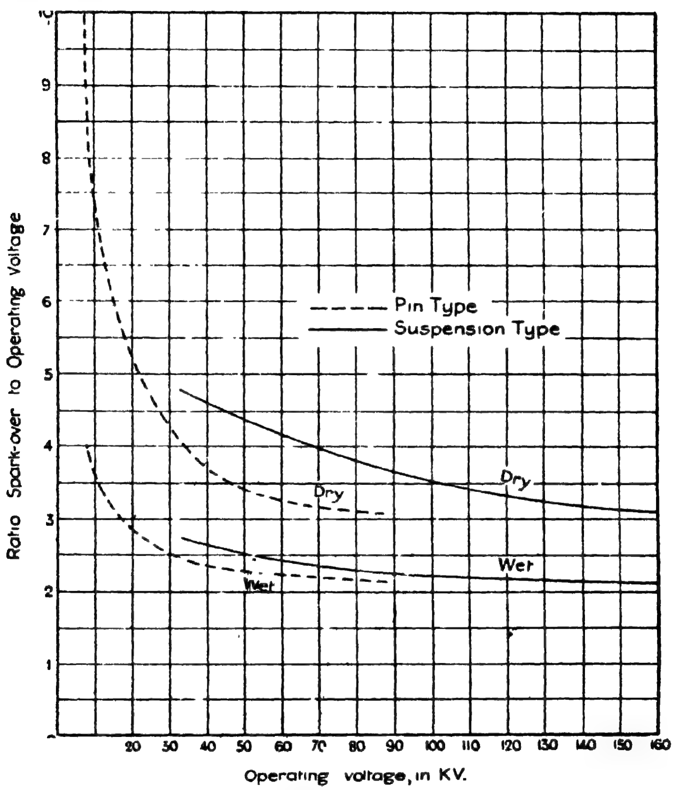

Fig. 48.—Typical safety factors for insulators.

the safety factor as above defined. The puncturing power of high-frequency impulses is, however, greater than that of 50-cycle potentials, and so, beside a high impulse ratio, insulators should possess a correspondingly high dielectric strength.

The flash-over voltage of insulators is lowered by a decrease of air density produced by atmospheric temperature or pressure

changes. In a uniform dielectric field the reduction of voltage would be strictly proportional to the decrease in the air density, but with the non-uniform dielectric field of an insulator the reduction depends somewhat upon the particular type and design. In any case, however, the departure from the theoretical relation for uniform dielectric fields is small, and the correction for atmospheric conditions can safely be made by assuming the flash-over voltage to be directly proportional to the air density. For instance, if the flash-over voltage of a pin-type insulator is found to be 100 kilovolts at sea level (barometric pressure 29·9 inches), then at an elevation of 5 000 feet (barometric pressure 24·6 inches) it would be liable to flash-over at a pressure of

$$100 \times \frac{24{\cdot}6}{29{\cdot}9}$$
$$= 82{\cdot}3 \quad \text{kilovolts.}$$

Due allowance must be made for this reduction of flash-over voltage when selecting insulators for lines erected at a high altitude.

87. Testing of Insulators.—In the course of manufacture insulator tests are made for one of two purposes:—

1. Design tests to ascertain the electrical and mechanical performance of a particular type of insulator.
2. Routine tests to weed out defective insulators.

These two categories are indicated by the letters 'D.' and 'R.' respectively in the list given below. Until recently there was a lack of uniformity among manufacturers both as to the tests employed and the precise methods of carrying out the same. Some or all of the following tests have been used:—

50-cycle Dry Flash-over Voltage (*D. and R.*).—The dry flash-over voltage is determined by applying a gradually increasing voltage between the electrodes of the insulator until the surrounding air breaks down with formation of a sustained arc. As a routine test, vigorous flash-over voltage is impressed for two or three minutes on all shells before assembly, and afterwards on the complete insulator.

50-cycle Wet Flash-over Voltage (*D.*).—This is determined by gradually applying voltage between the electrodes of the insulator the latter being sprayed with water. The standard precipitation conditions are 0·2 inch per minute at an angle of 45 degrees, the

water having a resistance at normal atmospheric temperature of about 10 000 ohms per centimetre cube.

Puncture Voltage (*D. and R.*).—For this test the insulator is totally immersed in oil, the voltage being increased gradually to a certain specified figure, or until puncture occurs. In a routine test, the average puncture strength of each batch of insulators is estimated by selecting a small number of units (about 5 per thousand) out of those which have passed the flash-over voltage test successfully, and carrying them to destruction under oil.

Impulse Ratio (*D.*).—To obtain the flash-over voltage at impulse frequency, and hence the impulse ratio, the lightning generator developed by Peek is employed. By means of this apparatus a steep wave front voltage having an equivalent frequency of several hundred kilocycles per second can be impressed on the insulator, the voltage being measured by a sphere gap.

High-frequency Oscillator (*R.*).—This is a valuable method of testing introduced by Creighton,[10] in which a potential having a frequency of the order of 200 kilocycles per second is impressed on the insulator. The oscillator itself consists of a resonating circuit with a spark gap, which produces a damped high-frequency wave train every time the 50-cycle supply voltage passes through its maximum value. Due to time lag, a voltage about 30 % higher than the 50-cycle flash-over voltage can be applied to the insulators, and the defective units are rapidly picked out. Furthermore, the discharge tends to spread over the insulator surfaces and may thus detect flaws in the petticoats more readily than the 50-cycle test. The high-frequency test has also a logical basis inasmuch as the electrical stresses due to lightning are of a similar high-frequency nature. Owing, however, to the difficulty of standardisation, it does not appear so suitable for a design test as the test at operating frequency.

Mechanical Strength (*D. and R.*).—To ascertain the ultimate mechanical strength of a pin-type insulator it is mounted on a steel pin, and a gradually increasing load applied to the tie-wire groove by means of a steel bridle until failure occurs. Suspension insulators are tested in pure tension. Alternatively insulators may be specified to withstand without failure a high mechanical load and the simultaneous application of a voltage approximately 75 % of the dry flash-over voltage. Routine mechanical tests are usually

made by subjecting all insulators to a specified load, say, 25 % above the maximum working load, without injury.

Porosity (*R.*).—The best method of determining the degree of vitrification of the porcelain is by the fuchsine penetration test. Samples of every batch of recently-fired insulators are broken into pieces, and immersed under pressure in a solution of fuchsine dye in alcohol. After a sufficient period of time has elapsed the specimens are removed from the testing vessel and examined. The slightest degree of porosity is indicated by a deep penetration of the dye into the body of the specimen.

Temperature-cycle (*D.*).—In this test the insulator is subjected to several temperature cycles by alternate immersion in hot and cold water. No damage must be caused to the insulator or the glaze, and after drying the insulator must withstand the usual flash-over test.

The actual conditions under which the various insulator tests are carried out has an important influence on the results obtained, and it should be noted that ratings and tests for insulators operating at 3 300 volts and above have been standardised by the British Standards Institution. For full details reference should be made to this specification.[11]

88. Formation of Corona.—When the pressure between the conductors of an overhead transmission system exceeds a certain definite value—the visual critical voltage of the line—a hissing noise is heard, and the conductors are found to be surrounded by a luminous envelope to which the name of corona has been applied. The luminous envelope is composed of air which has broken down and become temporarily conducting under the high electrostatic stress, and its effect is equivalent to increasing the diameter of the conductors. The break-down starts first near the surface of the conductor, as the electrostatic stress or potential gradient has its maximum value there, and the thickness of the conducting layer of air increases with increase of pressure. If the conductors are very close together (the interaxial distance being less than fifteen times the diameter of the conductors) the formation of corona involves an increase in the potential gradient between them; the corona spreads farther, and so on, until flash-over occurs. In this case immediate disruptive discharge takes place, and no stable corona can be formed. With the wide spacings adopted for commercial lines, the formation of corona has the precisely opposite

effect of decreasing the stress between the conductors. The corona extends until the increase in the effective diameter of the conductor is sufficient to bring the potential gradient at the edge of the corona down to the disruptive gradient of air, and the corona can then spread no farther at the particular voltage under consideration.

Corona is accompanied by a loss of energy which increases very rapidly once the visual critical voltage is exceeded. This energy is dissipated in the form of light, heat, sound, and chemical action.

89. Dielectric Strength of Air and Disruptive Critical Voltage.—The potential gradient at which complete disruption of a dielectric occurs, is called the disruptive strength or dielectric strength of the material. With air at a barometric pressure of 76 cm., and temperature of 25° C., it is $g_0 = 30$ kilovolts per centimetre. The dielectric strength of air is proportional to its density over a very wide range, and thus directly proportional to the barometric pressure, and inversely proportional to the absolute temperature. Taking the density of air under the above standard pressure and temperature conditions as being equal to unity, the relative density at a pressure of b cm., and temperature of t° C., is

$$\delta = \frac{3{\cdot}92b}{273 + t},$$

and the dielectric strength under these conditions is $g_0\delta$.

In a uniform dielectric field such as exists between parallel plates the potential gradient has a constant value at all points. If the voltage is gradually increased, as soon as the break-down gradient of 30 kilovolts per centimetre is attained (assuming standard pressure and temperature conditions), the air breaks down, and flash-over occurs short-circuiting the plates. If the dielectric field is not uniform, as for instance the field between two parallel wires or two spheres, then with increasing voltage the break-down gradient will not be reached simultaneously throughout the entire field, but is first attained at the surface of the wires or spheres.

In the case of parallel wires it has been shown that the potential gradient at the surface of the conductors is given by

$$g_{\pi} \quad \frac{E}{r \log_e \frac{d}{r}}, \qquad \qquad (21)$$

where E is the voltage to neutral, both g_{π} and E being expressed in R.M.S. values.

When the disruptive gradient of air is reached at the conductor surface

$$g_0 = \frac{E_0}{r \log_e \frac{d}{r}},$$

or

$$E_0 = g_0 r \log_e \frac{d}{r},$$

E_0 being known as the disruptive critical voltage.

In practice, corrections have to be applied to this formula for air density and surface conditions of the conductor. Noting also that for sine waves the R.M.S. value of g_0 is 21·1 kilovolts per centimetre, the complete formula is

$$E_0 = 21{\cdot}1 m_0 r \delta \log_e \frac{d}{r} \quad \text{kV. to neutral (R.M.S)}, \quad . \quad (117)$$

where

air density factor (1 at 76 cm. barometer, 25° C.)

$$= \frac{3{\cdot}92b}{273 + t},$$

b = barometric pressure in centimetres,
t = temperature in degrees C.,
r = radius of conductor in centimetres,
d = spacing between centres of conductors in centimetres,
m_0 = 1 for polished wires
= 0·98 – 0·93 for roughened or weathered wires
= 0·87 – 0·80 for stranded conductors (concentric lay).

In the case of stranded conductors the variation in m_0 is due to the number of strands employed, and the varying ratio between the strand diameter and overall diameter in different sizes of conductors. It also depends on the surface conditions of the conductor, often improving after a conductor has been in service for a time, and become weathered and the roughness and abrasions oxidised away. Any mutilation of the conductor during construction or stringing lowers m_0 considerably, but in the absence of any special features 0·85 seems a safe value to use in calculations.

The value of E_0 as given above is the disruptive critical voltage for fair-weather conditions. Bad atmospheric conditions, such as fog, sleet, rain, and snowstorms, lower E_0 considerably and increase the corona losses.

90. Visual Critical Voltage.—In the case of parallel wires it is found that visual corona does not begin at the voltage E_0 at which the disruptive gradient of air g_0 is reached, but at a higher voltage E_v called the visual critical voltage. As a result, when corona begins, the potential gradient g_v at the conductor surface is higher than the disruptive gradient g_0. Contrary to what might be expected, g_v, the apparent strength of air, is not a constant but depends on the size of the conductors, air being apparently stronger at the surface of small conductors than large ones. Peek[12] has

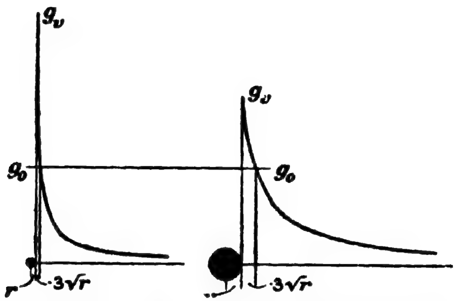

FIG. 49.—'g_v' and 'g_0' for small and large wires.

shown that the relation between the apparent strength of air and the radius of the conductor can be expressed by the formula

$$g_v = g_0\left(1 + \frac{0{\cdot}3}{\sqrt{r}}\right).$$

Since we have also

$$g_v = \frac{E_v}{r \log_e \dfrac{d}{r}},$$

it follows that

$$g_0 = \frac{E_v}{(r + 0{\cdot}3\sqrt{r}) \log_e \dfrac{d}{r}}.$$

Comparing this with formula (20) it is evident that g_0 is the gradient $0{\cdot}3\sqrt{r}$ centimetre from the surface. Hence the stress at the conductor surface when corona appears is not the same for all diameters, as already stated, but is always constant and equal to g_0 at a distance $0{\cdot}3\sqrt{r}$ centimetre from the surface. This is shown diagrammatically in Fig. 49.

According to Peek, this phenomenon is due to the fact that dielectric break-down of air requires a finite volume of over-stressed air. In other words, a finite amount of dielectric energy is necessary to cause rupture or start corona. Hence the potential gradient at the conductor surface must exceed the elastic limit g_0, or be increased to g_v, in order to supply the necessary rupturing energy between the conductor surface and the finite radial distance in space away ($0\cdot3\sqrt{r}$ cm.) where the stress is g_0 and break-down occurs.

The equation for the visual critical voltage may now be written .

$$E_v = g_v r \log_e \frac{d}{r}$$

$$g_0\left(1 + \frac{0\cdot3}{\sqrt{r}}\right) r \log_e d$$

where E_v is the R.M.S. value in kilovolts to neutral, g_0 having the value of $21\cdot1$.

If the energy-distance theory is true, g_v, unlike g_0, should not vary directly with the relative air density δ, and as a matter of fact it is found by experiment that

$$g_v = g_0\delta\left(1 + \frac{0\cdot3}{\sqrt{\delta r}}\right).$$

The visual critical corona point is quite sharp and definite for wires, but not so in the case of stranded conductors. With the latter, corona, after it first appears, increases gradually for a considerable range of voltage until a certain definite voltage is reached, after which the increase is very rapid. The first point is called the local corona point, and the second the decided corona point.

The complete formula for the visual critical voltage is thus

$$E_v = 21\cdot1 m_v \delta r\left(1 + \frac{0\cdot3}{\sqrt{\delta r}}\right) \log_e \frac{d}{r} \text{ kV. to neutral (R.M.S.)}, \quad (118)$$

where

$$\begin{aligned} m_v &= m_0 \text{ for wires } (1 - 0\cdot93) \\ &= 0\cdot72 \text{ for local corona on stranded conductors} \\ &= 0\cdot82 \text{ for decided corona on stranded conductors,} \end{aligned}$$

the other constants being expressed as in formula (117).

91. Corona Power Loss.—The formula for loss of power in fair weather as derived from experimental data by Peek [13] is

$$P_c = \frac{390}{\delta}(f+25)\sqrt{\frac{r}{d}}(E-E_0)^2 10^{-5} \text{ kW. per mile per phase,} \quad (119)$$

where E = R.M.S. kilovolts to neutral,
f = frequency in cycles per second,

the other constants having their former values.

The approximate loss under storm conditions is obtained by taking E_0 as 0·8 times its fair-weather value.

As a matter of fact, with perfectly smooth and cylindrical conductors no loss occurs until the visual critical voltage is reached, when the loss suddenly takes a definite value equal to that calculated by the quadratic law with E_v as the applied voltage. It then follows the quadratic law for all higher voltages. With the roughened and weathered conductors used in practice, however, the quadratic law is approximately followed over the whole range of voltage starting at E_0, but the loss is dependent to such an extent on the irregularity factor, m_0, that accurate calculations are impossible.

92. Practical Corona Formulæ and their Application.—The formulæ required for the determination of the corona characteristics of transmission lines expressed in terms of inch units and common logarithms are:—

Disruptive Critical Voltage,

$$E_0 = 123 m_0 r \delta \log_{10}\frac{d}{r} \quad \text{kV. to neutral (R.M.S.).} \quad (120)$$

Visual Critical Voltage,

$$E_v = 123 m_v \delta r\left(1 + \frac{0{\cdot}189}{\sqrt{\delta r}}\right)\log_{10}\frac{d}{r} \text{ kV. to neutral (R.M.S.).} \quad (121)$$

Power Loss,

$$P_c = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}(E - E_0)^2 10^{-5} \text{ kW. per mile per phase,} \quad (122)$$

where

E = R.M.S. kilovolts to neutral,
δ = air density factor (1 at 29·9 in. barometer, 77° F.)
$= \dfrac{17{\cdot}9b}{459 + t}$.

b = barometric pressure in inches,
t = temperature in degrees F.,
r = radius of conductor in inches,
d = spacing between conductor centres in inches,
f = frequency in cycles per second.

For approximating storm loss consider E_0 = 0·8 of its fair-weather value.

m_0 = 1 for polished wires
= 0·98 – 0·93 for roughened or weathered wires
= 0·87 – 0·80 for stranded conductors (concentric lay).
m_v = m_0 for wires (1 – 0·93)
= 0·72 for local corona on stranded conductors
= 0·82 for decided corona on stranded conductors.

In order to illustrate the use of the formulæ the following example will be taken :—

Problem 5.—Find the corona characteristics of a three-phase line 100 miles long, consisting of three 0·10 sq. in. stranded copper conductors, spaced in an eight-foot delta arrangement. Temperature taken as 80° F. Altitude of line 1 000 feet, corresponding to an approximate barometric pressure of 28·8 inches. Frequency 50 cycles.

From wire tables the diameter of the conductor is found to be 0· 408 inch.

Then

$$\frac{d}{r} = \frac{96}{0\cdot 204} = 470\cdot 6$$

$$\log_{10} \frac{d}{r} = 2\cdot 673$$

$$\sqrt{\frac{r}{d}} = 0\cdot 0461$$

$$\delta = \frac{17\cdot 9 \times 28\cdot 8}{459 + 80} = 0\cdot 956$$

$$m_0 = 0\cdot 85$$

$$m_v = 0\cdot 72 \quad \text{or} \quad 0\cdot 82$$

$$1 + \frac{0\cdot 189}{\sqrt{\delta r}} = 1 + \frac{0\cdot 189}{\sqrt{0\cdot 956 \times 0\cdot 204}} = 1\cdot 428.$$

The disruptive critical voltage is

$$E_0 = 123 m_0 r\delta \log_{10}\frac{d}{r} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (120)$$

$$= 123 \times 0{\cdot}85 \times 0{\cdot}204 \times 0{\cdot}956 \times 2{\cdot}673$$
$$= 54{\cdot}5 \quad \text{kV. to neutral.}$$

Visual corona first appears at

$$E_v = 123\, m_v \delta r\left(1 + \frac{0{\cdot}189}{\sqrt{\delta r}}\right)\log_{10}\frac{d}{r} \quad . \quad . \quad . \quad . \qquad (121)$$

$$= 123 \times 0{\cdot}72 \times 0{\cdot}956 \times 0{\cdot}204 \times 1{\cdot}428 \times 2{\cdot}673$$
$$= 65{\cdot}9 \quad \text{kV. to neutral,}$$

and more decidedly at

$$E_v = 123 \times 0{\cdot}82 \times 0{\cdot}956 \times 0{\cdot}204 \times 1{\cdot}428 \times 2{\cdot}673$$
$$= 75{\cdot}1 \quad \text{kV. to neutral.}$$

Supposing now that the line is operated at 110 000 volts (63·5 kV. to neutral), the fair-weather power loss is

$$P_c = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}(E - E_0)^2\, 10^{-5} \quad . \quad . \quad . \qquad (122)$$

$$= \frac{390}{0{\cdot}956} \times 75 \times 0{\cdot}0461(63{\cdot}5 - 54{\cdot}5)^2\, 10^{-5}$$
$$= 1{\cdot}14 \quad \text{kW. per mile per phase,}$$

and the approximate storm loss is

$$P_c \quad \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}(E - 0{\cdot}8E_0)^2\, 10^{-5}$$

$$- \frac{390}{0{\cdot}956} \times 75 \times 0{\cdot}0461(63{\cdot}5 - 43{\cdot}6)^2\, 10^{-5}$$
$$= 5{\cdot}59 \quad \text{kW. per mile per phase.}$$

In actual practice, the voltage, and therefore the corona loss varies at different parts of the line. In many cases also the altitude of the line varies considerably at different points. These effects can be allowed for in a long line by calculating the loss per mile at a number of points. A curve is then plotted with the loss per mile as ordinates, and length of line as abscissæ; the area of the curve representing the total loss on the line.

When the line conductors are not spaced in an equilateral triangle, but as is often the case, unsymmetrically in a plane, corona

will start at a lower voltage on the centre conductor where the stress is greatest than on the outside conductors. The estimation of the power loss in this case is more difficult, as it is necessary to evaluate the potential gradient round each individual conductor and use this as the basis of calculations for the loss therein.[14]

93. Voltage Limitations of Line.—The critical voltage limit of a line can be raised by increasing either the spacing or the size of the conductors, but the latter method is preferable as the spacing must be kept down to a minimum value in order to save tower costs, and avoid excessive reactance drop in the line. For increasing the size of the conductors, stranded conductors with hemp centres have occasionally been employed, but have not proved satisfactory from a mechanical point of view owing to the hemp

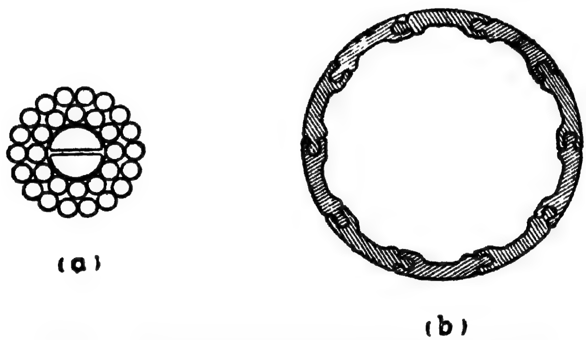

FIG. 50.—Types of conductor used to reduce corona loss.

deteriorating rapidly. Steel-cored aluminium conductors have a much greater diameter than copper ones of the same conductivity, and this consideration often leads to the choice of steel-cored aluminium for systems operating near the corona limit

In a special conductor construction introduced by the Anaconda Wire and Cable Co., one or more layers of copper strands are spiralled round a core of twisted copper, I-beam in shape, as shown in Fig. 50 (*a*). Thus strength is added to the hollow conductor without the addition of dead weight or sacrifice of conductivity or durability. Another design coming into use consists of a number of tongued and grooved rectangular copper sections which are spiralled along the length of the conductor to form a hollow tube (see Fig. 50 (*b*)).[15]

In general, it is not advisable to operate a line above its fair-

weather disruptive critical voltage E_0 (determined for 25° C. and average barometric conditions). If the operating voltage happens to be just below this value the corona losses in fine weather will be negligible. They may, however, have a fairly high value under storm conditions, but, since storms are only experienced at intervals in most districts, it is usually more economical to pay for these losses for small parts of the year than try to eliminate them absolutely by using heavy conductors.

94. Triple-harmonic Currents Due to Corona.—The practice of designing the line so as to avoid corona formation under normal conditions is also desirable in order to eliminate triple-harmonic voltages or currents which may be introduced in the system thereby. The formation of corona starts at a point on the ascending portion of the voltage wave, and disappears when the wave has passed through a maximum and reached the same value on the descending portion. This occurs every half-cycle, and produces a pulsation in the voltage wave having three times the generator frequency. In an earthed system this triple-frequency voltage causes a triple-frequency current to flow through the capacitance of the system to earth and back through the earthed neutral. The effect is accentuated by the fact that the capacitance of each conductor is not constant but also pulsates at triple-frequency due to the increase and decrease of effective conductor diameter caused by the corona. In tests made by Lewis [16] the magnitude of the triple harmonics found were of the order of 40 °/₀, with neutral earthed at one end of the line. In such a case the capacitance current in the conductors, and the rise in pressure along the line at times of light load, may be much greater than their calculated values.

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CHAPTER VIII.

VOLTAGE CONTROL BY SYNCHRONOUS PHASE MODIFIERS.

95. Importance of Voltage Control.—One of the most troublesome features associated with the operation of overhead transmission systems is the inherent variation of voltage at the receiving end, due to changes in the load. These fluctuations of voltage must be prevented from reaching the distributing network for the following reasons:—

1. In the case of a lighting load, the lamp characteristics are very sensitive to changes of pressure. For example, a 5 % decrease in the applied voltage results in a decrease of 15-20 % in the illuminating power of metallic-filament lamps. On the other hand, if the voltage is 5 % above the correct value, the lamps deteriorate at a rapid rate, their life being shortened by about 60 %.

2. Similar fluctuations of voltage are undesirable in the case of a power load consisting of induction motors. For, if the voltage is above normal, the motor operates with a saturated magnetic circuit, with consequent large magnetising current and heating and lower power factor. If the voltage, on the other hand, is low, its effect is to very considerably reduce both the starting torque and pull-out torque.

3. In the case of distribution transformers also, the impressed voltage must be kept within fairly close limits in order to avoid excessive heating, and the introduction of prominent third harmonics in the voltage or current wave.

96. Methods of Voltage Control.—In many cases, and particularly with short lines, the method adopted to deliver a steady voltage at the consumer's terminals is to change the voltage impressed on the line at the sending end *pari passu* with the load. The line is designed so that the regulation or inherent voltage variation at the receiving end, from no load to full lagging load and including the effect of raising and lowering transformers, is

kept to a value not exceeding, say, 20 to 25 %. Automatic voltage regulators, compounded for line drop, then vary the excitation of the generators so as to keep the fluctuations of the receiving-end voltage within close limits. Finally, the voltage in the distributing network is kept steady by using potential regulators, or boosting transformers, on the feeders of the distributing system.

Whilst satisfactory for relatively short lines, this method of working is not applicable to lines of great length because the difference in receiving-end voltage between no-load and maximum-load conditions becomes excessive. It should be noted, moreover, that by this method of control the variations of pressure are not eliminated, but merely transferred from one end of the line to the other. In some cases, however, a considerable amount of power has to be supplied to consumers in the vicinity of the generating station, and so voltage variation at this end involves the same disadvantage as variation at the receiving end. It is only in comparatively recent years that the problem of voltage control has been adequately solved by the introduction of the constant-voltage system of transmission. In this method, specially designed synchronous motors are installed at the receiving end of the line to change the power factor of the system as the load changes and thus always preserve a constant pressure drop along the line. By this means, not only can the sending- and receiving-end voltages be held absolutely constant, but other important advantages are secured.

97. Synchronous Motors.—It is well known that by altering the excitation of a synchronous motor it may be made to take either a lagging or leading current from the line. These changes are clearly shown by the '*V*' curves of the machine. The curve given in Fig. 51 relates to a particular synchronous motor when running light or carrying no mechanical load. The minimum value of armature current, between the lagging-current and leading-current branches of the curve, indicates that the motor is then working at unity power factor, and the power input is just sufficient to meet the no-load losses of the machine. For this particular motor the excitation must be increased from 112 amperes at unity power factor to 155 amperes at full kVA. output leading; the range of excitation being 1·4 to 1. For operation between full lagging and full leading kVA. output, the range of excitation is 67 to 155 amperes or 2·3 to 1.

Idle-running synchronous motors were first employed in connection with power plants to correct for low power factor of the load, and thus reduce the current and power losses in the generators and feeders. For this purpose they should be installed on the distributing system network as near the centre of gravity of the load as possible, so that the benefits of power-factor correction may be

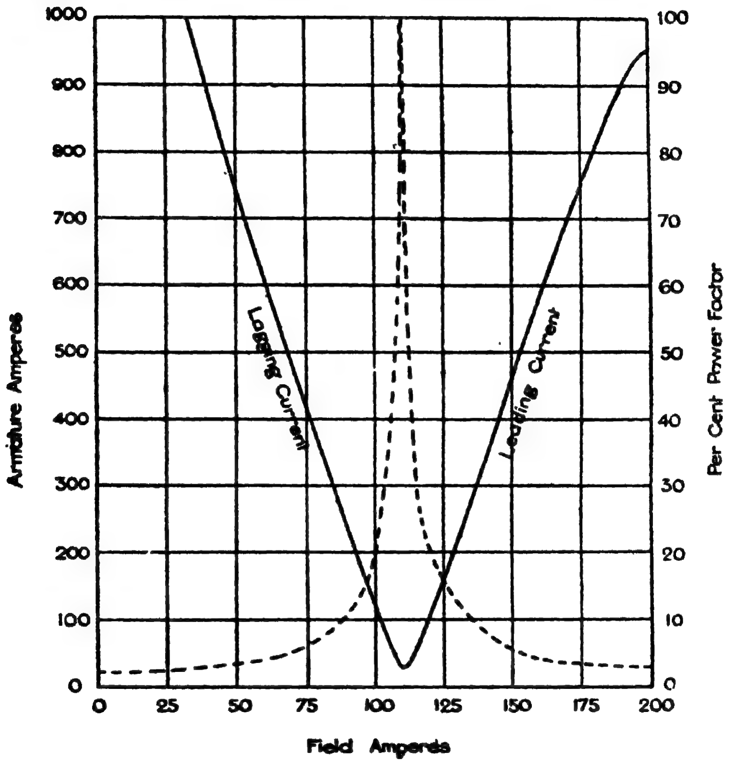

FIG. 51.—'V' curves of a synchronous motor.

felt over the entire generating, transforming and transmitting apparatus. When required for this duty synchronous machines are usually referred to as 'synchronous condensers' since they are always required to take a leading current and hence have the electrical characteristics of a condenser. In the case under consideration, where idle-running synchronous machines are used for voltage-regulating purposes, they are required to run part of the

time with leading current and part of the time with lagging current, and thus the term 'synchronous phase modifier' appears to be more appropriate.

Synchronous phase modifiers differ from synchronous motors intended to be used for mechanical loads inasmuch as they are built for the highest economical speeds, and equipped with smaller shafts and bearings. Furthermore, special attention is given to securing a high overall efficiency. With modern improvements and refinements of design, the total losses at full-load leading power factor expressed in terms of the kVA. output are approximately 4 % for a 2 000 kVA. machine, falling down to 2 % for machines of 20 000 kVA. and over. As usually built, synchronous phase modifiers are designed to give their full-load output at leading power factor, and can then carry about 50 % of their rated capacity lagging. Larger lagging loads result in unstable operation on account of the weakened field. Machines can, however, be designed to operate at full rating both leading and lagging, but they are larger, have a greater loss, and are more expensive than standard machines.

98. Power Factor Control of Line Voltage.—The action of synchronous phase modifiers in controlling the terminal voltage of a line can be easily understood by considering a modification of the fundamental vector diagram which was given in Fig. 14. Fig. 52 is the same as Fig. 14 except that the current in the line at the receiving end is split up into its two components, I_p in phase with E_r, and I_q in quadrature with E_r and lagging behind it for usual loads. Then we have

RC = ohmic pressure drop due to I_p (drawn parallel to OR).
CH = reactive pressure drop due to I_p (drawn 90° ahead of OR).
HG = ohmic pressure drop due to I_q (drawn parallel to ND).
GS = reactive pressure drop due to I_q (drawn 90° ahead of ND).

The voltage drop along the circuit is, to a first approximation,

$$I_pR + I_qX. \quad . \quad . \quad . \quad . \quad (123)$$

Now the component I_p cannot be altered as this represents the actual power taken by the receiving-end load. By drawing a leading wattless current from the line, however, I_q can be decreased or reversed if necessary. This alters the size of the impedance triangle HGS. For as I_q is reduced more and more by the leading current taken by the synchronous phase modifiers, the point S travels along SH until it reaches H. If the leading current taken by the

synchronous phase modifiers is further increased, I_q is reversed and the impedance triangle also, the point S travelling along SH produced. It is easy to see that by this means, no matter what the load or power factor, the difference between the sending- and receiving-end pressures can be adjusted so as to be absolutely constant.

For instance, assume a line with a constant impressed voltage of 110 000 volts at the sending end. Under normal conditions the receiving-end pressure at no load rises to, say, 120 000 volts due to capacitance effects, while on full lagging load the receiving-end pressure may fall to about 80 000 volts. Synchronous phase modifiers might be installed at the receiving end of the line to

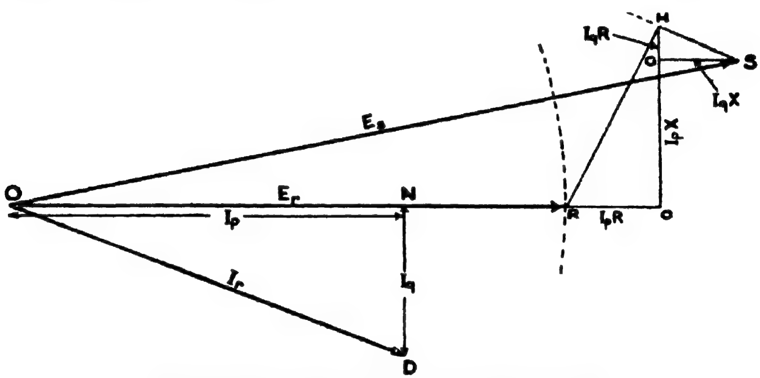

Fig. 52.—Modified vector diagram for a short line.

maintain a constant terminal pressure of 100 000 volts. At times of light load the machines would be operated under-excited, drawing a lagging current to pull down the voltage from 120 000 to 100 000, while under load they would be over-excited and draw a leading current to raise the terminal pressure from 80 000 to 100 000 volts. This constant voltage regulation is usually made automatic by installing voltage regulators operating on the field circuit of the synchronous phase modifiers, and having their control magnets actuated by the phase voltage of the circuit in which the pressure is to be maintained constant.

99. Influence of Synchronous Phase Modifiers on Carrying Capacity of Line.—One of the greatest advantages of the constant-voltage system is that it allows the load of a transmission line to be made two or three times as great as could otherwise be carried,

thus reducing line costs per kilowatt transmitted. In any non-regulated overhead system, the voltage variation at the line terminals fixes a limit to the amount of power that can be transmitted. With the lagging loads obtained in practice, this voltage variation is determined chiefly by the reactance of the line as shown by formula (123). Now with high-voltage lines, the reactance is usually very much greater than the resistance, hence the maximum permissible regulation is reached while the line losses, which are determined chiefly by the resistance, are still very light. Thus the line is unnecessarily efficient from an economic point of view. If, however, synchronous phase modifiers are installed to maintain a steady voltage at the line terminals, the line losses can be made to assume any value which is shown to be desirable from economic considerations.

Practical experience indicates that non-regulated systems cannot be operated satisfactorily when the voltage variation from no load to full lagging load is more than 15 °/ₒ for the high-tension circuit. Taking into account also the pressure drop in the sending- and receiving-end transformers, this would mean a total regulation of 20 to 25 °/ₒ. When, as a result of developing load, the regulation of the system has reached this value, it has been customary to install a duplicate line or alternatively to increase considerably the operating voltage of the system. The installation of synchronous phase modifiers is usually a cheaper means of achieving the same object. By using these machines loads can be transmitted which would produce a voltage regulation of 40 to 80 °/ₒ in a non-regulated line.

The influence of synchronous phase modifiers in increasing the power rating of overhead lines is best illustrated by taking a few numerical examples. Assume that power has to be transmitted a distance of 100 miles to supply a load of 80 °/ₒ power factor; the receiving-end line voltage being 100 000 volts, 50 cycles. Taking a 12-foot spacing between conductors, the smallest size of stranded copper conductor it is possible to use, from consideration of corona, is 0·10 sq. in. The resistance and reactance of this, and four larger stranded conductors, are given in the second and third columns of Table 11. Under the condition that the load of a non-regulated line is limited by a regulation of 15 °/ₒ due to the line impedance only, the maximum amounts of power that can be transmitted over these conductors are shown in the fourth column, and

TABLE 11.—*Carrying Capacities of Regulated and Non-regulated Overhead Lines.*

Regulated lines—Limiting Factor, 10 % Power Loss.

Non-regulated Lines—Limiting Factor, 15 % Voltage Variation in Transmission.

$E_r = \frac{100\,000}{\sqrt{3}}$ volts. Distance of Transmission = 100 miles.

Cos ϕ_r = 80 % lagging. Spacing = 12′. Frequency = 50 cycles.

Size of Conductor.	Resistance.	Reactance.	Non-regulated Line.		Regulated Line.	Ratio of Carrying Capacities.
			Carrying Capacity in Kilowatts.	Efficiency of Transmission.	Carrying Capacity in Kilowatts.	
0·10 sq. in.	43·5 ohms	69·6 ohms	15 700	90·6 %	25 500	1·63
0·20 „	21·6 „	66·0 „	21 100	93·6 „	49 500	2·35
0·30 „	14·3 „	63·4 „	24 300	95·0 „	73 000	3·00
0·40 „	10·8 „	61·9 „	25 700	96·0 „	93 000	3·62

the transmission efficiencies in the fifth column. It will be seen that the line losses are unduly small, particularly with the heavier conductors. Suppose now that synchronous phase modifiers are installed at the receiving end to control the voltage, and that the maximum loads on the lines are determined by the condition that the line losses must not exceed 10 %. The carrying capacities of the same sets of conductors with this method of control ($E_s = \frac{110\,000}{\sqrt{3}}$ volts) are given in the sixth column of the table. The last column shows the carrying capacity of the constant-voltage line expressed as a ratio of the carrying capacity of the non-regulated line, and it will be noticed that very substantial increases of load are permissible. In fact, by this method, loads can be increased until limited by the cost of the extra power losses in the line, or by the cost of the synchronous machinery needed to maintain constant voltage.

100. Effect on Power Factor.—The advantages of operating alternating-current machinery and systems on unity power factor are well known and do not need elaboration here. Briefly, the bad effects of low power factor are felt both in the initial cost of electrical equipment and in the expense of operating the same. Thus low power factor results in—

1. Relatively large and costly electrical apparatus—including

generators, line conductors, transformers and switchgear—the dimensions of which are governed by the kVA., rather than by the kW., output.

2. Reduced efficiency for the whole of the equipment owing to the copper losses for a given kW. load being inversely proportional to the square of the power factor.

Although, as previously stated, synchronous phase modifiers are primarily installed for purposes of voltage control, and not for power factor improvement, it is interesting to see what effect they have on the latter. At times of light load the synchronous phase modifiers will be working under-excited, dragging a lagging current from the line, and thus lowering the power factor to some extent. At times of moderate or heavy loads on the line the machines will be working over-excited, drawing a leading current from the line, and raising the power factor. Generally speaking, then, the operation of synchronous phase modifiers has a beneficial influence on the power factor of the system, as at such times when a high power factor is most desirable the machines play the double *role* of voltage regulators and power factor correctors.

101. Influence on Installation of Reactors.—In the event of a short-circuit occurring on a line, particularly near the generating end, the heavy rush of current which takes place often causes serious damage to transformers and circuit-breakers. With the increase in size of generating stations it has become necessary to provide means for limiting these currents to a safe value, and so reactance coils are frequently installed on the low-tension side of the line transformers. Alternatively, the transformers may be designed with a high internal reactance so that they can safely withstand short-circuiting without extra help. Unfortunately, the use of reactors on a non-regulated line has the disadvantage of directly reducing the power rating, since the voltage variation which determines the power rating is thereby increased. On the other hand, with the constant-voltage system of operation a high terminal reactance can be employed, with consequent increase in the reliability of service.

102. Summary of Results.—Before summarising the results of the preceding discussion, it is necessary to consider two or three disadvantages which are introduced by the employment of synchronous phase modifiers for voltage control. First of all, there is always the possibility of synchronous machines falling out of step, and thus

producing large accidental variations of voltage since the line pressure is so dependent on these machines. Interruptions to supply may also be caused which are best avoided by installing no auxiliary apparatus which it is possible to do without. Another disadvantage of synchronous machines is that they tend to increase the current flowing into a short-circuit. It should be remembered, however, that the machines are usually a long distance from the generating station, and their influence on the short-circuit current is felt chiefly at the receiving end of the line. The most dangerous short-circuits on the system are those which occur close to the generating station, and the synchronous machinery at the other end of the line cannot increase these appreciably owing to the large intervening line reactance.

It has been pointed out that the constant-voltage system allows of far more power being transmitted along an overhead line than could be dealt with satisfactorily by a non-regulated line. In other words, the number of lines for any given project must be reduced if the economies of the constant-voltage system are to be fully realised. Hence, in case of line trouble, the consequences are liable to be more serious with the constant-voltage system than with the non-regulated system. For instance, supposing that with the non-regulated system four sets of conductors are required to transmit a certain amount of power, then, with one set out of action, each of the other three would be overloaded by 33 °/ₒ. With the constant-voltage system employing only two sets of conductors to transmit the same total load, on failure of one set of conductors the load on the other set would be liable to an increase of 100 °/ₒ. There is a growing tendency, however, to interconnect transmission systems in districts where this is possible, thus providing an insurance against failure of supply. A large reserve of lines is not such a *desideratum* under these circumstances.

The chief advantages and disadvantages of the constant-voltage system of transmission may thus be summarised as follows:—

Advantages.—1. Steady voltage maintained at both ends of line at all loads.

2. Increase of power that can be dealt with by one set of conductors thus permitting of considerable financial saving, particularly in long-distance heavy-power transmissions.

3. Improvement of power factor at times of moderate and heavy loads.

4. High terminal reactance can be employed, thus giving better protection.

Disadvantages.—1. Possibility of synchronous phase modifiers dropping out of step, or causing interruption to supply.

2. Increase of short-circuit current of system.

3. Lower reserve of lines in case of line trouble.

When considering whether to install synchronous phase modifiers as an integral part of a transmission system it is necessary to consider the whole problem both from an economic and technical point of view, particularly with short or moderate distances of transmission. On the longest-distance transmissions now employed in various parts of the world the constant-voltage system of operation offers such overwhelming advantages that it may now be considered standard practice.

103. Calculation of Synchronous Phase Modifier Capacity.—A formula for calculating the synchronous phase modifier capacity necessary for the maintenance of constant voltage on a long line is easily derived from the relationship

$$\dot{E}_s = \dot{E}_r A + \dot{I}_r B. \quad . \quad . \quad . \quad . \quad (76)$$

$\dot{I}_r$ is now the actual current at the receiving end of the line, and consists of the load current combined with the reactive current taken by the synchronous machinery.

Let $\dot{I}_r$ be split up into its two components, I_p the in-phase, and I_q the quadrature, component respectively, so that

$$\dot{I}_r = I_p \pm jI_q.$$

I_q is taken as positive when leading, and negative when lagging.

Substituting in equation (76), and resolving into rectangular co-ordinates, gives

$$\dot{E}_s = \dot{E}_r(A_1 + jA_2) + (I_p + jI_q)(B_1 + jB_2),$$

or $$E_s^2 = (E_r A_1 + I_p B_1 - I_q B_2)^2 + (E_r A_2 + I_p B_2 + I_q B_1)^2. \quad (124)$$

Substituting $\dfrac{1\,000P}{E_r}$ for I_p, where P is the power delivered at the receiving end in kW. per phase (E_r being the value to neutral), and solving for I_q this becomes

$$I_q = E_r\left(\frac{A_1B_2 - A_2B_1}{B^2}\right) - \sqrt{\left(\frac{E_s}{B}\right)^2 - \left(\frac{1\,000P}{E_r} + E_r\frac{A_1B_1 + A_2B_2}{B^2}\right)^2}. \quad (125)$$

The alternative solution for I_q obtained by employing the + sign in front of the radical should not be used, as it represents a condition of unstable operation.

Having found I_q, the reactive kVA. at the receiving end of the line is then given by

$$Q = \frac{E_r I_q}{1\,000} \text{ kVA. per phase.} \quad . \quad . \quad (126)$$

The kVA. required from the synchronous phase modifier is obtained by algebraically subtracting from Q the reactive kVA. of the load, the latter being

$$Q_1 = -P \tan \phi_r \text{ kVA. per phase.} \quad . \quad . \quad (127)$$

Hence the capacity of the synchronous machinery necessary for voltage regulation is

$$\begin{aligned} Q_{pm} &= Q - Q_1 \\ &= \frac{E_r I_q}{1\,000} + P \tan \phi_r \text{ kVA. per phase.} \quad . \quad (128) \end{aligned}$$

It should be noted in connection with these formulæ that Q and Q_{pm} may be either leading or lagging according to whether the sign is + or −.

The following example will be taken to illustrate the method of calculation :—

Problem 6.—The 300-mile line whose constants have been previously calculated is to be operated on the constant-voltage system, both the sending-end and receiving-end pressures being maintained constant at 154 000 volts. Determine the synchronous phase modifier capacity necessary when delivering a load of 50 000 kVA. (42 500 kW. at 85 °/。 power factor lagging).

Referring to Art. 46 we have

$$\begin{aligned} A_1 &= 0{\cdot}869780, \\ A_2 &= 0{\cdot}035418, \\ B_1 &= 47{\cdot}937, \\ B_2 &= 180{\cdot}786, \end{aligned}$$

and from the data of the problem

$$\begin{aligned} E_r &= 88\,912 \quad \text{volts to neutral,} \\ P &= \frac{42\,500}{3} \\ &= 14\,166{\cdot}7 \quad \text{kW. to neutral.} \end{aligned}$$

Hence

$$E_r\left(\frac{A_1B_2 - A_2B_1}{B^2}\right) = 395{\cdot}364,$$

$$\left(\frac{E_s}{B}\right)^2 = 225\,995,$$

$$\left(\frac{1\,000P}{E_r} + E_r\frac{A_1B_1 + A_2B_2}{B^2}\right)^2 = 79\,291{\cdot}2,$$

and

$$I_q = 395{\cdot}364 - \sqrt{225\,995 - 79\,291{\cdot}2}$$
$$= +\,12{\cdot}345 \quad \text{amperes.}$$

The reactive kVA. at the receiving end of the line is

$$Q = \frac{12{\cdot}345 \times 88\,912}{1\,000}$$
$$= +\,1\,097{\cdot}6 \quad \text{kVA. per phase (leading),}$$

and the reactive kVA. of the load is

$$Q_1 = 14\,166{\cdot}7 \times \tan 31{\cdot}78^\circ$$
$$= -\,8\,779{\cdot}7 \quad \text{kVA. per phase (lagging).}$$

Hence the capacity of synchronous phase modifier necessary is

$$Q_{pm} = 1\,097{\cdot}6 + 8\,779{\cdot}7$$
$$= 9\,877{\cdot}3 \quad \text{kVA. per phase (leading);}$$

the total capacity for the three-phase system being

$$3Q_{pm} = 29\,632 \quad \text{kVA. (leading).}$$

The electrical conditions when no load is being supplied is readily found by substituting $P = 0$ in formulæ (125) and (128).

The reactive current at the receiving end of the line is

$$I_{q0} = E_r\left(\frac{A_1B_2 - A_2B_1}{B^2}\right) - \sqrt{\left(\frac{E_s}{B}\right)^2 - \left(E_r\frac{A_1B_1 + A_2B_2}{B^2}\right)^2}$$
$$= 395{\cdot}364 - \sqrt{225\,995 - 14\,946{\cdot}0}$$
$$= -\,64{\cdot}04 \quad \text{amperes,}$$

and the capacity of synchronous machinery necessary at times of zero load is therefore

$$Q_{pm0} = \frac{-\,64{\cdot}04 \times 88\,912}{1\,000}$$
$$= -\,5\,693{\cdot}9 \quad \text{kVA. per phase (lagging),}$$

or a total capacity for the three-phase system of

$$3Q_{pm0} = -\,17\,082 \quad \text{kVA. (lagging).}$$

Thus a synchronous phase modifier rated at 30 000 kVA. and of standard design would be capable of dealing with both load, and no-load, conditions.

Having obtained the actual current in the line at the receiving end $\underset{\bullet}{I_r} = I_p + jI_q$, the remaining electrical characteristics of the line may be calculated by applying the customary formulæ.

Thus

$$\underset{\bullet}{E_s} = \underset{\bullet}{E_r}A + (I_p + jI_q)B$$
$$= 88\ 912 \underline{/21 \cdot 472^\circ} \quad \text{volts to neutral,}$$

and

$$\underset{\bullet}{I_s} = (I_p + jI_q)A + \underset{\bullet}{E_r}C$$
$$= 193 \cdot 01 \underline{/45 \cdot 066^\circ} \quad \text{amperes,}$$

and other numerical results obtained therefrom are

$$\phi_s = 23 \cdot 594^\circ \quad \text{leading.}$$
$$\cos \phi_s = 91 \cdot 64 \text{ per cent.}$$
$$\text{kW}_s = 15\ 726 \quad \text{kW. per phase.}$$
$$\text{Loss} = 1\ 559 \quad \text{kW. per phase.}$$
$$\text{Efficiency} = 90 \cdot 09 \text{ per cent.}$$

104. Graphical Determination of Synchronous Phase Modifier Capacity.—The synchronous phase modifier capacity necessary for the voltage regulation of a long line can be conveniently determined from the voltage and current vector diagrams of the line. These diagrams have been given previously, but it is now desirable to examine their properties in rather more detail. In the voltage diagram shown in Fig. 53, OR represents E_r, the receiving-end voltage, which is taken as reference, and OP leading OR by a small angle is E_rA, the sending-end voltage when the line is on open-circuit. When load is being supplied, the impedance consumed by the line will raise or lower the sending-end voltage according to the power factor of the load. This effect is shown by the vector PS which represents I_rB, and the resultant sending-end voltage under load is thus OS. The position of the point S, which locates the sending-end voltage vector, depends on the character of the load. With a unity power factor load at the receiving end, the point S travels along PA. At any other constant power factor load, the locus of S is a straight line drawn from P leading or lagging behind PA by the angle ϕ_r. If the amount of power delivered is kept constant while the power factor is varied, S travels along a line perpendicular to PA. With a constant kVA.

load, S moves along the arc of a circle drawn with P as centre. Finally, for the condition that the sending-end voltage must be kept constant as the load on the line varies, the point S must follow the arc of a circle drawn with O as centre and having a radius equal to the given sending-end voltage E_s.

The current diagram shown in Fig. 54 is similar to the voltage diagram in that it consists of two vectors, one of which is proportional to the receiving-end voltage, and one which is proportional to the receiving-end current. In the diagram, OR again represents E_r, the receiving-end voltage, and OM drawn at an angle of just over 90 degrees to OR is E_rC, the charging current at the sending

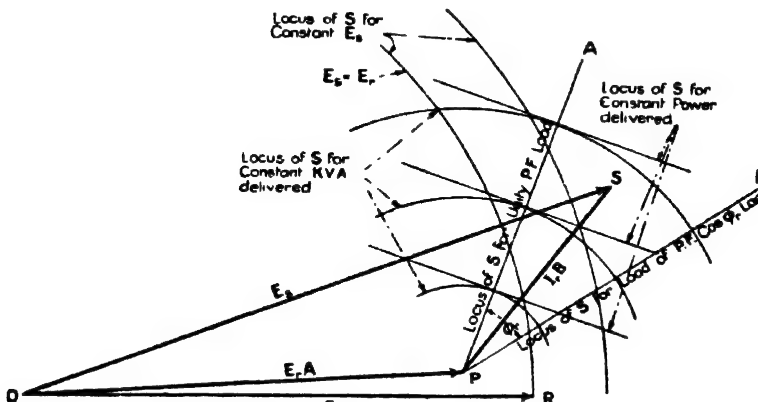

FIG. 53.—Voltage vector diagram for long line.

end when the line is on open-circuit. As the line is loaded, a current MN representing I_rA is added to the charging current, thus giving the sending-end current ON. For a unity power factor load, N travels along the line MA leading OR by a small angle. At any other power factor, the locus of N is a straight line drawn from M leading or lagging behind MA by the angle ϕ_r. With constant power delivered and varying power factor, N moves along a line perpendicular to MA. Similarly, the locus of N for a constant kVA load will be an arc of a circle drawn with M as centre.

On comparing the two diagrams it is seen that the line PA in the voltage diagram and its associated system of lines being the loci of the sending-end voltage, is exactly similar to MA in the current diagram and its associated loci for the sending-end current.

Any point on the voltage diagram such as S represents a definite amount of power delivered at the receiving end of the line, and a corresponding point N can always be found on the current diagram which represents exactly the same load conditions. OS and ON are the sending-end voltage and current respectively for this particular load. It is obvious that concentric arcs of circles may also be drawn on the current diagram as shown, giving the loci of I_s for any given constant sending-end voltage E_s.

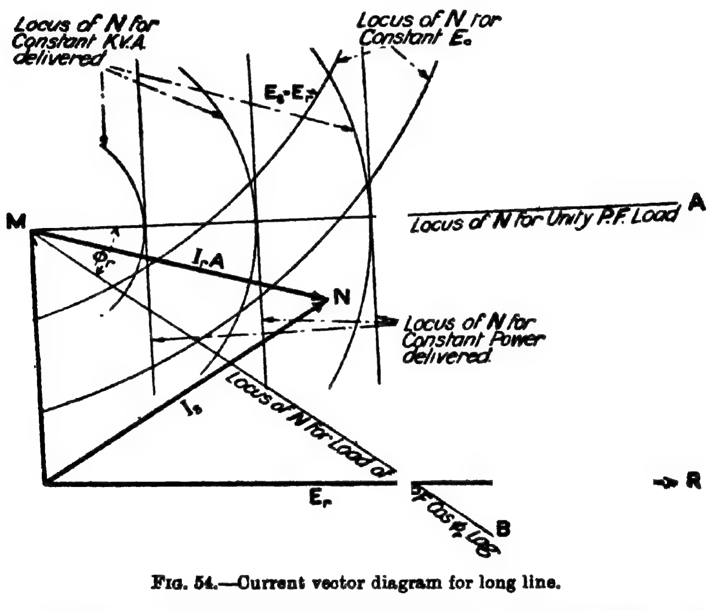

FIG. 54.—Current vector diagram for long line.

The use of these diagrams to solve Problem 6 graphically will now be explained.

Choosing a scale of, say, 1 inch = 10 000 volts, OR and OP are drawn as previously described (Art. 47) to represent E_r and E_rA respectively (see Fig. 55). In order to get a kVA. scale take a load of 50 000 kVA. at the receiving end at unity power factor, I_r will then equal 187·5 $\angle 0°$ amperes per conductor, and

$$I_rB = 187{\cdot}5 \underline{/0°} \times 187{\cdot}0 \underline{/75{\cdot}15°}$$
$$= 35\,060 \underline{/75{\cdot}15°} \quad \text{volts.}$$

From P draw the unity power factor line at an angle of 75·15° to OR, measure up along it a distance $PQ = 3\cdot 506$ inches, and through Q draw a line perpendicular to PA. This is the 50 000 kW. line, and other lines can then be drawn parallel to it to show the loci of the sending-end voltage corresponding to other loads. With O as centre draw an arc of radius equal to the scale value of the sending-end voltage which it is decided to maintain constant by means of synchronous phase modifiers. In the problem under consideration $E_s = E_r$.

To find the conditions at the receiving end when a load of 50 000 kVA. (42 500 kW. at 85 °/ₒ power factor lagging) is being delivered, draw on the diagram the power line corresponding to 42 500 kW., and also PB lagging behind PA by $\cos^{-1} 0{\cdot}85$ or

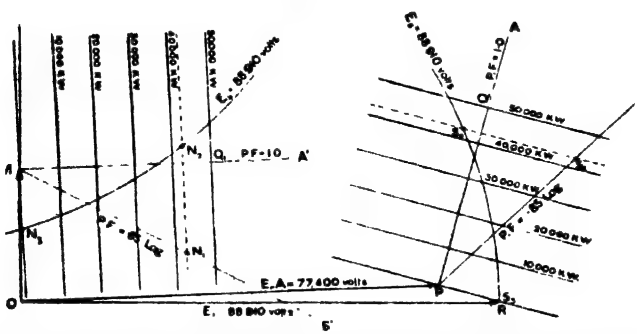

FIG. 55.—Voltage and current solutions for various receiving-end conditions.

31·79°. The point of intersection S_1 of these two lines locates the end of the sending-end voltage vector for the non-regulated line, which vector has a value of 106 000 volts. The sending-end voltage is actually maintained at 88 910 volts, and to reduce the voltage from 106 000 to 88 910 requires an amount of phase modifier capacity represented by the distance S_1S_2 or 30 000 kVA. leading.

Under zero-load conditions, to keep E_s constant at 88 910 volts requires a phase modifier capacity represented by PS_3 or 17 000 kVA. lagging.

The voltage diagram is all that is needed to determine the phase modifier capacity for any given load, but by drawing the current diagram the remaining characteristics of the line can be found. The current diagram is most conveniently drawn on the same sheet, and to the same scale, as the voltage diagram. Both I_rB and I_rA

are proportional to I_r, so if the current scale is chosen so as to make $I_rB = I_rA$, the same power scale will apply to both diagrams.

For 50 000 kVA. at unity power factor

$$I_rA = 187{\cdot}5 \underline{/0^\circ} \times 0{\cdot}8705 \underline{/2{\cdot}33^\circ}$$
$$= 163{\cdot}2 \underline{/2{\cdot}33^\circ} \quad \text{amperes,}$$

and for equality of power scale this current must be represented on the diagram by the same length which represents I_rB, or 3·506 inches. Hence

$$163{\cdot}2 \quad \text{amperes} = 3{\cdot}506 \quad \text{inches,}$$

or the current scale is 1 inch = 46·55 amperes.

Now from O draw OM at an angle of 90·72 degrees to represent

$$E_rC = 88\ 910 \times 0{\cdot}001349 \underline{\diagdown 90{\cdot}72^\circ}$$
$$= 119{\cdot}9 \underline{\diagdown 90{\cdot}72^\circ} \quad \text{amperes,}$$

and from M draw the unity power factor line MA' at an angle of 2·33 degrees to OR. The power scale can then be readily transferred from the voltage diagram on to the current diagram by means of dividers.

With centres M and Q_1, and radii equal to PO and QO respectively, strike two arcs and the point of intersection will be the centre from which to draw the curve showing the locus of sending-end current for a constant sending-end voltage E_s. The points N_1, N_2, and N_3 on the current diagram correspond to S_1, S_2, and S_3 on the voltage diagram. Thus ON_1 is the sending-end current when no phase modifiers are in use and is equal to 146 amperes. When the sending-end voltage is held constant at 88 910 volts, the sending-end current is $ON_2 = 193$ amperes.

The sending-end phase-angle is the angle $S_2ON_2 = 23{\cdot}5^\circ$, and the power factor 91·7 °/ₒ leading.

The total kVA. input to the line can now be calculated and therefrom the line losses and efficiency of transmission.

105. The Power Circle Diagram.—This diagram may be defined as a graph showing the reactive power at one end of the line (usually the receiving end) for various loads transmitted, assuming definite sending- and receiving-end voltages. It thus forms a very convenient method of finding the requisite synchronous phase modifier capacity for all possible loads, and not for one particular load only. The circle diagram for a short line was described by

Philip,[1] and later on was extended by Dwight [2] to take into account the effect of capacitance, that is a diagram for a long line.

The data required for drawing the diagram in the latter case are obtained from equation (124)

$$E_s^2 = (E_rA_1 + I_pB_1 - I_qB_2)^2 + (E_rA_2 + I_pB_2 + I_qB_1)^2,$$

which is the equation of a circle and can be reduced to the form

$$\left(I_p + E_r\frac{A_1B_1 + A_2B_2}{B^2}\right)^2 + \left(I_q - E_r\frac{A_1B_2 - A_2B_1}{B^2}\right)^2 - \left(\frac{E_s}{B}\right)^2 = 0. \qquad (129)$$

In this equation voltages are measured to neutral, hence by multiplying throughout by $\frac{3E_r}{1\,000}$, a circle is obtained which is specially adapted to a three-phase system as it shows the relation between the *total* reactive and active kVA. at the receiving end of the line.

The co-ordinates of the centre of this circle are a and b, and the radius is c where

$$a = -\frac{3E_r^2(A_1B_1 + A_2B_2)}{1\,000B^2} \text{ kW.},$$

$$b = +\frac{3E_r^2(A_1B_2 - A_2B_1)}{1\,000B^2} \text{ kVA.},$$

$$c = +\frac{3E_rE_s}{1\,000B} \text{ kVA.}$$

The receiving-end power circle diagram for the 300-mile line, assuming that the sending- and receiving-end line voltages are both held constant at 154 000 volts, is shown in Fig. 56. In this particular case the numerical values of a, b, and c are

$$a = -32\,610 \text{ kW.}$$
$$b = +105\,460 \text{ kVA.}$$
$$c = +126\,800 \text{ kVA.}$$

Any point in the fourth quadrant of the diagram represents a definite load supplied at a definite power factor. To determine the kVA. required from the synchronous phase modifier at this load, the reactive kVA. of the load must be subtracted from the corresponding ordinate of the circle which represents the total kVA. in the line at the receiving end. As the reactive kVA. of the load is lagging, and thus negative, this operation is equivalent to arith-

metically adding the reactive kVA. of the load to the corresponding kVA. of the circle diagram.

Thus, supposing that the load although varying in magnitude has a constant power factor of 85 °/。 lagging, a straight line can be drawn at an angle cos $^{-1}$ 0·85 below the base line. By means of

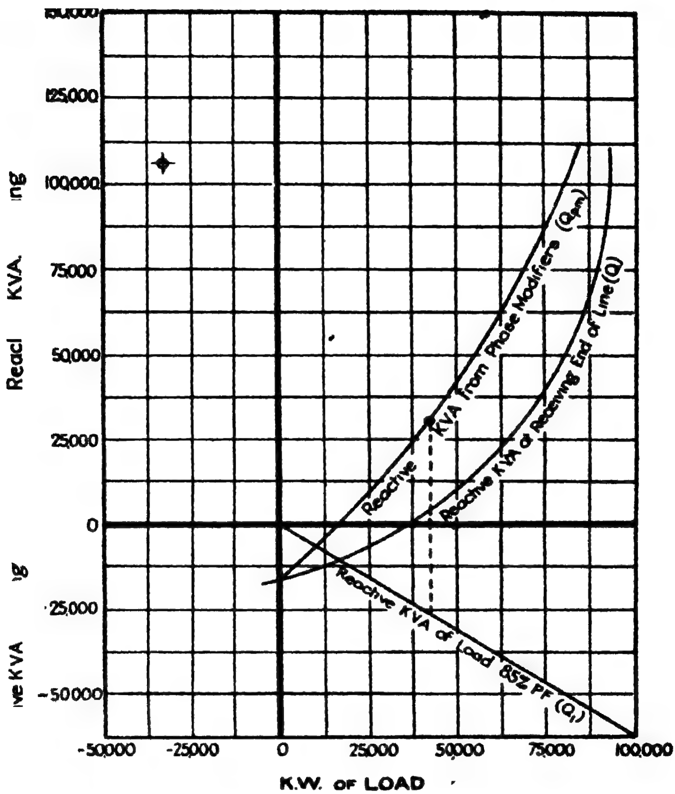

FIG. 56.—Receiving-end power circle diagram for constant-voltage transmission line.

a pair of dividers the reactive kVA. of the load can then be added to the corresponding ordinate of the circle, thus plotting the curve of kVA. required from the synchronous phase modifier; this latter curve being an ellipse.

It will be noted that there is a theoretical limit to the amount of load in kilowatts that can be transmitted, which is easily read

from the diagram as it is the farthest distance to the right reached by the circle. The maximum possible load is thus

$$P_{max} = c + a,$$

which is numerically less than c since a is negative.

The phase modifier capacity required for transmitting the load of 42 500 kW. at 85 % power factor is indicated in the diagram by the intersection of the dotted line with the phase modifier curve, and agrees with the results of previous calculations. The maximum amount of power that can be transmitted by the line at the voltages under consideration is

$$P_{max} = 126\,800 - 32\,610$$
$$94\,190 \quad \text{kW.}$$

It will be noticed that there is a very rapid increase in the amount of phase modifier capacity required when loads in the neighbourhood of the theoretical maximum are reached.

In making the calculations for a long transmission line it is often desirable to include the effect of the raising and lowering transformers, and methods of accomplishing this have been described by Dwight[3] and Evans and Sels.[4]

106. Determination of Synchronous Phase Modifier Capacity in Case of Short Line.—Hitherto the determination of synchronous phase modifier capacity has been discussed in relation to long transmission lines, as this is the more important case occurring in practice. It is interesting, however, to note the modification of the preceding formulæ when applied to the case of short lines.

In the case of a line where capacitance effects may be ignored the fundamental equation corresponding to (124) is

$$E_s^2 = (E_r + I_pR - I_qX)^2 + (I_pX + I_qR)^2, \quad . \quad (130)$$

from which

$$I_q = \frac{E_rX}{Z^2} - \sqrt{\left(\frac{E_s}{Z}\right)^2 - \left(\frac{1\,000P}{E_r} + \frac{E_rR}{Z^2}\right)^2}, \quad . \quad (131)$$

where the current is leading when positive and lagging when negative.

The capacity of synchronous phase modifiers necessary for voltage regulation can then be obtained as before,

$$Q_{pm} = \frac{E_rI_q}{1\,000} + P\tan\phi_r \quad \text{kVA. per phase.} \quad . \quad (132)$$

Also by writing equation (130) in the form

$$\left(I_p + \frac{E_r R}{Z^2}\right)^2 + \left(I_q - \frac{E_r X}{Z^2}\right)^2 - \left(\frac{E_s}{Z}\right)^2 = 0$$

it is readily seen that, in the case of a short line, the co-ordinates of the centre of the circle are

$$a = -\frac{3E_r^2 R}{1\ 000Z^2},$$

$$b = +\frac{3E_r^2 X}{1\ 000Z^2},$$

the radius of the circle being

$$c = +\frac{3E_r E_s}{1\ 000Z}.$$

107. Choice of Sending-end Voltage.—Although in the constant-voltage system of operation both the sending-end and receiving-end voltages are maintained constant, it is not essential that they should be equal. Frequently it happens that a less synchronous phase modifier capacity is needed by operating with E_s higher than E_r. This can readily be seen from the following considerations. The minimum phase modifier capacity for any given transmission scheme is obtained when the lagging current taken by the machines with the line open-circuited at the receiving end, is equal to the leading current taken when the line is carrying full inductive load. Now a lagging current in the line at zero load produces a direct loss of pressure, that is, requires a higher sending-, than receiving-end, pressure. Hence in the case of a short line, E_s should be chosen higher than E_r. As the length of line increases, the lagging current taken by the synchronous phase modifier at zero load is more and more neutralised by the leading charging current of the line. Hence for minimum phase modifier capacity, E_s approaches nearer and nearer to E_r, and in the case of very long high-voltage lines the two terminal voltages may be practically equal. The circle diagram has the valuable characteristic that, with an alteration of E_s, the only diagram constant which is affected is the radius of the circle, which varies directly with E_s. Hence concentric circles can be drawn with practically no additional trouble to show the results for different values of the sending-end voltage.

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CHAPTER IX.

UNDERGROUND CABLES—TYPES AND ELECTRICAL CHARACTERISTICS.

108. Introductory.—The chief use of underground electric cables for many years was in connection with urban distributing systems at low or comparatively moderate pressures. Recent improvements in design and manufacture, however, have led to the development of cables suitable for use at the highest voltages associated with modern transmission systems, and underground lines are now often employed for the straight transmission of power for short or moderate distances, or for sections of overhead lines in built-up areas. In this and the following chapter only high-voltage cables suitable for transmitting energy in bulk will be primarily considered, although much of the discussion is also applicable to all classes of cable.

Compared with overhead systems, cables have the following advantages:—

1. Greater freedom from interruption of service and damage due to thunderstorms, severe weather conditions, or extraneous interference. On overhead lines short-circuits and earths due to flash-over, breaking of conductors or insulators, objects falling across the wires, or other causes have always to be reckoned with.

2. Removal of liability of accident to public.

3. Avoidance of unsightly appearance of line supports and overhead conductors in the neighbourhood of cities.

4. Freedom from way-leave troubles, the cable route usually following the roadways.

The above considerations often result in the installation of underground lines in populous districts, despite the fact that the first cost is much higher than that of an overhead line.

109. Materials.—Notwithstanding the many different types of cable in use for high-voltage power transmission they all consist essentially of a copper conductor or conductors, insulated with impregnated paper, and provided with an overall lead sheath. The

adoption of impregnated paper as an insulating medium results in a cable of superior durability, high dielectric strength, low capacitance and dielectric power factor, and low cost.

Copper.—The bulk of the copper used for electrical conductors is electrolytically refined in order to obtain a high degree of purity, and the actual copper content of the product is of the order of 99·95 %. Extremely small quantities of impurities lower the electrical resistance very greatly, but with care there is no difficulty in obtaining commercial copper of a minimum conductivity of 100 %.

Stranded conductors are, of course, always employed for cables, and the degree of stranding is higher than that used for overhead conductors in order to provide the greater degree of flexibility required.

Insulating Materials.—For many years paper for cable insulation was made from manilla fibre, but wood-pulp paper is now in more general use. The physical characteristics, such as tensile strength, elasticity, porosity, and ability to absorb liquids, are of extreme importance in determining the choice of a suitable quality of paper.

Since the paper is applied in layers to secure flexibility, it must have good mechanical strength and elasticity to permit the core being wrapped without undue breakage of paper, and to facilitate the cable being bent in service without tearing the paper tapes.

Paper itself is a hygroscopic, fibrous material, and requires treating with an oily compound to render it fit for use in underground cables. High oil absorption and porosity are therefore essential for the efficient impregnation of paper cores of considerable thickness.[1]

The compound normally used for impregnation purposes is a heavy-grade mineral oil mixed with resin, the actual proportions of oil and resin depending on the particular manufacturer. The chief non-electrical properties to be considered in choosing a compound are its lubricating properties, viscosity, chemical stability, and freedom from acids.

Good lubricating properties and a low set point are necessary so as to facilitate the layers of paper sliding over one another without tearing when the cable is being installed.

The viscosity of the compound used in cables manufactured by the normal process, and in particular the variation of viscosity with temperature are also of extreme importance. The viscosity of the

compound must be sufficiently low at the impregnating temperature to permit thorough impregnation of the paper, but on the other hand it must be sufficiently high at normal working temperature to prevent drainage from one part of the cable to another after installation. The higher the resin content of the compound the greater the viscosity at a given temperature.

A low coefficient of thermal expansion is also a desirable characteristic, but unfortunately there is little scope for regulation of this feature among the otherwise suitable materials available.

From an electrical point of view the compound should have high values of resistivity and dielectric strength; thus there must be an entire absence of absorbed moisture. A low power factor is also desirable in order to keep down the dielectric losses in the cable, and here again the resin content plays an important part in fixing the magnitude of these losses and their variation over the working temperature range.[2]

As regards dielectric strength, the 50-cycle breakdown stress of dry paper, which is 60 to 80 kV. per cm. when tested in fairly thin layers, is raised by impregnation to a value of 500 to 600 kV. per cm. These are momentary or short-time values, the dielectric strength depending to a great extent on the length of time during which voltage is applied before breakdown takes place. Further consideration, however, will be given at a later stage in this chapter to the electrical phenomena occurring in impregnated paper when forming the insulation of a cable and subject to actual service conditions.

Lead.—The lead sheathing applied over the insulating material has the two-fold purpose of preventing access of moisture to the paper and forming a mechanical protection against handling. Ductility and durability are essential features of the sheathing, and to secure this all impurities in the metal must be eliminated. A minimum standard of 99·9 % purity is usually specified, and cable manufacturers have no difficulty in working to this standard.

Although extremely ductile, pure lead is liable to fail by cracking when exposed to mechanical vibration such as may occur on bridge crossings, near railway tracks, or in submarine cables subjected to tide action. In these circumstances, binary and ternary lead alloys which have a considerably higher tensile strength than pure lead, particularly under alternating stress, are now often used. The fatigue limit for pure lead, for example, is of the order of

± 0·18 ton per sq. in. for 10 million reversals, but the addition of 0·85 % antimony raises this to ± 0·60 ton, whilst the addition of 0·5 % antimony and 0·25 % cadmium increases the fatigue limit to about ± 0·74 ton per sq. in. The ductile properties of these alloys are inferior to pure lead, but not sufficiently so as to prevent extrusion in the normal manner.

The composition of four anti-vibration alloys and chemical methods of testing lead and lead-alloy sheaths have been standardised by the British Standards Institution.[3]

110. Types and Manufacture.—There are two chief types of cable used for power transmission at high voltages, *viz.* single-core and three-core. The former type has the advantage of simplicity of construction, consisting essentially of a stranded copper conductor, a belt of impregnated paper insulation and an overall lead sheath. Plain lead-covered cables are often used for work inside generating stations, etc., but when buried direct in the ground the sheath is generally protected by a serving of hessian tapes or jute soaked in a preservative compound of a bituminous nature.

In three-core cables the three separate conductors are placed inside one lead sheath. The great advantage is thereby obtained that steel armouring may be used to protect the cable, whereas with single-core cables carrying alternating currents the armouring would usually cause excessive voltage drop and power losses.

The manufacture of a three-core cable is briefly as follows:—

Each of the conductors is built up on a stranding machine, and is then passed through a paper-lapping machine where the paper in the form of tapes, 3-6 mils. thick, is wrapped on spirally to the required thickness. Each layer of paper is laid so as to break joint with the layer beneath, and the width of the tapes must not be excessive or the flexibility of the cable will be impaired. The three cores are then laid spirally together, the interstices between them being packed with paper or jute fillers so that a truly circular shape is produced, and round this is wrapped other layers of paper to form what is known as the belt insulation. In another type of cable the belt insulation is omitted, each core being wrapped with a thin metallic screen, and after laying up in the usual manner a thin copper-reinforced cotton tape is applied overall to hold the cores together during succeeding processes.

The reel containing the technically wet paper cable is then transferred to the impregnating plant and heated in steam vessels

at a temperature well over 100° C., first in air and later in a high vacuum, till the last traces of moisture have been extracted. Impregnating oil is now allowed to flow into the tank and a pressure of the order of 50 lbs. per sq. in. maintained for a considerable period. Subsequently the temperature is allowed to drop down to 65° C. before removing the reel from the tank in order to prevent the formation of cavities in the insulation. The whole period of drying, impregnation, and cooling occupies from one to three weeks, according to the voltage of the cable and consequent thickness of insulation involved.

After impregnation is complete, the cable is sheathed by being passed through the lead press which extrudes round it a closely-fitting lead tube, the latter whilst still hot and unoxidised being then usually treated to a bath of hot bituminous compound.

In the case of armoured cables, the armouring (which may be a layer of galvanised steel wires or two layers of steel tape laid so as to break joint) is then applied either direct on the lead or over a bedding of compounded hessian tapes or tarred jute yarn, the cable being then finished off with an overall serving of similar material. The armouring and textile materials are applied in one operation, and prior to the application of each layer the cable is passed through a bath of hot bituminous compound which helps to form an efficient protection to the sheath and armour against the action of water and chemical acids in the soil. For work inside stations or for cables to be drawn into ducts, bare armouring is often employed, any coverings of tape or yarn being placed between sheath and armour.

Sector, or oval-shaped, conductors are usually employed in three-core cables, as this design permits of a smaller overall diameter for a given voltage rating, and so leads to a saving in cost and weight. The reduction in overall diameter is a function of conductor size, but owing to non-standardisation of cross-section its value varies to a certain extent with cables of different manufacture.

These special cross-sections of conductor are usually obtained by rolling or hammering a circular stranded conductor to the desired shape, and special care has to be taken to avoid sharp edges which might give rise to excessive local electrostatic stresses. Occasionally, special stranding round a central flat, or triangular, core of wires is employed. The use of any special shape necessitates

a continuous twist in the conductor as it is laid up, owing to the fact that it must occupy a geometrically similar relation to the cable axis at all points along the cable. A recent improvement imparts this twist to the conductor in the stranding machine immediately after shaping, so that the core papers are not distorted or creased during the process of laying up, and all internal mechanical stresses are removed from the cable.

The method of impregnation described above is the one generally used in this country, but the prior-impregnation process developed by Beaver is also of importance. In this process the paper is impregnated in sheet form before cutting into tapes, thus securing absolute uniformity in the treatment of the dielectric, and permitting the use of a compound which is of high viscosity at the maximum working temperature of the cable. Due to this feature and the use of non-fibrous fillers, long vertical runs of cable are possible, which is not the case with cables impregnated by the normal method owing to compound migration, and the development of excessive hydrostatic pressure at the lower end of the run. A notable example is an installation of 22 000-volt, three-core cables at the Crown Mines, South Africa, the total vertical run in this case being 4 740 ft.[4]

Dimensions and voltage tests for paper-insulated cables up to and including pressures of 22 000 volts have now been standardised by the British Standards Institution.[5]

III. Capacitance and Insulation Resistance of Single-core Cables.—In the transmission of power by single-core, lead-sheathed cables the three cables forming a three-phase system are laid close together in the ground or in multi-way ducts, the lead sheaths being bonded together and earthed at one or more points. The three sheaths are thus maintained at zero potential, and in determining the capacitance of a conductor it is only necessary to consider the dielectric field existing between the conductor and sheath. Let Fig. 57 represent a section of a single-core cable having a conductor radius of r centimetres, and an internal sheath radius of R centimetres. If a charge of q units per centimetre length is given to the

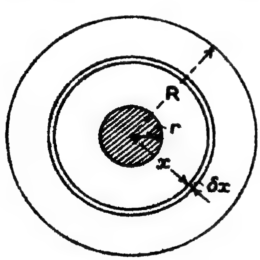


FIG. 57. — Capacitance and insulation resistance of a single-core cable.

conductor, lines of dielectric flux pass out radially, and produce at a distance x centimetres from the axis of the cable a flux density of

$$D = \frac{4\pi q}{2\pi x}$$

$$= \frac{2q}{x} \text{ lines per square centimetre.}$$

If the permittivity of the insulating material is ϵ, the electrostatic force at the point is

$$F = \frac{D}{\epsilon} = \frac{2q}{\epsilon x} \text{ dynes,} \quad . \quad . \quad . \quad (133)$$

and the potential difference between the conductor and sheath is

$$E \quad \int_r^R \frac{2q}{\epsilon} \frac{dx}{x}$$

$$= \frac{2q}{\epsilon} \log_e \frac{R}{r} \text{ ergs.} \quad . \quad . \quad . \quad (134)$$

The capacitance per centimetre length of the cable is

$$C = \frac{q}{E} = \frac{q}{\frac{2q}{\epsilon} \log_e \frac{R}{r}}$$

$$= \frac{\epsilon}{2 \log_e \frac{R}{r}} \text{ electrostatic units,}$$

or expressed in terms of practical units,

$$C_0 = \frac{\epsilon \times 2{\cdot}54 \times 12 \times 5\,280}{2 \times 2{\cdot}303 \log_{10} \frac{R}{r} \times 9 \times 10^{11}}$$

$$= \frac{0{\cdot}0388\epsilon}{\log_{10} \frac{R}{r}} \text{ microfarads per mile.} \quad . \quad . \quad . \quad (135)$$

Considering again a 1 centimetre length of cable, the insulation resistance of a thin layer at a radial distance x centimetres from the cable axis is

$$dR' = \frac{\rho dx}{2\pi x},$$

where dx is the thickness of the layor, and ρ its volume resistivity.

Hence the insulation resistance per centimetre length of cable is

$$R' = \frac{\rho}{2\pi}\int_r^R \frac{dx}{x}$$
$$= \frac{\rho}{2\pi}\log_e \frac{R}{r},$$

or expressed in mile units,

$$R' = 2{\cdot}28\rho \log_{10}\frac{R}{r} \times 10^{-12} \quad \text{megohms per mile.} \qquad (136)$$

Average values of ϵ and ρ for impregnated paper insulation are

$$\epsilon = 3{\cdot}3 \text{ to } 3{\cdot}8,$$
$$\rho = 5 - 8 \times 10^{14} \quad \text{ohms},$$

these figures being taken at about 15° C.

Temperature has a comparatively small effect on the permittivity of impregnated paper, but the resistivity decreases rapidly with increase of temperature. This variation in resistivity may be represented closely by the expression

$$\rho_t = \rho_0 e^{-at},$$

where the coefficient a has a value of about 0·04 or 0·05.

112. Sheath Phenomena in Single-core Cables.—In the case of power transmission by single-core cables, the presence of a separate lead sheath around each conductor has a considerable influence on the electrical characteristics of the circuit. In such a system of cables the electromagnetic fields of currents flowing through the conductors induce e.m.fs. in the sheaths, and under certain conditions heavy currents may be set up therein. The actual current flowing along the sheath depends, amongst other things, on the magnitude and frequency of the current in the conductor, the arrangement and spacing of the cables, the sheath resistance, and whether the sheaths are bonded or unbonded.

Following the classification of Cramp and Calderwood,[6] the induced sheath currents may be divided into two kinds :—

1. Currents whose outward and return paths lie entirely in the sheath of one cable (sheath eddies).

2. Currents whose outward and return paths are formed by the sheaths of separate cables (sheath-circuit eddies).

If the lead sheaths of the cables are open-circuited, either by being entirely insulated from each other or by being interconnected only at one end of the run, then the sheath-circuit eddies cannot exist. The sheath eddies, however, are able to circulate, and in

addition voltages are induced in the sheaths. If the sheaths are short-circuited by bonding together at each end of the run, these bonds are in effect a short-circuit of the sheath e.m.fs. In this case, both sheath eddies and sheath-circuit eddies are able to flow.

In order to fix ideas consider in what follows that the three cables forming a three-phase circuit are arranged symmetrically in triangular formation, that is with each cable occupying the edge of an equilateral prism.

Lead Sheaths Open-circuited at One or Both Ends.—In this case the only currents which can flow in the sheaths are sheath eddies which are produced by the proximity of the return conductors, *i.e.* the current in the sheath of any one cable is induced by the electro-magnetic fields of the currents flowing in the conductors of the other two. For example, the sheath of cable A is cut by the resultant of the magnetic fluxes due to the conductor currents of cables B and C, and since the intensity of this flux varies over the cross-section of the sheath there is a similar variation in the induced e.m.f. Hence, equalising currents are set up which pass along one side of the sheath and return along the other, the integral of these sheath currents over the cross-section of the sheath being zero. The magnitude of the sheath eddies and the power loss due to them is greatest when the sheaths are close together, as in this case there is the greatest variation in the flux density over the cross-section of the sheaths. As the interaxial spacing of the cables is increased, the magnitude of the effect rapidly diminishes.

The following formula given by Arnold[7] enables the sheath losses to be estimated fairly closely:—

$$\text{Sheath losses} = I^2\left\{\frac{78\omega^2}{R_s}\left(\frac{R_m}{d}\right)^2 \times 10^{-9}\right\}\text{watts per phase,} \quad (137)$$

for the particular length of run to which R_s applies.

Also the ratio

$$\frac{\text{Sheath losses}}{\text{Conductor losses}} \quad \frac{78\omega^2}{RR_s}\left(\frac{R_m}{d}\right)^2 \times 10^{-9}, \quad . \quad (138)$$

where I = current per conductor,

$\omega = 2\pi \times$ frequency,

R = conductor resistance in ohms,

R_s = sheath resistance in ohms,

d = interaxial spacing

and R_m = mean radius of sheath $\Big\}$ in same units.

At a frequency of 50-cycles, $78\omega^2 \times 10^{-9} = 0{\cdot}0077$, so that for this particular case

$$\text{Sheath losses} = \frac{0{\cdot}0077\, I^2}{R_s}\left(\frac{R_m}{d}\right)^2 \text{ watts per phase,} \qquad (139)$$

and

$$\frac{\text{Sheath losses}}{\text{Conductor losses}} = \frac{0{\cdot}0077}{RR_s}\left(\frac{R_m}{d}\right)^2. \quad . \quad . \quad (140)$$

Under the worst circumstances, *i.e.* with the sheaths in contact, the power loss in open-circuited sheaths is quite small and can generally be neglected. For example, with a three-phase system of 66 000-volt, 0·40 sq. in. cables the open-circuited sheath loss will not be greater than 2 per cent. of the conductor loss.

More important in practice than the phenomenon described above is the voltage induced in the open-circuited sheaths by transformer action between the conductors and sheaths. The value of this induced voltage depends on the flux interlinked with the sheaths, and so increases as the interaxial spacing of the cables is increased.

Numerically, the voltage induced in each sheath

$$e_{sh} = IX_m, \quad . \quad . \quad . \quad . \quad (141)$$

where I = current per conductor,

and $X_m = \omega M$ = mutual reactance between conductor and sheath.

The value of M, the mutual inductance between conductor and sheath, can be determined with sufficient accuracy by an analogous method to that used for calculating the self-inductance of the conductor, except that instead of integrating the flux between the limits d and r (where d is the interaxial spacing and r the conductor radius—Art. 13), it is integrated between d and R_m, where R_m is the mean radius of the sheath. From analogy, therefore, with formula (9), the mutual inductance between any one conductor and sheath can be written down as

$$M_o = 0{\cdot}741 \log_{10}\frac{d}{R_m} \text{ millihenries per mile.} \qquad (142)$$

For the particular case of cables laid touching each other the value of $\frac{d}{R_m}$ varies within comparatively narrow limits, so that the induced voltage per mile of cable is approximately constant, and as a rough guide may be taken as about 9 volts per mile for 100

amperes current in the conductor. This is, of course, the voltage to neutral, the voltage between sheaths being $\sqrt{3}$ times this value or about 15 volts per mile. It will be seen, therefore, that in the case of a long run of cable, and particularly when heavy currents are being transmitted, the normal voltage existing between sheaths may reach dangerously high values. Furthermore, in case of short-circuit occurring on some part of the system when the current flowing may be of the order of fifty times its normal value, arcing may be caused between sheaths resulting in serious damage.

Lead Sheaths Short-circuited at Each End.—It is usually considered the best practice to suppress the induced sheath voltages by bonding the sheaths together at each end of the run, and accept the extra power losses due to the sheath-circuit eddies which are now able to flow. Unlike the sheath eddy losses, these losses decrease as the cable spacing is decreased, so they can usually be kept to a reasonable magnitude by laying the cables as close to one another as possible.

The reactance of the sheath circuit is numerically equal for all practical purposes to X_m, the mutual reactance between conductor and sheath, so that the sheath-circuit impedance is $\sqrt{R_s^2 + X_m^2}$, and the actual sheath current flowing with short-circuited sheaths is

$$I_{sh} = \frac{IX_m}{\sqrt{R_s^2 + X_m^2}} \qquad (143)$$

Both the numerator and denominator of this formula are proportional to the length of the run, so that cross-bonding at jointing positions, or the practice of running plain lead sheathed cables together in metallic cleats, does not sensibly affect the value of the sheath current.

It should be noticed that the sheath eddy current is superimposed on the sheath-circuit eddy current I_{sh}, so that the actual current flowing in the sheath when short-circuited is not uniformly distributed around the sheath circumference. The effect is to increase the current density in those parts of the sheaths which are nearest together, but the un-uniformity is small and the consequent alteration in the sheath losses can be ignored.

From an electrical point of view the effect of currents in the sheath can be most conveniently treated mathematically by assuming that there is an additional increase in the conductor resistance of such a value that when multiplied by the square of the current

flowing through the conductor the losses in the sheath are obtained The reactance of the conductor, on the other hand, is decreased, but the change is not so important as that in the conductor resistance.

To determine the magnitude of the effect, reference should be made to Fig. 58 which represents diagrammatically a transmission circuit composed of single-core cables. The arrangement is equivalent to a 1/1 ratio current transformer with zero reactance in the secondary, this secondary being formed by the lead sheath of resistance R_s. The primary of the transformer has the same resistance R as the conductor, and the leakage reactance X_c is due to the flux between the lead sheath and the conductor and is therefore quite small. The primary admittance of this equivalent transformer is $\frac{1}{X_m}$, where X_m is the mutual reactance between conductor and sheath.

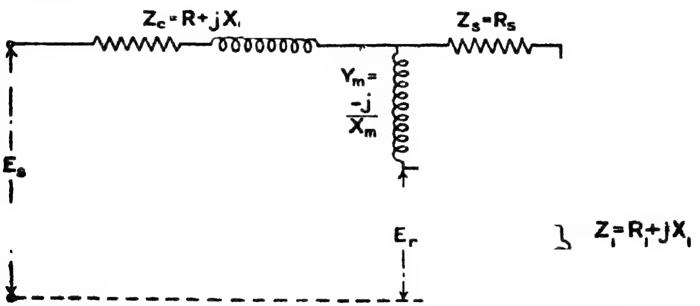


Fig. 58.—Single-core, lead-covered cable represented as an inductive circuit.

Let E_s = impressed voltage from conductor to neutral,
$Z_1 = R_1 + jX_1$ = impedance of load.

The joint admittance of the two parallel circuits of impedance X_m and R_s respectively is $\frac{-j}{X_m} + \frac{1}{R_s}$, and the corresponding impedance

$$= \frac{1}{\frac{-j}{X_m} + \frac{1}{R_s}}$$

$$= \frac{X_m R_s}{-jR_s + X_m}$$

$$= \frac{X_m R_s (X_m + jR_s)}{R_s^2 + X_m^2}.$$

The total line impedance

$$
\begin{aligned}
Z_{\text{eff}} &= Z_c + Z_2 \\
&= R + jX_c + \frac{X_m R_s (X_m + jR_s)}{R_s^2 + X_m^2} \\
&= R + \frac{X_m^2 R_s}{R_s^2 + X_m^2} + j\left(X_c + \frac{X_m R_s^2}{R_s^2 + X_m^2}\right). \qquad (144)
\end{aligned}
$$

Hence, with short-circuited sheath, the resistance of the conductor is increased from R to the effective value

$$R_{\text{eff}} = R + \frac{X_m^2 R_s}{R_s^2 + X_m^2}. \qquad (145)$$

That is to say, this value of resistance, when multiplied by the square of the current flowing in the conductor, gives the total power loss per phase in both the conductor and the sheath.

The losses in the sheath itself are

$$\frac{I^2 X_m^2 R_s}{R_s^2 + X_m^2} \text{ watts per phase,} \qquad (146)$$

for the particular length of run for which the values of R_s and X_m apply, and the ratio

$$\frac{\text{Sheath losses}}{\text{Conductor losses}} = \frac{X_m^2 R_s}{R(R_s^2 + X_m^2)}. \qquad (147)$$

Also, due to the mutual inductive effect of the sheath, the conductor reactance is somewhat decreased. For with open-circuited sheath $R_s = \infty$, and the reactance has its normal value

$$X = X_c + X_m, \qquad (148)$$

whereas with short-circuited sheath the effective value is

$$X_{\text{eff}} = X_c + \frac{X_m R_s^2}{R_s^2 + X_m^2}, \qquad (149)$$

and substituting for X_c from (148) gives

$$X_{\text{eff}} = X - \frac{X_m^3}{R_s^2 + X_m^2}. \qquad (150)$$

To illustrate the influence of the sheath, the effective resistance and inductance, and also the sheath losses, have been calculated for a system of 66 000-volt, 0·40 sq. in., single-core cables laid in triangular formation at various spacings. The results, both with open-circuited and short-circuited sheaths, are shown in Table 12.

Taking the normal full-load current of these cables as 350

TABLE 12.—*Sheath Losses and Effective Resistance and Reactance per Mile of 0·40 sq. in., Single-core Cable.*

$r = 0\cdot418$ in. Temp. of conductor = 45° C.
$R = 1\cdot08$ " Temp. of sheath = 30° C.
$R_1 = 1\cdot23$ " $\rho_s = 23\cdot2 \times 10^{-6}$ ohms.
$R_2 = 1\cdot38$ " Frequency = 50 cycles.

Interaxial Spacing.	Sheath Open-circuited.			Sheath Short-circuited.		
	Resistance, R.	Reactance, $X = X_c + X_m$.	Voltage Induced per Mile, IX_m ($I = 350$ amperes).	Resistance, $R + \frac{X_m^2 R_s}{R_s^2 + X_m^2}$.	Reactance, $X - \frac{X_m^3}{R_s^2 + X_m^2}$	Increase in Power Losses Due to Sheath.
2·77 in.	0·116 ohms	0·274 ohms	30·8 volts	0·130 ohms	0·272 ohms	12 %
6 "	0·116 "	0·352 "	58·1 "	0·163 "	0·337 "	41 "
12 "	0·116 "	0·422 "	83·6 "	0·206 "	0·382 "	77 "
24 "	0·116 "	0·492 "	107·0 "	0·248 "	0·416 "	114 "
60 "	0·116 "	0·584 "	139·0 "	0·303 "	0·440 "	165 "

amperes, the induced voltage to neutral with open-circuited sheaths would be 30·8 volts per mile when the cables are touching each other. The sheath current for the case of short-circuited sheaths would be 57 amperes, corresponding to an increased power loss of 12 %. At wider spacings all these figures are much increased.

With cables having large conductor sections the power losses in short-circuited sheaths assume serious proportions. At normal supply frequencies and with cables laid fairly close together as in practice, X_m^2 is small compared with R_s^2. Hence, from (14

$$\frac{\text{Sheath losses}}{\text{Conductor losses}} = \frac{X_m^2}{RR_s} \text{ (approximately).} \qquad (151)$$

Now as the conductor section increases in size, both R and R_s decrease, whilst X_m remains sensibly constant. Hence, the ratio of sheath losses to conductor losses increases rapidly, the condition being eventually reached where the sheath losses are actually greater than the conductor losses. For example, the sheath losses on a system of 66 000-volt, single-core cables spaced at 6-in. centres are greater than the losses in the conductors for all sizes of conductor above 0·85 sq. in.

The problem of heavy sheath losses has not arisen to any serious extent in this country, as conductor sections are usually of moderate size, and cables are laid closely together in the ground. In America, however, where large conductor sections and wider spacings are employed, due to the installation of cables in ducts, the combined influence of these factors and the higher supply frequency has resulted in the development of special methods of cross-bonding at each jointing position. These methods eliminate the sheath currents, and limit the induced voltage in the sheaths to a value equal to, or in some cases rather less than, that corresponding to the length of cable between joints. At the same time they do not prevent the return of fault current along the sheaths which would be the case if the sheaths were simply left open-circuited at each joint.[8]

So far the cables have been assumed to be arranged symmetrically, but in practice the three cables forming a three-phase system may be arranged in the same plane. In this case, there is a great difference between the sheath losses of the middle and each of the outer cables, the ratio when the cables are laid close together being about 1 to 4. Furthermore, there is a transfer of power

from one phase to another, which results in an unbalancing of the impedance drop in the system. Transposition of the cables is highly desirable, and if this is carried out at each jointing position —the sheaths being cross-bonded only at every third joint—equalisation of the sheath currents is effected, and the losses in each cable sheath can be calculated as if the three cables were equidistant at the geometric mean spacing, 1·26 d.

It should be noted that although the impedance out-of-balance is eliminated with the flat transposed arrangement of cables the total sheath losses will be higher, being in the case of close spacing about twice the sheath losses for the symmetrical arrangement. The symmetrical, triangular arrangement of cables with transposition is thus to be preferred wherever possible, and this is also the best arrangement for reducing to a minimum the inductive interference with neighbouring pilot or telephone cables.[9]

113. Electrostatic Stresses in Single-core Cables.—A single-core cable can be represented as in Fig. 57 by concentric cylinders, the radius r of the inner cylinder corresponding to the conductor radius, and the internal radius R of the outer cylinder corresponding to the internal radius of the lead sheath. Assume that the potential of the inner cylinder is E, and that of the outer one is zero. Then the electrostatic force or potential gradient is radial in direction, and at radius x is

$$g = \frac{2q}{\epsilon x}, \qquad . \quad . \quad . \quad . \quad . \qquad (133)$$

and from (134)

$$q = \frac{E\epsilon}{2 \log_e \frac{R}{r}}.$$

Hence the potential gradient or electrostatic stress at radius x is

$$g = \frac{E}{x \log_e \frac{R}{r}}. \qquad . \quad . \quad . \quad . \qquad (152)$$

It has its maximum value at the surface of the inner cylinder

$$g_{max} = \frac{E}{r \log_e \frac{R}{r}}, \qquad . \quad . \quad . \quad . \qquad (153)$$

and its minimum value at the surface of the outer cylinder

$$g_{min} = \frac{E}{R \log_e \frac{R}{r}}. \qquad . \quad . \quad . \quad . \qquad (154)$$

so that

$$\frac{g_{max}}{g_{min}} = \frac{R}{r}. \qquad . \quad . \quad . \quad . \quad . \quad (155)$$

It should be noticed that formula (153) for the stress at the conductor surface is obtained on the assumption of a smooth cylindrical conductor, and thus only supplies a nominal value for this stress. With the ordinary stranded conductors in common use the stresses near the conductor are increased due to the greater curvature of the surface of the individual wires. Deutsch[10] has obtained a stress formula for stranded conductors, which appears to be in substantial agreement with figures given by Atkinson[11] and other investigators, showing that the effect of stranding in a typical case is to increase the maximum stress on the dielectric by about 20 °/ₒ.

For a given voltage and overall diameter of single-core cable there is one particular radius of conductor which produces the least stress at the conductor surface. A small conductor will allow of a greater thickness of insulation, but on the other hand, the smaller radius of curvature tends to increase the stress; while the effect of too large a radius of conductor is to cause an increase of the stress through reduction of the total thickness of insulation. The stress at the conductor surface is

$$g_{max} = \frac{E}{r \log_e \frac{R}{r}} \qquad . \quad . \quad . \quad . \quad (153)$$

and if E and R are constant this stress has a minimum value when

$$\log_e \frac{R}{r} = 1, \qquad . \quad . \quad . \quad . \quad . \quad (156)$$

or

$$R = 2{\cdot}718r.$$

in which case the insulation thickness

$$R - r = 1{\cdot}718r.$$

Hence the actual stress at the conductor surface

$$g_{max} = \frac{E}{r} \qquad (157)$$

For high-voltage cable the value of r determined by the above method would, in general, give a conductor of cross-section much larger than required from the point of view of current-carrying

capacity. In such a case aluminium might be used instead of copper as a conductor material, or if this does not bring the conductor radius up to the required limit, the conductor may be built up of copper wires stranded round a hemp centre.

A cable conductor constructed on the latter principle and in actual use on a 86 kV., three-phase circuit of single-core cables is shown in Fig. 59. The conductor is of jute, 0·75 in. diameter, wrapped with four layers of paper and surrounded by 41 copper wires, the total cross-sectional area of conductor being 0·132 sq. in. The tubular conductor is wrapped with a thin copper tape, the whole assembly being then impregnated with compound and covered with a lead sheath 0·06 in. thick, bringing the overall diameter up to 1·04 in. The dielectric thickness of the cable is 0·95 in. and the corresponding maximum stress 34 kV. per cm. If an ordinary conductor had been used and the same thickness of dielectric, the nominal maximum stress would have been 48 kV. per cm. This special construction of conductor has the further advantage of presenting a smooth cylindrical surface to the dielectric instead of the corrugated surface of the stranded conductor.

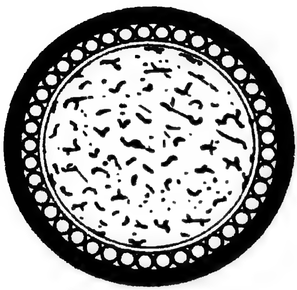


FIG. 59.—Special conductor construction for high-voltage, single-core cable.

In general, however, the theoretical relation between R and r for minimum stress is seldom observed in the design of high-voltage cable. Other factors besides stress values have to be taken into consideration, and it is customary to employ the same thickness of insulation for all sizes of conductor, the actual thickness of course depending on the operating voltage. Furthermore, g_{max} is sensibly constant at values of $\frac{R}{r}$ between 2 and 4, the stress variation in this range being only about 6 %.

114. Grading of Cables for Alternating Pressures.—It has been shown above that the electrostatic stress in a single-core cable decreases towards the sheath, its value at a distance x from the axis being

$$g = \frac{2q}{\epsilon x} \quad . \quad . \quad . \quad . \quad (133)$$

In a cable of homogeneous insulation, the voltage which may be safely applied is limited by the stress at the inner conductor, and if the dielectric is strong enough at this point it is too strong and needlessly expensive further away. If, however, by some means the stress can be so distributed that its value in the outer layers of the dielectric is increased without increasing the stress near the conductor, a less thickness of insulation will be required for any given operating voltage, and an important economy effected. This result may be achieved in practice by grading the cable in one of two ways:—

1. By forming the dielectric in layers of insulating material having different values of permittivity.

2. By using the same material for all layers, but inserting one or more metallic intersheaths in the dielectric concentric with the conductor. These intersheaths are not intended to carry any part of the working current but only to fix the potentials at different points in the insulation.

These methods are known as 'capacitance' grading, and 'intersheath' grading respectively.

Capacitance Grading.—It will be seen from an examination of formula (133), that if it were possible to vary the insulation of a cable so that its permittivity at every point was inversely proportional to the distance from the axis, the stress in the dielectric would be absolutely constant. This is the principle of the graded cable, first suggested by O'Gorman.[12] Such perfect grading is practically impossible, since the permittivities of materials which are suitable for cable insulation do not vary to a great extent. However, very good results can be obtained by using two or three layers of dielectric.

As an illustration, consider first of all the design of a 0·30 sq. in., non-graded cable for a pressure of 60 000 volts between conductor and lead. If the stress in the dielectric must not exceed 40 000 volts per centimetre, the radius of the conductor for a cable of minimum overall diameter would be

$$\frac{E}{g_{max}} = \frac{60\,000}{40\,000}$$
$$= 1{\cdot}5 \text{ cm.}$$

A normal stranded conductor of this radius would have a sectional area nearly three times as large as desired, and a special

conductor constructed on the lines of Fig. 59 might be adopted. The internal radius of the sheath would be

$$R = 2{\cdot}718 \times 1{\cdot}5$$
$$= 4{\cdot}08 \quad \text{cm.}$$

Consider now the case of a cable having the same overall dimensions, but graded by means of two layers of material having permittivities ϵ_1 and ϵ_2 respectively. In order to secure the same value of maximum stress in each layer

$$\frac{2q}{\epsilon_1 r} = \frac{2q}{\epsilon_2 r_1}, \quad . \quad . \quad . \quad . \quad (133)$$

or

$$\epsilon_1 r = \epsilon_2 r_1,$$

where r_1 is the internal radius of the second layer. A 0·30 sq in., stranded conductor has an overall radius of 0·915 centimetre, and assuming $\epsilon_1 = 5$ and $\epsilon_2 = 3$ it follows that

$$r_1 = \frac{0{\cdot}915 \times 5}{3}$$
$$= 1{\cdot}525 \quad \text{cm.}$$

The voltage across the inner layer of dielectric is

$$E_1 = g_{\max} r \log_e \frac{r_1}{r},$$

and the voltage across the outer layer is

$$E_2 = g_{\max} r_1 \log_e \frac{R}{r_1}.$$

Hence the total voltage which can be safely applied to the cable is

$$E = E_1 + E_2$$
$$= g_{\max}\left(r \log_e \frac{r_1}{r} + r_1 \log_e \frac{R}{r_1}\right). \quad . \quad . \quad (158)$$

Taking as before a maximum stress of 40 000 volts per centimetre,

$$E = 40\,000\left(0{\cdot}915 \log_e \frac{1{\cdot}525}{0{\cdot}915} + 1{\cdot}525 \log_e \frac{4{\cdot}08}{1{\cdot}525}\right)$$
$$= 40\,000(0{\cdot}4674 + 1{\cdot}5008)$$
$$= 78\,730 \quad \text{volts.}$$

Hence by grading the dielectric in two layers, and without increasing the overall dimensions of the cable, the safe operating

voltage has been raised from 60 000 volts to 78 730 volts, being an increase of about 31 °/。 Figs. 60 (*a*) and (*b*) show diagrammatically the two cables and the stresses in each case.

In the above discussion, the insulating materials have been assumed to have the same dielectric strength. If this is not so, the maximum stress in each layer should be made proportional to the dielectric strength of that layer, thus leading to a uniform factor of safety for the complete cable. Hence, in general, if ν_1, ν_2, etc., are the dielectric strengths of the different layers, the radii of the layers can be calculated from the relationship

$$\nu_1\epsilon_1 r = \nu_2\epsilon_2 r_1 = \nu_3\epsilon_3 r_2. \quad . \quad . \quad . \quad (159)$$

Intersheath Grading.—In this method of grading, suggested by Morris,[13] the same material is used for insulation throughout the total thickness of the cable. but is divided into two or more sections

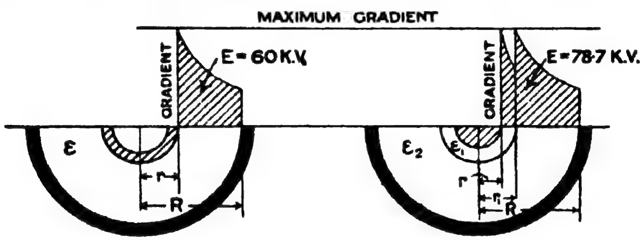


FIG. 60. –Ungraded and capacitance-graded cable.

by means of metallic cylinders. Each section can be made to take its proper share of the total voltage by applying a definite potential from an outside source to the intersheaths. The intersheaths can be made of lead tubes as this provides a smooth surface in contact with the dielectric, and the sole current carried by any intersheath is the difference between the charging currents taken by the sections on each side of it. The most obvious way of anchoring the potentials of the various intersheaths is by connecting them to tappings taken from the secondary windings of the supply transformers.

As an illustration of the method, suppose that the 0·30 sq. in. cable (r = 0·915 centimetre, R = 4·08 centimetres) is to be graded by means of one intersheath. If the lead intersheath is 0·125 centimetre in thickness, and is situated half-way between conductor and sheath, its internal and external radii will be r_1 = 2·435

centimetres, and $r_2 = 2\cdot 560$ centimetres respectively. The voltage across the first section is

$$E_1 = g_{max}\left(r \log_e \frac{r_1}{r}\right),$$

and the voltage across the second section is

$$E_2 \qquad {}_{x}\left(r_2 \log_e \frac{R}{r_2}\right).$$

Assuming as before a maximum stress of 40 000 volts per centimetre, and substituting numerical values

$$\begin{aligned} E &= E_1 + E_2 \\ &= 40\,000\left(0\cdot 915 \log_e \frac{2\cdot 435}{0\cdot 915} + 2\cdot 560 \log_e \frac{4\cdot 08}{2\cdot 560}\right) \\ &= 83\,550 \quad \text{volts.} \end{aligned}$$

Hence by means of one intersheath, without increasing the overall dimensions of the cable, its voltage rating has been increased by about 39 °/ₒ (see Fig. 61).

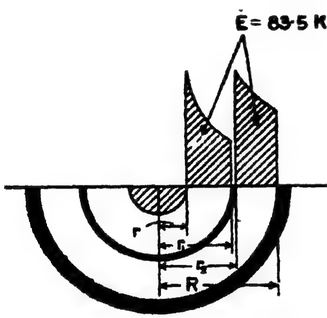


FIG. 61.—Intersheath-graded cable.

Where long lengths of cable have to be dealt with, the cross-sectional area of the intersheaths must be ample to carry the charging current of the system. Let the capacitance of the condenser formed by the conductor and the intersheath be c_1 farads, and the capacitance between the intersheath and the outer sheath of the cable be c_2 farads. Then, considering the inner condenser, a capacitance current of $\omega c_1 E_1$ amperes flows out along the conductor and returns by the intersheath. Similarly, for the outer condenser, a capacitance current $\omega c_2 E_2$ flows out along the intersheath and flows back along the outer sheath. Hence the actual current carried by the sheath is

$$\omega(c_2E_2 - c_1E_1); \tag{160}$$

this is of course the sending-end value. At the receiving end the capacitance current is zero, and at a point distant x from the

sending end of a cable of length l the intersheath current is approximately

$$\omega(c_2E_2 - c_1E_1)\left(1 - \frac{x}{l}\right).$$

Referring to the 0·30 sq. in. graded cable

$$c_1 = \frac{0\cdot 0388 \times 3\cdot 8}{\log_{10}\frac{2\cdot 435}{0\cdot 915}} \times 10^{-6}$$

$$= 0\cdot 3469 \times 10^{-6} \quad \text{farads per mile,}$$

$$c_2 = \frac{0\cdot 0388 \times 3\cdot 8}{\log_{10}\frac{4\cdot 08}{2\cdot 560}} \times 10^{-6}$$

$$= 0\cdot 7284 \times 10^{-6} \quad \text{farads per mile.}$$

Hence for a 20-mile, 50-frequency transmission, the capacitance current in the intersheath at the sending end is

$$2\pi \times 50(47\,730 \times 0\cdot 7284 - 35\,820 \times 0\cdot 3469)10^{-6} \times 20 \qquad (160)$$
$$= 140 \text{ amperes.}$$

115. Practical Difficulties Involved in Grading.—Although from a theoretical standpoint there are considerable advantages to be gained by grading, it has not yet met with any success in practice. This is chiefly due to the development of new types of cable, and improvements in the quality of insulating materials resulting in the adoption of higher working stresses—values up to 140 kV. per cm. now being used. Furthermore, there are difficulties in the practical application of methods of grading.

The chief handicap of the capacitance method of grading is due to the fact that there are such few high-grade insulating materials of reasonable cost, whose permittivities can be made to vary over the necessary range. Impregnated paper could be given values of permittivity ranging from about 3·0 to 4·0. Higher values than this might be obtained by loading rubber with various chemicals as, according to Jona,[14] its permittivity can thereby be varied from 4·2 to 6·1. The cost of such a rubber dielectric, however, would be high, and probably prohibitive for this reason. Furthermore, in a composite dielectric of rubber and paper, beside, the risk of chemical action taking place between the two layers, there is the possibility that the individual materials would not retain their permittivities unchanged from year to year. If such a progressive increase or diminution in the values of these constants took place, the grading of the cable

would be spoiled, and breakdown might occur at a lower pressure than with the ungraded cable. For these reasons it seems distinctly preferable to have the insulation a homogeneous, compact mass of impregnated paper.

The chief disadvantage of the intersheath method of grading is the relatively flimsy character of the intersheath, and the danger of damaging or interfering with its continuity during transportation and installation. High local potentials might then concentrate at the defective place, resulting in failure. Furthermore, failure of the dielectric on one side of an intersheath would lead to an immediate concentration of stress on the dielectric on the other side, with the risk of simultaneous breakdowns at other points along the cable. The length of transmission is also unduly limited by the safe current-carrying capacity of the intersheaths

116. Capacitance of Three-core, Belted-type Cables.—In overhead systems the capacitance to earth is generally negligible,

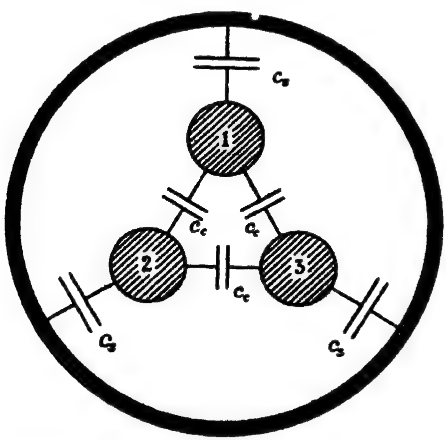


FIG. 62.—Representation of capacitances in three-core, belted-type cable by condensers.

but in an underground transmission by multicore cables the condensers formed by the comparatively small thickness of insulation between conductors and earth must be reckoned with. Russell[15] has shown that in a three-core cable nine different capacitances can be obtained, corresponding to various combinations of the three conductors and the sheath. It is sufficient here, however, to

imagine that the three conductors and sheath form an arrangement of condensers as shown in Fig. 62. The capacitance of each of the condensers connected between the conductors and sheath is c_s, and the capacitance of the condensers connecting the conductors to one another is c_e. It is apparent that the capacitance c_s is to earth or neutral. Also the mesh-connected capacitances c_e may be converted to equivalent star-connected capacitances by multiplying by three. Therefore the effective capacitance of each conductor to neutral is

$$C = c_s + 3c_e. \qquad . \qquad . \qquad . \qquad . \qquad (161)$$

Owing to variation in the permittivity of impregnated paper and the more general use of special shapes of conductor, it is preferable to obtain capacitance figures from test data. In the routine testing of cables the standard measurement of capacitance is between one conductor (1) and the other two conductors and sheath (2, 3, and S). In addition, the value of the capacitance between the three conductors connected together (1, 2, 3) and the sheath (S) is often measured. Denoting these measured capacitances by (a) and (b) respectively, the following relations exist :—

$$\text{Capacitance } (a) = c_s + 2c_e$$

and

$$\text{Capacitance } (b) = 3c_s.$$

Hence

$$c_e = \frac{\text{Cap. } (a)}{2} - \frac{\text{Cap. } (b)}{6}$$

and

$$c_s = \frac{\text{Cap. } (b)}{3}$$

so that the effective capacitance of one conductor to neutral, in terms of the measured values is

$$C = \frac{9 \times \text{Cap. } (a) - \text{Cap. } (b)}{6}. \qquad . \qquad . \qquad (162)$$

If capacitance (b) is not available, the capacitance to neutral can be obtained to a close approximation by taking C as equal to 1·2 times capacitance (a).

In the absence of test figures altogether, the following empirical formula due to Simons,[16] and derived from experimental work on cable models (see Art. 117), can be employed :—

$$\frac{0{\cdot}048\epsilon}{\log_{10}\left[\left\{0{\cdot}52\left(\frac{t}{T}\right)^2 - 1{\cdot}70\frac{t}{T} + 3{\cdot}84\right\}\frac{T+t}{d} + 1\right]} \quad . \quad (163)$$

microfarads per mile,

where t = belt-insulation thickness,
T = conductor-insulation thickness,
and d = conductor diameter,

all in the same units. The extra length due to coring is provided for in this formula.

An analogous formula to (163) can also be derived for the insulation resistance of a three-core cable, but is not of much practical use, as the power losses due to leakage with three-phase voltage are negligible compared with other sources of dielectric losses.

117. Electrostatic Stresses in Three-core, Belted-type Cables.—The electrostatic stress, at any point in the dielectric of

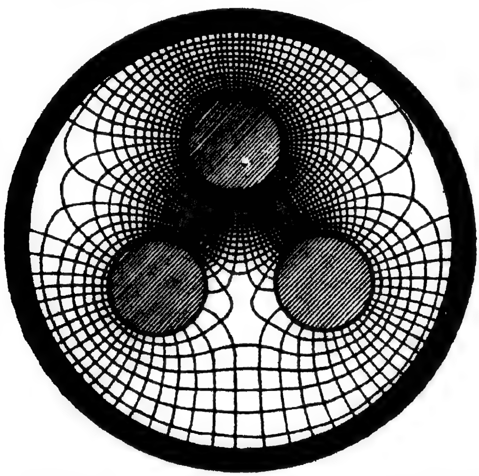


FIG. 63.—Electrostatic field of three-core, belted-type cable at a particular instant.

a single-core cable, can be calculated in terms of the applied voltage and the cable dimensions by a simple mathematical formula. In the case of a three-core cable, the magnitude of the dielectric stresses cannot, in general, be determined in this manner, and it is necessary to fall back on experimental data. This data has been chiefly obtained by measurements on cable models consisting of three metal tubes representing the three conductors surrounded by a fourth tube representing the sheath, the whole arrangement being immersed in a tank of electrolyte. Both Atkinson[17] and Emanueli[18] have used this method, and the former has also employed tinfoil models consisting of sheets of tinfoil on which were soldered four

copper rings to represent the conductors and sheath. When the three inner electrodes of either type of model are connected to a three-phase supply, the lines of current flow correspond exactly to the distribution of dielectric flux in a cable having a geometrically similar cross-section. Hence, the electrostatic stresses at any point in the cable can be found by measurements of the potential gradient existing at the corresponding point in the model.

Of course, in a three-core cable the electrostatic field is polyphase, and rotates at uniform angular velocity round about the geometric centre of the cable, in a manner analogous to the rotation of the magnetic field in an induction motor. Fig. 63 shows the stress distribution and equipotential lines in a typical cable at the particular instant when the top conductor is at its maximum potential.

In general, the stresses in a multicore cable depend upon the absolute dimensions of the cable and upon the voltage applied, but it is evident that the stresses will be the same in cables which are geometrically similar, if the applied voltages are proportional to the sizes of the cables. In other words, the ratio of maximum to average stress is a constant for cables which are geometrically similar. The highest stresses occur in the triangular section of the cable formed by joining the centres of the conductors, and it has been found that their magnitude is independent of the belt-insulation thickness. These stresses vary according to the ratio of conductor-insulation thickness to conductor diameter, and may be completely determined if the ratio of actual stress to average is plotted as a function of the ratio of conductor-insulation thickness to conductor diameter.

The stress of greatest importance is the maximum stress in the cable dielectric (*A* in Fig. 64), and this occurs at the conductor surface at the point nearest to the centre of the cable. Another stress of interest is that at the conductor surface on the line joining conductor centres (*B* in Fig. 64). With the usual thicknesses of cable insulation this stress is very nearly as great as the maximum stress, and, furthermore, can be calculated by means of a formula due to Russell.[19]

These two stresses and various others at representative points in the dielectric are best expressed in terms of the average stress between conductors, denoting by this latter quantity the numerical average obtained by dividing the voltage between conductors by the total intermediate thickness of insulation. The ratio of actual stress

to average can then be plotted as a function of the ratio of conductor-insulation thickness to conductor diameter, and the results obtained are reproduced in Fig. 64. In this diagram stress B has been calculated by Russell's formula, and the remaining curves are the experimental results obtained by Atkinson. By the use of these curves the determination of the stresses in three-core cables is made as simple as in the corresponding single-core case. It should be

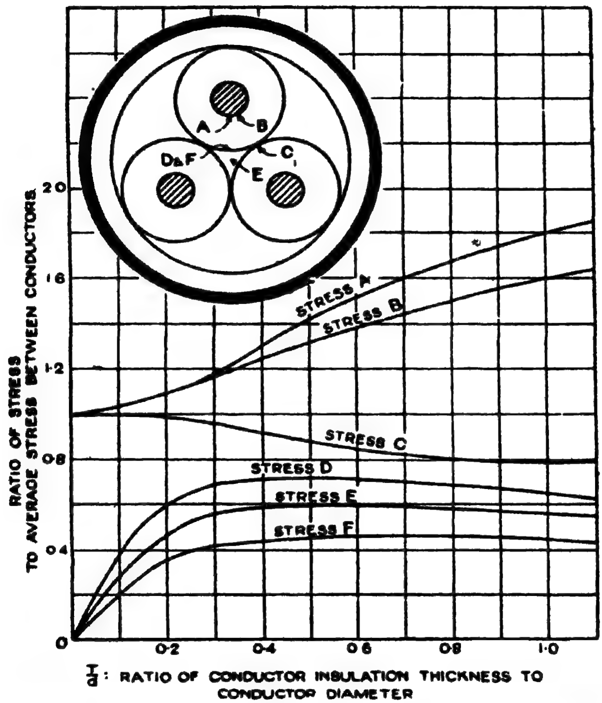


FIG. 64.—Electrostatic stresses in three-core, belted-type cables.

noted that all the stresses shown in Fig. 64 are radial to the conductor surface with the exception of stress F, which is a tangential stress measured along the circular boundary of the conductor insulation.

In addition to the maximum stress and various other stresses in the centre triangle of the cable, the stress at the conductor surface and toward the sheath is also of interest. Its magnitude depends very largely upon the thickness of the belt insulation. In the

of a cable having a thin belt, the stress towards the sheath can be made the same as the maximum stress, and in these circumstances the stresses around the entire circumference of the conductor are almost uniform. If the cable, as is the usual practice with high-voltage cable, has a belt-insulation thickness of the same order as that of the conductor insulation, the stress towards the sheath will be considerably less than the maximum. Here again Russell has given a method of calculating the stress, but there are several variables in the formula, and an exact solution for all conditions would involve an extensive series of curves. It is possible,

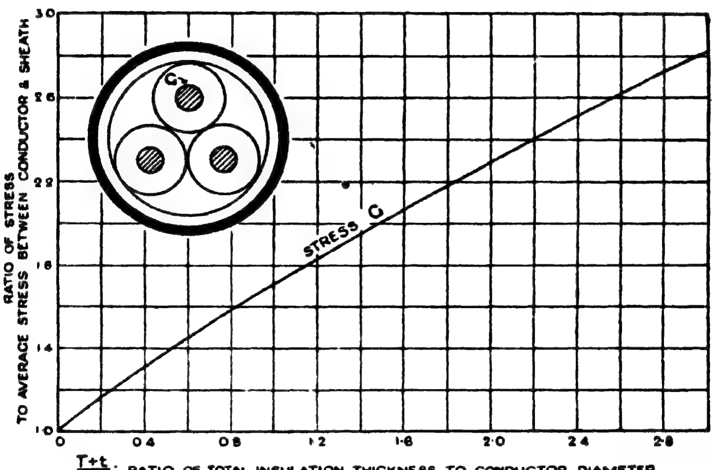


FIG. 65.—Electrostatic stress in three-core, belted-type cables at conductor surface and towards sheath.

however, by taking somewhat similar co-ordinates to those previously employed, to plot one curve shown in Fig. 65 which is correct within about 1 °/₀ for any point. It should be noted in connection with Fig. 65 that the stress is a function, not only of the conductor insulation, but also of the belt insulation, and it must therefore be expressed not in terms of the average stress between conductors, but of the average stress between conductor and sheath. Similarly, as abscissæ must be taken the ratio of total-insulation thickness to conductor diameter.

To illustrate the use of the preceding data, the electrostatic stresses will be calculated for the case of a 33 000-volt, three-core,

belted-type, 0·20 sq. in. cable. The conductor diameter is 1·47 centimetres, the thickness of the conductor insulation is 0·635 centimetre, and the belt-insulation thickness is also 0·635 centimetre.

Hence for the purpose of determining the stresses in the inner triangle of the cable the average stress is

$$\frac{33}{1{\cdot}27} = 26{\cdot}0 \quad \text{kV. per cm.,}$$

and the ratio of conductor-insulation thickness to conductor diameter is

$$\frac{0{\cdot}635}{1{\cdot}47} = 0{\cdot}43.$$

With 0·43 as abscissæ, a series of factors can be read off from the curves of Fig. 64, which factors, when multiplied by the average stress 26·0, give the actual stresses at the various points in the cable.

The maximum stress in the cable is

$$\text{Stress } A = 1{\cdot}34 \times 26{\cdot}0 = 34{\cdot}8 \quad \text{kV. per cm,}$$

and the other values are

$$\begin{aligned} \text{Stress } B &= 1{\cdot}27 \times 26{\cdot}0 = 33{\cdot}0 \quad \text{kV. per cm.} \\ C &= 0{\cdot}91 \times 26{\cdot}0 = 23{\cdot}7 \quad \text{kV. ,, ,,} \\ D &= 0{\cdot}69 \times 26{\cdot}0 = 17{\cdot}9 \quad \text{kV. ,, ,,} \\ E &= 0{\cdot}58 \times 26{\cdot}0 = 15{\cdot}1 \quad \text{kV. ,, ,,} \\ F &= 0{\cdot}43 \times 26{\cdot}0 = 11{\cdot}2 \quad \text{kV. ,, ,,} \end{aligned}$$

The stress at the surface of the conductor in the direction of the sheath is similarly derived from Fig. 65.

The average stress in the total thickness of insulation between conductor and sheath is

$$\frac{33}{\sqrt{3} \times 1{\cdot}27} = 14{\cdot}9 \quad \text{kV. per cm.}$$

and the ratio of total-insulation thickness to conductor diameter is

$$\frac{1{\cdot}27}{1{\cdot}47} = 0{\cdot}86.$$

Then with 0·86 as abscissa, the corresponding ordinate is found as 1·62. Hence the stress at that point on the conductor surface which is nearest to the sheath is

$$\text{Stress } G = 1{\cdot}62 \times 14{\cdot}9 = 24{\cdot}1 \quad \text{kV. per cm.}$$

It must be borne in mind that the stress data given above are for the condition of homogeneous dielectric. If, in an actual cable, the permittivity of the filler material differs very much from that

relating to the main insulation, the stresses may be somewhat modified thereby. Also no account has been taken of the effect of stranding in increasing the stress at the surface of the individual wires in the conductor so that stresses A, B and G as obtained are nominal values only. This, however, does not detract from the value of the method as a means of comparing the electrostatic stresses in various designs of cable, or in various parts of the same cable.

118. Breakdown Voltage.—At normal supply frequency the breakdown strength of the dielectric in good quality, high-voltage cable is about 400 kV. per cm. Its value is greatly influenced, however, by the length of time during which the voltage is applied, the above figure being the short-time value, that is the figure obtained from tests where breakdown has occurred within the course of a few minutes. This time effect in insulation failure is, of course, a fairly general phenomenon, applying to many solid materials and not merely to impregnated paper forming part of the structure of a cable.

A typical curve showing the breakdown strength starts at a value of about 400 kV. per cm., falling fairly rapidly at first and later more slowly to an asymptotic value of 160 to 180 kV. per cm., which is usually reached after 50 to 100 hours. The lower curve shown in Fig. 66 is characteristic and can be represented by a formula of the type

$$\text{Breakdown voltage} = E_0\left(1 + \frac{a}{\sqrt[n]{t}}\right), \tag{164}$$

where E_0 is the voltage which the dielectric will withstand indefinitely, t is the time of application of voltage in hours, a and n are constants.

The values of a and n, however, are liable to considerable variation being dependent upon manufacturing details, test conditions and previous history of the sample. For example, the effect of heating cycles may lower the curve considerably if an excessive temperature range has been used. It is not surprising, therefore, that various investigators have obtained different inverse roots in the fundamental equation ranging up to the 20th root, so that this figure has apparently an appreciable range.

The most important point to be noted is that short-time values of breakdown voltage are not very definite, and cannot legitimately

be used for comparing different cable samples. It is quite possible for one material having a lower short-time breakdown voltage than another to be ultimately the stronger at operating voltages, hence comparison should be on the basis of the long-time or asymptotic values.

The effect of mechanical pressure on the breakdown voltage of impregnated paper is extremely important, and the two curves shown in Fig. 66 show the result of subjecting cables to an external pressure of 15 atmospheres per sq. in.[30] It will be seen that there is a substantial improvement in the breakdown voltage, and this

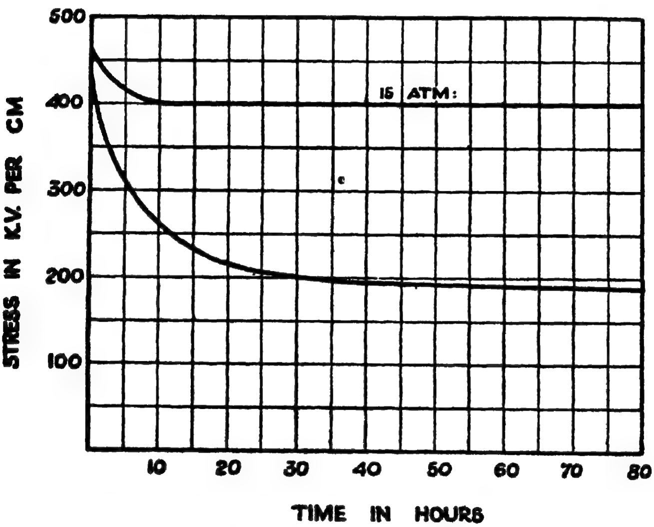


Fig. 66.—Breakdown stress of cable at normal atmospheric pressure, and when subjected to an external pressure of 15 atmospheres.

applies particularly to the long-time value which is increased by over 100 %.

119. Dielectric Losses.—The energy losses occurring in the dielectric of cables are due to leakage and so-called dielectric hysteresis. The former loss is the I^2R loss of the current passing by conduction through the resistance of the dielectric, and is independent of frequency, so that it occurs both with direct and alternating voltages. The leakage current is proportional to the impressed voltage, hence the loss is proportional to the square of the impressed voltage.

With alternating pressures there is a further loss of energy usually styled hysteresis loss, and which under the usual operating conditions is very much larger than the leakage loss. Apparently some energy is consumed in reversing the stresses in solid dielectrics, and this appears as heat, causing a rise in temperature of the dielectric. At normal working stresses the dielectric hysteresis loss is proportional to the square of the electrostatic stress, that is to the square of the impressed voltage.

In order to take into account these energy losses, the charging current of a cable may be considered as made up of two components, one being the true capacitance current of which the phase is exactly 90 degrees in advance of the impressed voltage, the other being the

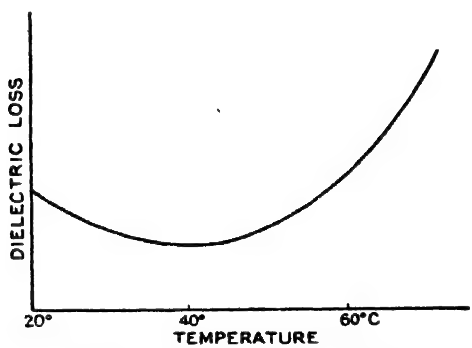


FIG. 67.—'V' curve of dielectric loss.

'energy' component in phase with the voltage. The dielectric losses are equal to the product of the voltage and the in-phase component of the charging current. Hence if in any cable

W_l = energy losses in watts per phase due to leakage,
W_h = energy losses in watts per phase due to dielectric hysteresis,
E = impressed voltage to neutral,
C = capacitance per conductor,
and ϕ_d = angle of advance of charging current,

the charging current is wCE, and the dielectric losses per phase are

$$W_d = W_l + W_h = wCE^2 \cos\phi_d, \qquad (165)$$

where $\cos\phi_d$ is known as the dielectric power factor.

Ordinarily, the dielectric losses of paper-insulated cables are small, but in the case of cables for the highest voltages, where the stresses are necessarily pushed to their extreme limit, they may have to be taken into consideration in fixing the current rating of the cable.

A very important and interesting relationship in the study of cable dielectrics is the variation of dielectric loss with temperature. It has been shown by a number of investigators during the past few years that, over the working temperature range, the dielectric loss in most cables falls to a minimum at about 40° C. and then rises again. The variation when plotted gives rise to what is known as the 'V' curve of dielectric loss, and the minimum temperature point corresponds very closely to the temperature of liquefaction of the resinous impregnating compound. A typical curve for paper-insulated cable is exhibited in Fig. 67. Since the variation of capacitance current with temperature is small, the dielectric power factor / temperature curve is of the same general shape as the dielectric loss / temperature curve. In this connection, it seems preferable to exhibit dielectric-loss data in terms of dielectric power factor rather than in power loss per mile of cable in order to facilitate comparison between different types and sizes of cables.

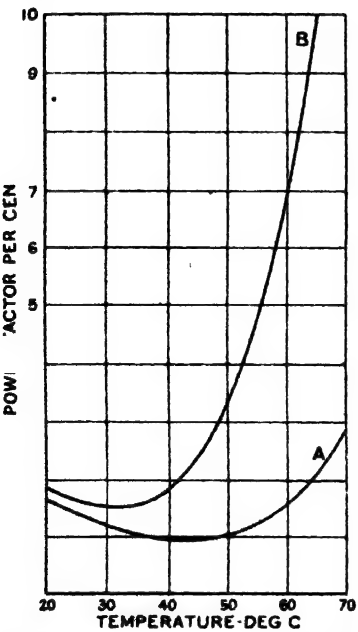


Fig. 68.—Effect of type of paper on power factor / temperature curve.

The actual shape of the 'V' curve depends on several factors and may be deliberately changed by the use of different papers, impregnating compounds, and methods of manufacture. For example, the two curves of Fig. 68, due to Dunsheath,[21] were obtained on two single-core cables of exactly similar dimensions, impregnated with the same compound, and manufactured under

identical conditions. The only difference between them was that the papers employed for the two lengths were composed of different fibres. Fig. 69, taken from a paper by Del Mar and Hanson,[22] shows the results obtained on another pair of similar cables in which both had the same insulating paper and treatment, but in which different impregnating compounds were employed. Other factors, such as inadequate drying of the paper, increase the dielectric losses and change the shape of the curve very considerably.

Contrary to what might be thought, the dielectric power factor is not independent of frequency. As the frequency of the impressed

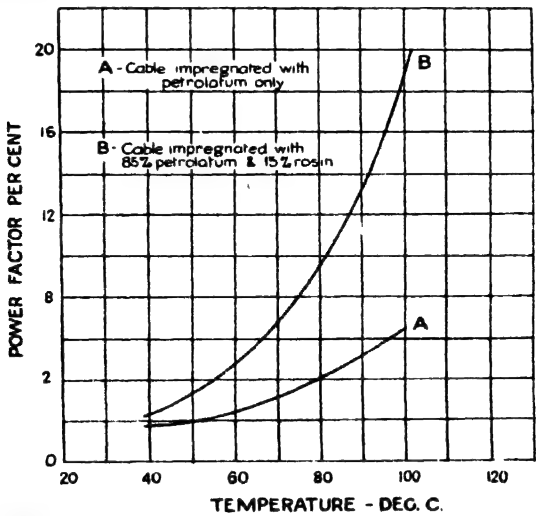


Fig. 69.—Effect of different impregnating compounds on power factor / temperature curve.

voltage is increased, the dielectric losses of the cable also increase, but at a slower rate than the frequency, so that the losses, say, at 50 cycles, are not twice the value of those at 25 cycles. In other words, as the charging current is proportional to the frequency, the dielectric power factor diminishes when the frequency increases. These remarks apply to cables running at the usual operating temperatures of 50° to 60° C. or over. At ordinary atmospheric temperatures the dielectric power factor *increases* with increase of frequency. These phenomena are clearly illustrated by the curves of Fig. 70 given by Emanueli,[23] and it will be seen that the

minimum points of the 'V' curves appear to lie on a line parallel to the abscissae.

It should be noted that none of the curves exhibited in Figs. 68-70 are typical of modern cables, being given here only to show the nature of some of the difficulties incident to the manufacture of high-voltage cables. Present-day cables have dielectric power factors of about 0·3 % at ordinary atmosphere temperatures, and the variation over the working temperature range is often comparatively insignificant.

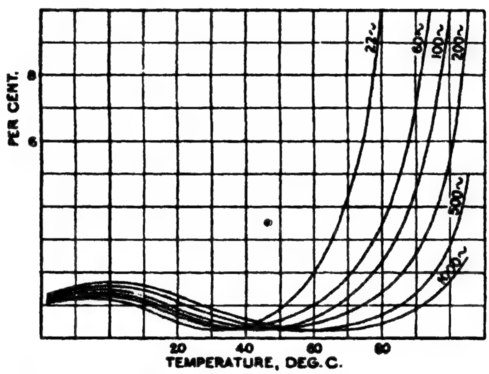


Fig. 70.—Power factor / temperature curves at various frequencies.

120. Ionisation Phenomena.—In an actual cable the proportionality between dielectric loss and the square of the impressed voltage does not always hold good when the electrostatic stresses are high. Under these conditions additional dielectric loss often occurs due to the development of ionisation, so that there is an apparent increase in the conductance and dielectric power factor. Ionisation may be caused by moisture in the dielectric, but more generally is the result of the presence of voids, a void being a space, which, instead of being filled with impregnating compound is occupied by air or other gases, often at low pressure. Such a void may exist between the mass of paper and the sheath, or it may lie as a more or less flat film between one layer of paper and the next superposed layer. Such a space is liable to ionisation, partly because owing to its low permittivity it is subjected to a relatively high electrostatic stress, and partly because the dielectric strength is very small compared with that of impregnated paper.

Fig. 71 shows the general effect of voltage on a cable containing occluded air or gas in the dielectric. Up to a certain critical value of electrostatic stress the power factor and conductance are sensibly constant, beyond that point the curve bends upwards due to ionisation. The voids near the conductor surface where the maximum value of stress occurs are the first to be broken down and ionisation then spreads progressively through the dielectric, the voids nearest the sheath being the last to be affected. At yet higher values of stress the curve flattens out again, the gases being more or less completely ionised.[24]

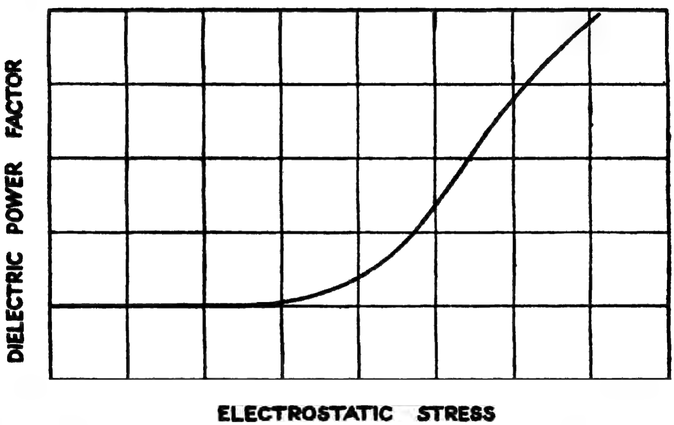


FIG. 71.—Variation in dielectric power factor of cable due to ionisation.

121. Mechanism of Void Formation.—The formation of voids in the dielectric of a cable may be due to—

(1) Faulty materials or methods of manufacture.

(2) The effect of loading cycles in service.

The elimination of voids during the course of manufacture is by no means an easy matter for many reasons. For example, although great care is taken to de-gasify the impregnating compound before use, there is a definite limit to the amount of gas that can be extracted under commercial manufacturing conditions. Also the vacuum applied to the impregnating tanks prior to the admission of compound cannot in practice be less than about 2 mm absolute, so that there must be a certain amount of air left in the cable dielectric. During the impregnating process this

residual air or gas may be driven inwards to be entrapped between the papers. Any imperfection in the dielectric due to loosely wrapped or creased paper on the cores, or bad contact between cores and fillers in the case of three-core cables, will facilitate the formation of these voids.

The physical properties of the compound and paper and their variation with temperature also play an important part in deciding whether or not voids will form during impregnation.

It is very important, therefore, to ensure that the cable dielectric is impregnated as thoroughly as possible, and complete cooling allowed before sheathing so that the maximum amount of compound is absorbed. In the process of manufacture rigid control of the drying, impregnating and cooling periods is necessary, and this is often effected by taking continuous measurements of capacitance and power factor whilst the cable is in the tank, the curves showing the variations in these quantities being a guide to the completeness or otherwise of each phase of the process.[25]

Consider now the case of an initially well-impregnated cable subjected to heating and cooling cycles. On heating, the compound expands, and owing to the temperature coefficient of the compound being so much greater than that of lead, the sheath is distended. When the cable cools down during periods of light load the compound contracts, but the lead sheath does not contract with it owing to its inelasticity, and voids are thus formed in the dielectric. These voids occur most readily where the insulation is more lightly packed, so that in the case of a three-core cable they tend to form in the fillers, or between the fillers and the cores, where good contact is often difficult to achieve. In the case of a single-core cable where the dielectric is more or less homogeneous, the voids may form anywhere in the interior of the dielectric depending on such factors as the viscosity of the compound and rate of cooling, etc.

Alternate heating and cooling, therefore, tends to encourage void formation, so that if there is ionisation before heating this may be accentuated afterwards, whilst if originally there is no sign of ionisation the phenomenon may be introduced by the loading cycles. The magnitude of the effect is dependent to a great extent on the values of maximum and minimum temperature adopted.[24]

122. Stability Limits.—The liability of a cable to develop ionisation in service may be estimated by subjecting it to loading

cycles, a series of ionisation curves being taken at room temperature before and after each cycle. Each curve is often found to lie above the one previously obtained, the effect of successive heating in the case of faulty cables being to change the shape of the curve from being more or less horizontal to one of the type shown in Fig. 71.

Another method of showing the results, which has advantages when the number of loading cycles is large, is to plot the room-temperature power factor at a particular voltage against the number of cycles. This method has been adopted in Fig. 72 which shows dielectric power factor values taken at 40 kV. on a 33 kV., three-core, screened-type cable, the temperature of

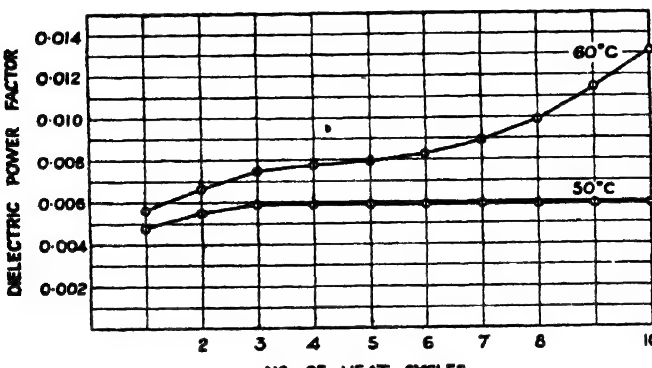


FIG. 72.—Stability test on cable.

measurement in each case being 30° C. Initially the temperature cycles were taken up to a maximum limit of 50° C., and the lower curve shows that apart from a slight rise during the first heat cycle no deterioration was observed. When the upper temperature limit of the cycles was raised to 60° C. a state of instability arose, the power factor becoming worse after each cycle as shown by the upper curve. For this particular cable the stability limit was thus about 50° C.[20]

123. Cable Breakdown.—The development of voids, with consequent ionisation, has undoubtedly been one of the chief causes of cable deterioration and subsequent breakdown. Where these gaseous spaces occur, the ionic bombardment produced by a strong electrostatic field causes polymerisation of the impregnating

compound, and results in the formation of a kind of wax with liberation of hydrogen and other gases. Although the wax appears to be harmless the hydrogen tends to produce water by reaction with some of the other gases evolved. A further step is the development of carbonisation which occurs along the interfaces and between the gaps of the paper tapes, ultimately creating discharge paths and destroying the insulation. These chemical and heating effects of ionisation explain the rise in power factor with time found to occur with some cables when repeated heating and cooling is carried out between extreme temperature limits.

Assuming also that the long-time breakdown voltage of a cable is controlled by ionisation, the shape of the time/breakdown-voltage curve is explained and the remarkable manner in which this is affected by mechanical pressure. On the removal of load from cables of normal structure the hydrostatic pressure may fall below atmosphere, and in some cases partial vacua may exist at some points of the dielectric, while other points are under pressure. These regions of low pressure are easily broken down with production of ionisation. Conversely, by increasing the mechanical pressure on the dielectric (see Fig. 66) ionisation can be eliminated to a great extent, with a consequent increase in the long-time dielectric strength of the cable.

There have been established three distinct types of cable breakdown :—

1. Disruptive.
2. Thermal instability.
3. Tracking.

1. Disruptive breakdown is a type of fault in which a discharge occurs between conductor and sheath at some relatively weak place in the dielectric producing a clean hole with no sign of carbonisation. It is the kind of fault which may occur when short-time testing with high voltage in the laboratory.

2. Breakdown due to thermal instability has often been encountered during development work on the production of cables for the highest voltages. The dielectric losses at these voltages are necessarily high, and if the power factor / temperature curve is such as to rise steeply at operating temperatures it is easy to visualise conditions where temperature stability is impossible. With an increase of load the dielectric temperature starts to rise; due to this increase in temperature the dielectric losses will also

increase, and so the temperature mounts still higher. If the external conditions are such that heat dissipation cannot keep pace with heat generation the process is cumulative, and the temperature rises progressively till at some spot the dielectric is destroyed by burning.[26]

3. Breakdown due to tracking is the commonest form of high-voltage cable failure under service conditions. It is entirely due to the ionization phenomena described above, aud a complete account of the mechanism of this type of breakdown has been given by Robinson.[27] The first visible result of ionization is the formation of a minute carbon core in the oil between the fibres of the paper at the position of highest stress near the conductor surface. Carbonised tree formations arise from this core and gradually spread outwards, both longitudinally and radially, till the dielectric is bridged. The conducting path may be several feet in length, and requires a long time to build up, but breakdown is certain once the initial carbon core has been formed.

Special types of cable have been developed for operation at the highest voltages, and these owe their success to the methods adopted for eliminating void formation and thermal instability.[28]

124. Screened and S.L.-type Cables.—The belted type of three-core cable illustrated in Fig. 73 (*a*) has been found to be relatively unsuitable at pressures over 22 kV., failure often occurring after a considerable period of service. Dissection of the faulty lengths shows that the trouble starts at the contact surface between the cores and fillers, particularly in the central triangle between the three cores, and results in the outer core papers being badly charred, the deterioration gradually working inwards toward the conductor surface.

Höchstädter[29] attributed these failures to the tangential electrostatic stresses in the outer layers of the core insulation, the dielectric strength of paper being considerably less in the direction of the fibres than in a direction perpendicular to them. Voids are also formed between cores and fillers as it is almost impossible to maintain intimate contact between these under service conditions. To begin with, the fillers cannot be as tightly packed as the main core insulation during the process of manufacture. Thus any voids caused by heating cycles have a tendency to form at the boundary surface between cores and fillers, the trouble being accentuated by the lay of the cores and conductor expansion tending

to separate the cores as the temperature rises. It is evident, therefore, that failure has been due to the effect of tangential stress occurring at points in the cable where voids are most certain to be present.

An improved method of construction due to Höchstädter is the screened or H-type cable (Fig 73 (*b*)), in which no belt insulation is used, and a metallic conducting surface is provided over each of

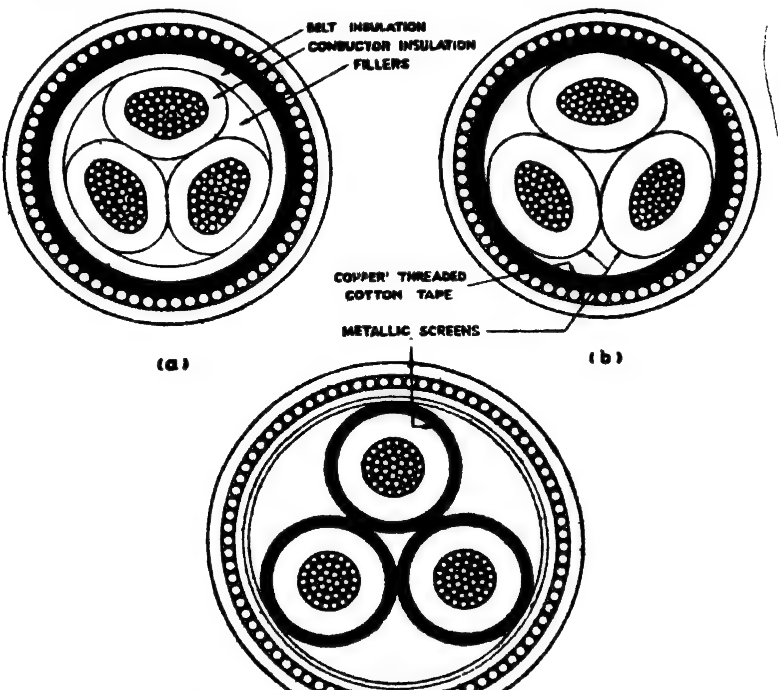


FIG. 73.—Various types of three-core cable (25 kV., 0·25 sq. in.).

the three cores. This metallic layer may consist of perforated aluminium foil of the order of 1 mil. in thickness or alternatively a 3-mil. copper tape applied with a narrow gap between turns. The three cores are laid up with jute or paper fillers in the normal manner, care being taken that the metallic layers are in good contact, and overall is wrapped a 15-mil. woven cotton tape to hold the cores together during the process of impregnation, etc. Good

electrical contact between the metallic layers and the sheath is ensured by means of fine copper wires stitched into the cotton tape, and by this means the outer core surfaces are maintained at earth potential.

The advantages of the screened type of cable may be summarised as follows :—

1. The electrical stress is purely radial since all tangential components are eliminated, and the relatively weak fillers are excluded from the electrostatic field.

2. Separation of the cores, by mechanical distortion of the cable or by thermal expansion, cannot introduce voids into the dielectric under stress.

3. The presence of the metallic screen prevents direct core-to-core faults, breakdown taking place to earth, which is an advantage from the operation point of view, as the fault current can be limited by earth resistance in the neutral.

4. The metallic screens facilitate the dissipation of heat from the central portions of the cable.

A further development of screening is the S.L.-type cable in which each core is provided with a separate lead sheath, and the cable is virtually an assembly of three lead-sheathed, single cores. In modern practice the cores are also covered with a layer of aluminium foil or copper tape (with or without the addition of the copper-reinforced cotton tape) under the lead sheath. By this means any voids formed between the lead sheath and the outside of the core are removed outside the influence of the electrostatic field. Cables provided with this screen are generally known as the H.S.L.-type. Fig. 73 (*c*) shows a typical arrangement, the three cores being laid up together with tarred jute yarns to a circular formation, the overall layers of cotton and hessian tapes being then applied to form a bedding for the armouring and serving.

Many of the advantages of the screened cable are also shared by the S.L.-type, but there is an important difference from the manufacturing point of view inasmuch as each of the cores in the latter type of cable is dried, impregnated and sheathed separately, and by this means it is considered that better impregnation can be secured. Also, due to the elimination of filler spaces containing compound, there is less tendency for oil drainage on hilly routes. It is also claimed that bending causes less distortion of the lead sheaths in the S.L.-type than in any other form of three-core

cable. On the other hand, owing to the diameter of the separate lead sheaths in S.L.-type cable being smaller than that of the overall lead sheath in the case of other types of three-core cables, the factor of safety against defects in the lead is greatly reduced.[30]

In practice the three sheaths are bonded together at jointing positions, so that circulating currents are produced therein as previously described in the case of single-core cables. Owing to the comparatively small diameter and high resistance of the sheaths, however, the additional losses due to these currents are quite small, being usually 2 to 3 °/₀ of the copper losses.

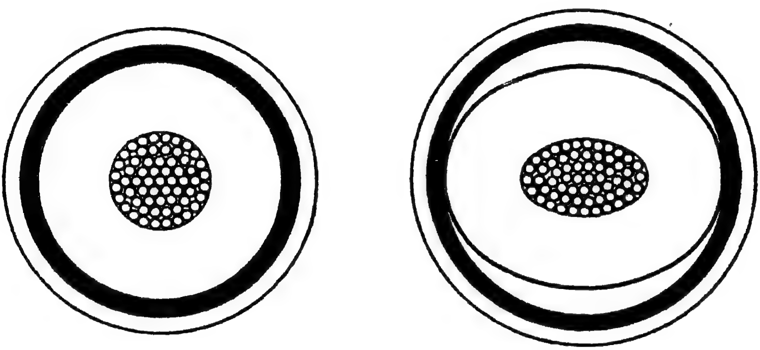
Screened or S.L.-type cables are now always used for pressures of 22 kV. and 33 kV., and even at 11 kV. they may be considered a better proposition than the belted type. For the customary range of conductor sizes, values of maximum working stress used in modern practice run up to 25 kV. per cm. for 11 kV. cables and 40 kV. per cm. for 33 kV. cables. In the case of shaped conductors, which are often used in screened cables, the electrostatic stresses at those portions of the conductor surface with the greatest curvature may be considerably higher than the figures given above.

125. Single-core Cables.—Three-core cables are generally used for pressures up to 33 000 volts, but at higher voltages considerations of weight and bulk may render them impracticable. For instance, a 66 000-volt, three-core, 0·30 sq. in., H.S.L.-type cable with the customary insulation thickness would have an overall diameter of 5·3 in., and a weight of about 84 lbs. per yard, so that the difficulty of transportation and installation would be considerable. Single-core cables are thus preferred for the highest pressures, and may also be used for lower pressures—where three-core cables would normally be employed—if a considerable amount of power has to be transmitted, thus necessitating a large section of conductor.

The disadvantage of single-core cables is, of course, that ordinary steel-wire armouring entails considerably increased power losses in the circuit, the additional losses due to the armouring being often equal to, or greater than, the conductor losses.[31] Non-ferrous armouring, on the other hand, whilst not affecting the losses to the same extent is expensive, and does not confer the same degree of protection, although this type of armouring has been used on certain submarine sections of single-core cables. In general, it appears practicable to use steel armouring with cables for the higher

pressures if a reduction in current-carrying capacity of 10 to 20 °/₀ is accepted. Hitherto, however, it has been considered that the extra factor of safety against mechanical damage due to the employment of armouring was not worth the reduction in rating, and in British practice reliance for protection is placed on a continuous layer of reinforced-concrete slabs laid directly above the cables.

Fig. 74 (*a*) shows a single-core, 0·50 sq. in. cable for use on a 66 000-volt, three-phase system, the actual pressure on the cable being 38·1 kV. The Cable Makers' Association's standard dielectric thickness for this voltage is 0·65 in., and before sheathing the core is wrapped with 1·5-mil. aluminium foil and 15-mil. copper-reinforced cotton tape, the purpose of this screening having been



(a) (b)

FIG. 74.—Types of single-core cable (66 kV., 0·50 sq. in.).

previously described in connection with H.S.L.-type cables. The sheath is composed of binary lead alloy containing 0·85 °/₀ antimony, and is served with three layers of compounded hessian tapes.

Fig. 74 (*b*) shows a later development, an oval-conductor, single-core cable, the advantage of this type being that the cable is enabled to follow temperature variations without the same degree of void formation. As the cable is loaded, the minor axis of the core increases in length whilst the major axis decreases, the core tending to assume a circular shape. Similarly, there is a contraction in volume with a decrease in temperature. In a circular-conductor cable it is possible to obtain hydrostatic pressures of the order of 80 lbs. per sq. in. under loading conditions, but with the oval-conductor cable, owing to the ready way in which the core

changes its shape to accommodate the expansion of the compound, pressures of the order of 5 lbs. per. sq. in. only are produced.

The maximum working stress in present-day 66 000-volt, circular-conductor cables is about 45 kV. per cm. for the usual range of conductor sizes, but with oval-conductor cables the stress varies at different points round the conductor surface, and the maximum stress, which occurs at the end of the major axis, may be about 20 °/$_0$ greater than the above values.

126. Oil-filled Cables.—The development of the oil-filled cable system has rendered possible the transmission of power by underground lines at the highest voltages in commercial use.

In the construction of oil-filled, single-core cables a channel is formed at the centre of the core by stranding the conductor wires round a cylindrical steel spiral; this channel is kept completely filled with oil by means of reservoir and feeding tanks at a pressure which is never allowed to drop below atmosphere at any point along the cable. This oil, which is of the same type as used for the initial impregnation, is a light mineral oil of very low viscosity, and the system is designed so that when the oil expands due to increase in cable temperature the extra volume, instead of distending the lead sheath, is accommodated by means of external reservoirs which return the oil to the cable when it contracts during periods of light load. By this means the formation of voids in the dielectric is rendered impossible.

A detailed description of the manufacturing process is outside the scope of the present work, but the following points should be noted :—

1. The customary drying of the cores in steam-heated tanks under vacuum is carried out, but this is only a preliminary operation, the final drying and impregnation of the paper being effected after lead sheathing. By this means much more perfect impregnation is secured because a higher degree of vacuum can be established in the cable than is possible by the normal method in large impregnating tanks. Furthermore, a valuable check is obtained during this process on the soundness of the sheathing.

2. The oil used for impregnating the cable is thoroughly filtered, dried and de-gasified before use, and after impregnation it is possible to check quantitatively the perfection of the process, and re-impregnate the cable if the necessary standard has not been reached.

3. The core papers are graded according to their porosity. It is well known that the dielectric strength of paper depends largely

on the density of the fibres, and therefore varies inversely as the porosity. In cables of normal structure a compromise must be effected between high dielectric strength on the one hand and reasonable porosity on the other. With oil-filled cables, however, the efficiency of the impregnating process is such that paper of low porosity and high dielectric strength can be used for the layers nearest the conductor, the total thickness of insulation being divided into sections which increase in porosity towards the sheath In this way the dielectric strength at any point in the insulation is approximately correlated to the electrostatic stress at that point resulting in a more uniform factor of safety as a whole.

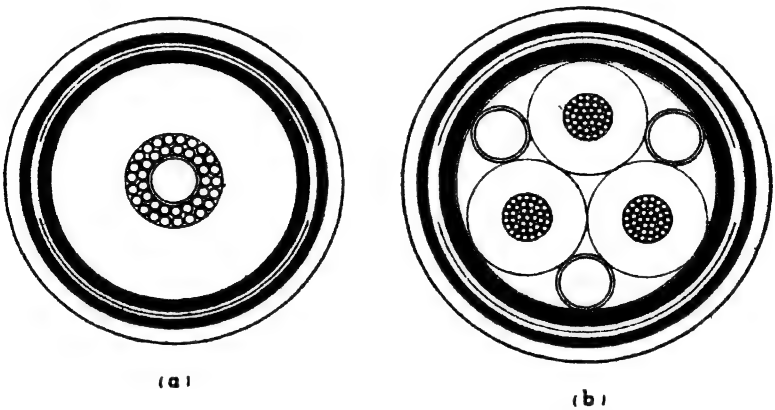


FIG. 75.—Types of oil-filled cable (132 kV., single-core, 0·40 sq. in., and 66 kV., three-core, 0·15 sq. in.).

Fig. 75 (*a*) shows a single-core, 0·40 sq. in., oil-filled cable for operation on a 132 000-volt, three-phase system, the dielectric thickness being 0·63 in. which is approximately the same thickness as used on a 66 000-volt, single-core cable of the solid type. For 66 000 volts the dielectric thickness of the oil-filled type would be 0·315 in. It will be noted that there are two lead sheaths, the inner one being made of pure lead. Around this sheath is wrapped a thin cloth tape to form a bedding for a reinforcing armour of plain brass tape 16-mils. thick, and over the armour a further thin layer of cloth and paper tapes. The outer lead sheath, which consists of a ternary lead alloy containing 0·5 °/。 antimony and 0·25 °/。 cadmium, is used for the purpose of protecting the armouring from chemical action in the soil. The

cable is finally served with a thin layer of paper and two compounded hessian tapes.

Fig. 75 (*b*) shows a 66 000-volt, three-core, 0·15 sq. in., oil-filled cable, the oil ducts in this case being accommodated in the filler spaces.

In an oil-filled cable installation the maintenance of oil pressure is of vital importance. On the one hand, it must not be allowed to fall below one atmosphere, and on the other, it must not be so great as to damage the lead sheath, thus fixing an upper limit of 3 to 4 atmospheres. Variation of hydrostatic pressure along the cable may be caused by differences in elevation of the route, or temperature changes due to varying loads which cause oil movement and loss of head by friction. It is evident that the above range of working hydrostatic pressure limits the length of cable that can be fed from one source, and it is necessary, therefore, for the line to be divided into sections. This division is accomplished by means of stop joints which connect two sections electrically and yet prevent any oil communication between them, allowing oil to be fed to one cable section independently of the other. It is a minor disadvantage of the oil-filled system that the cable route requires special study to determine suitable positions for stop joints and oil-feeding tanks, and the accommodation of the latter devices is not without difficulty in city areas.

There are many installations of single-core, oil-filled cables working successfully at 66 000 volts and 132 000 volts, and a considerable length of 220 000-volt cable is also in commercial operation. Three-core, oil-filled cables are in use on 33 000-volt and 66 000-volt systems.

Owing to the elimination of void formation, the maximum permissible stress in oil-filled cable is higher than in cable of ordinary construction. 66 000-volt cables are worked at a maximum stress of 70 kV. per cm. and 132 000-volt cables at a maximum stress of about 90 kV. per cm.

At the highest voltages oil-filled cables are the main solution at the moment. At voltages where oil-filled and solid-type cables are in competition, the extra cost of supplying and maintaining the oil-feeding stations must be considered in relation to the greater carrying capacity of the former type of cable. Oil-filled cables can be run at a considerably higher temperature than solid-type cables without damage, and have better heat-dissipating properties

due to the thinner insulation, so that much heavier loads can be transmitted for any given conductor cross-section.[32]

Brief mention may be made here of the oilostatic system developed in America. In this system the cables are provided with thin lead sheaths, which are stripped off as they are pulled into steel pipes. The pipes are then filled with oil, the pressure on which is maintained at about 200 lbs. per sq. in. Oil reservoirs and feeding tanks, etc., are necessary with this system as in the case of oil-filled cables.[28]

127. Pressure Cables.—The pressure cable, which has been recently introduced, is a serious rival to the oil-filled type, and is

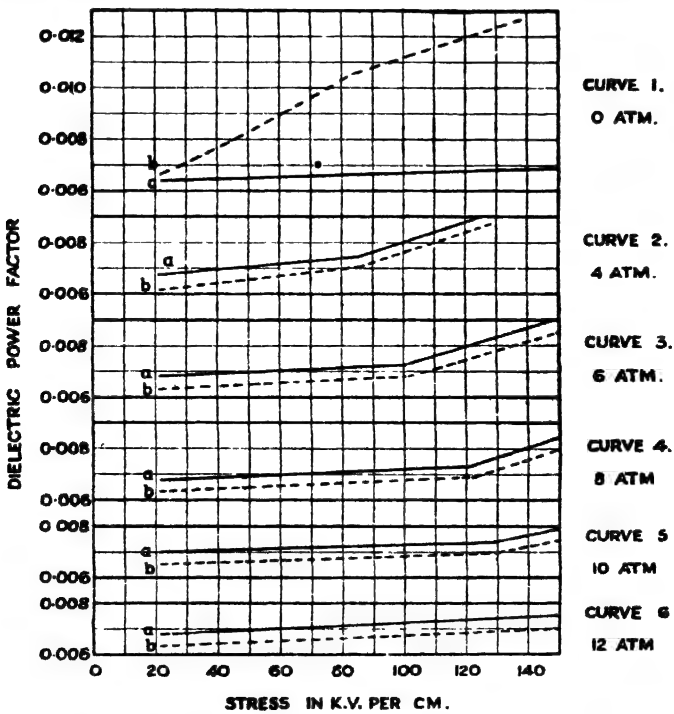


Fig. 76.—Variation of power factor with cable subjected to various external pressures.

being developed for the highest voltages. A cable manufactured and impregnated by the normal method is employed, but after installation it is subjected to radial compression so that any vacuous

spaces that tend to form are closed by the compression, or the pressure raised in them to such an extent that no ionisation can take place. Hence, the cable can be worked at stress values comparable with those employed in the case of oil-filled cables.

Fig. 76, due to Höchstädter and his associates,[20] shows the effect of varying external pressures on the ionisation curve of a three-core cable. The pressure was raised step by step from 0 to 12 atmospheres and at each step the cable was put through a heat cycle, being loaded to give a final temperature of 70° C. The power factor was measured before and after each cycle at room temperature, these measurements being indicated by (*a*) and (*b*) respectively. Test 1 shows clearly the instability of the cable at normal atmospheric pressure as a consequence of overheating. It is also clearly seen from tests 2 to 6 that with increasing pressure, in spite of the heating of the cable up to the same maximum temperature, the ionisation point is displaced further towards a higher voltage, and at a pressure of 12 atmospheres it disappears entirely out of the range of measurement.

The effect of external pressure on the time / breakdown-voltage curve has already been discussed and is equally remarkable, the long-time breakdown stress being raised from 180 kV. per cm. at normal atmospheric pressure to about 400 kV. per cm. at 15 atmospheres.

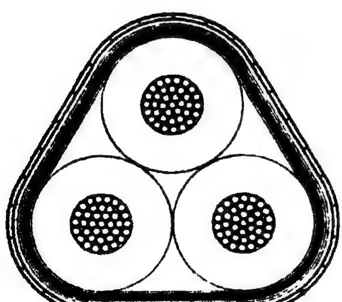


FIG. 77.—Pressure cable (66 kV., three-core, 0·15 sq. in., H.S.O. type).

Pressure cables can be divided into two distinct types according to whether the gas pressure is applied external or internal to the sheath.

In the first type, developed by Höchstädter, a triangular-shaped cable (H.S.O.-type) has been adopted, and Fig. 77 shows a section of a 66 000-volt, 0·15 sq. in. cable. This particular shape of cable has many advantages when compared with circular three-core cable, such as lower weight per yard and lower thermal resistance, but its particular suitability for the present purpose is due to the fact that this form of lead sheath acts extremely well as a pressure membrane. The insulation thickness for the cable illus-

trated is 0·306 in., and each core is wrapped with metallised paper before being laid up with fillers to triangular formation and bound with a copper tape. Impregnating and sheathing is carried out in the normal manner, the sheath being, of course, of triangular shape. The lead is wrapped with a steel tape applied over a bituminised paper, the cable being then served with hessian tapes and finished with a protective armour of narrow steel strips.

The cable is installed in a welded steel pipe, which forms a continuous gas-tight enclosure for cable and joints, a complete section of pipe being laid and thoroughly tested for leakage before the cable is drawn in. The pipe itself is laid in lengths of from 15 to 40 feet, adjacent lengths being welded together, and expansion joints being provided at each weld to accommodate any movement due to temperature variation. The outside of the pipe is coated with a bituminous compound, and wrapped with jute tapes impregnated with the same compound. After the entire length of cable is laid, the pipe is filled with nitrogen at a working pressure of 180 lbs. per sq. in. maintained by gas cylinders connected to each end of the line through automatic regulating valves.[33]

In a later development the welded steel pipe is dispensed with, the cable being provided with two sheaths, and the nitrogen under pressure is contained in the space between the sheaths. The whole cable is surrounded with a steel spiral band capable of withstanding the maximum gas pressure developed.

In the second type of pressure cable, developed during the last few years by various British cable manufacturers, the gas pressure is applied internal to the sheath. Individual designs vary to some extent, but the underlying principle is the same, depending essentially on the employment of an oversize lead sheath so as to provide a small annular space between dielectric and sheath, the latter being reinforced by non-magnetic metallic tapes to withstand internal pressure. Dried nitrogen is fed into this space from gas cylinders, and finding its way along the cable maintains the whole of the dielectric at a pressure of about 200 lbs. per sq in In operation, as the temperature rises, the compound expansion can be accommodated without distension of the sheath, and on cooling there is no possibility of void formation. For protection against corrosion, compounded servings are wrapped over the metallic reinforcement, or a second lead sheath may be used for the same purpose.

room temperature before commencement of the test and after each loading cycle. During the whole time a voltage, say 10 % above normal, is applied to the cable. The maximum temperature attained in each cycle may be gradually increased in order to determine the stability limit of the cable. A short-time test embracing several complete cycles can be carried out in a week or so, although no standardisation of the test conditions has yet been effected.

A long-time ageing test used by manufacturers for research purposes is carried out in a somewhat similar manner except that daily load cycles are applied for a period of twelve months or so, an over-voltage of from 10 % to 50 % being continuously maintained on the cable. This test is extremely searching, particularly when combined with visual examination of samples cut from the cable to see whether any deterioration has occurred. Although such tests can scarcely be considered to come within the scope of commercial acceptance tests, figures from type tests of this character are a useful guide to the purchaser.

129. Determination of Line Constants for Underground Transmission.—In the determination of line constants for underground cables, such factors as skin and proximity effects, sheath and armour losses, shape of conductors, etc., have to be considered.

Skin Effect.—This phenomenon has already been fully discussed in Art. 59, and the various formulæ therein given are also valid in the case of underground cables. The percentage increase of resistance at 50 cycles and a conductor temperature of 50° / 60° C. can be taken as $8A^2$, where A is the cross-sectional area of the conductor in square inches.

Proximity Effect.—The physical conditions of an overhead conductor and a conductor in an underground cable are quite different. In the latter case, the surfaces of the individual wires are uncontaminated, and being held closely together under load conditions by the increase of mechanical pressure on the conductor, the contact resistances between adjacent layers of wires must be extremely low. Spirality effect, also, must be almost completely eliminated. Experimental work by Arnold[34] shows that the increase of resistance in stranded cable conductors due to proximity effect can be most readily determined by assuming that the conductivity of the conductor to eddy currents is a

definite ratio of the direct-current conductivity. From the viewpoint of current rating it would not be safe to take the value of this ratio at less than 0·8. Probably the most convenient method of estimating the actual magnitude of the effect is by using the charts of proximity effect for parallel homogeneous conductors published by Costello,[35] but taking into account the correction factor given above. For practical sizes of conductors the percentage increase in resistance at 50 cycles and 50° / 60° C. working temperature can be taken as $24A^2(d/s)^2$, d being the conductor diameter, and s the interaxial spacing of conductors. The magnitude of the effect is therefore relatively more serious in three-core cables than in single-core cables of the same size and voltage class, due to the closer spacing of conductors in the former case.

Sheath and Armour Losses.—The effect of sheath currents on the resistance and reactance of single-core cables has already been discussed in Art. 112. A similar phenomenon occurs in the case of multicore cables, and is further complicated by eddy currents and hysteresis losses in the armouring. By assuming certain standard values for the magnetic constants of the armouring, formulæ have been derived for calculating all these losses.[3] Generally speaking, conductor sizes in high-voltage, three-core cables do not exceed 0·50 sq. in., and in this limited range the percentage increase of conductor resistance at 50 cycles due to sheath losses can be taken as $16A^2$, and due to armour losses $32A^2$, normal operating temperatures being assumed in each case.

As regards reactance, the net effect of eddy currents and magnetic induction in the armouring is to increase by about 15 % the values based on the fundamental formulæ for inductance (Art. 57).[37]

Shaped Conductors.—In the case of three-core, belted and screened cables with shaped conductors, values of inductance and capacitance cannot be stated exactly, as there is no standardisation of conductor cross-section. Speaking generally, the inductance is 5 % to 10 % less than for round conductors of the same area and separated by the same thickness of insulation, and the capacitance is increased by the same percentage. The smaller figure applies to the case of small conductors with large thicknesses of insulation, and *vice versa*.

From the above discussion it will be recognised that the calculation of line constants is a matter of some uncertainty,

particularly in the case of multicore cables, and where extreme accuracy is required these constants should be measured by A.C. bridge methods after the cable has been installed and at temperatures corresponding to operating conditions. For most practical purposes, however, these refinements are unnecessary.

In order to show clearly the derivation of underground-line constants the following numerical examples will be taken:—

Problem 7.—Find the constants of a 20-mile line of three single-core cables, 0·40 sq. in. (61/0·093″), circular conductors, for operation at 66 000 volts, 50-cycles, three-phase. Internal diameter of sheath 2·17 in., external diameter of sheath 2·47 in., thickness of overall serving 0·15 in. The cables are laid closely together in triangular formation, the sheaths being bonded at each end of the run.

From wire tables the resistance of the conductor at 15·6° C. is 0·104 ohm per mile, and taking the operating temperature as 45° C., the conductor resistance

$$\begin{aligned} R &= 0{\cdot}104(1 + 0{\cdot}004 \times 29{\cdot}4) \times 20 \\ &= 2{\cdot}32 \quad \text{ohms.} \end{aligned}$$

In order to determine the effective conductor resistance when the sheaths are short-circuited, the values of R_s and X_m must first be obtained.

Assuming the temperature of the lead sheath is 30° C., its resistivity ρ_s may be taken as $23{\cdot}2 \times 10^{-6}$ ohms, and the resistance of the sheath

$$\begin{aligned} R_s &= \frac{23{\cdot}2 \times 10^{-6} \times 5\,280 \times 12 \times 2{\cdot}54 \times 20}{\frac{\pi}{4}\{(2{\cdot}47)^2 - (2{\cdot}17)^2\} \times 2{\cdot}54 \times 2{\cdot}54} \\ &= 10{\cdot}6 \quad \text{ohms.} \end{aligned}$$

The interaxial spacing (d) is about 2·77 in., and the mean radius of the sheath (R_m) is 1·16 in., so that

$$\begin{aligned} X_m &= \omega M \\ &= 0{\cdot}741\omega \log_{10} \frac{d}{R_m} 10^{-3} \times 20 \\ &= 0{\cdot}741 \times 2\pi \times 50 \times \log_{10} \frac{2{\cdot}77}{1{\cdot}16} \times 10^{-3} \times 20 \\ &= 1{\cdot}76 \quad \text{ohms.} \end{aligned}$$

Hence the effective value of the conductor resistance is

$$R_{\text{eff}} = R + \frac{X_m^2 R_s}{R_s^2 + X_m^2} \quad . \quad . \qquad (145)$$

$$= 2{\cdot}32 + \frac{1{\cdot}76 \times 1{\cdot}76 \times 10{\cdot}6}{(10{\cdot}6)^2 + (1{\cdot}76)^2}$$

$$= \underline{2{\cdot}60 \quad \text{ohms.}}$$

(The increase of resistance due to skin and proximity affects would be about 0·04 ohm in this case, and may thus be neglected.)

The reactance with sheaths open-circuited would be

$$X = \left(0{\cdot}083 + 0{\cdot}741 \log_{10} \frac{2{\cdot}77}{0{\cdot}418}\right) 10^{-3} \times 2\pi \times 50 \times 20$$

$$= 5{\cdot}48 \quad \text{ohms,}$$

but with short-circuited sheaths this is reduced to

$$X_{\text{eff}} = X - \frac{X_m^3}{R_s^2 + X_m^2} \quad . \quad . \qquad (150)$$

$$= 5{\cdot}48 - 0{\cdot}05$$

$$= 5{\cdot}43 \quad \text{ohms.}$$

Assuming the dielectric permittivity to be 3·8, the capacitance

$$C_n = \frac{0{\cdot}0388 \times 3{\cdot}8}{\log_{10} \dfrac{1{\cdot}085}{0{\cdot}418}} \qquad (135)$$

$$= 0{\cdot}356 \quad \text{microfarad per mile,}$$

from which the susceptance

$$B = 0{\cdot}356 \times 10^{-6} \times 2\pi \times 50 \times 20$$

$$= \underline{0{\cdot}00223 \quad \text{mho.}}$$

Taking the dielectric power factor at operating temperature as 0·005, the conductance

$$G = 0{\cdot}00223 \times 0{\cdot}005$$

$$= \underline{0{\cdot}0000112 \quad \text{mho.}}$$

Problem 8.—Find the constants of a 10-mile line of three-core, belted-type cable, 0·25 sq. in. (37/0·093″), circular conductors, for operation at 22 000 volts, 50 cycles. Conductor-insulation thickness 0·225 in., belt-insulation thickness 0·175 in.

From wire tables the resistance of the conductor at 15·6° C. is 0·171 ohm per mile. Assuming an operating temperature of 55° C., the increase of resistance due to skin and proximity effects is 1 %, and due to sheath and armour losses 3 % (Art. 129), a total increase of 4 %. Also allowing 2 % for laying up the cores

$$R = 0{\cdot}171(1 + 0{\cdot}004 \times 39{\cdot}4) \times 1{\cdot}02 \times 1{\cdot}04 \times 10$$
$$= \underline{2{\cdot}10 \quad \text{ohms.}}$$

The inductance calculated by the usual formula, but allowing for coring and 15 % increase for the effect of armouring (r 0·325 in. ; interaxial spacing = 1·10 in.), is

$$L_0 = \left(0{\cdot}085 + 0{\cdot}741 \log_{10} \frac{1{\cdot}10}{0{\cdot}325}\right)10^{-3} \times 1{\cdot}02 \times 1{\cdot}15 \qquad (101)$$
$$= 0{\cdot}560 \times 10^{-3} \quad \text{henries per mile,}$$

and the reactance is consequently

$$X = 0{\cdot}560 \times 10^{-3} \times 2\pi \times 50 \times 10$$
$$= \underline{1{\cdot}76 \quad \text{ohms.}}$$

The capacitance is determined from Simons' formula

$$C_0 = \frac{0{\cdot}048\epsilon}{\log_{10}\left[\left\{0{\cdot}52\left(\frac{t}{T}\right)^2 - 1{\cdot}70\frac{t}{T} + 3{\cdot}84\right\}\frac{T + t}{d} + 1\right]} \qquad (163)$$

In this case

$$\frac{t}{T} = \frac{0{\cdot}175}{0{\cdot}225} = 0{\cdot}78,$$
$$\frac{T + t}{d} = \frac{0{\cdot}40}{0{\cdot}65} = 0{\cdot}62,$$

and taking ϵ as 3·8 gives

$$C_0 = \frac{0{\cdot}048 \times 3{\cdot}8}{\log_{10}[\{0{\cdot}316 - 1{\cdot}326 + 3{\cdot}84\}0{\cdot}62 + 1]} \qquad (163)$$
$$= 0{\cdot}415 \quad \text{microfarads per mile.}$$

Hence the susceptance is

$$B = 0{\cdot}415 \times 10^{-6} \times 2\pi \times 50 \times 10$$
$$= \underline{0{\cdot}00130 \quad \text{mho.}}$$

Assuming that the dielectric power factor at the operating temperature is 0·005, the conductance

$$G = 0{\cdot}00130 \times 0{\cdot}005$$
$$= 0{\cdot}0000065 \quad \text{mho.}$$

The derivation of the constants for three-core screened and S.L.-type cables follows on the same lines as above except that the capacitance values are calculated by the ordinary logarithmic formula (135). In the case of the latter type of cable the effect of the separate lead sheaths on the resistance and reactance values can be calculated by the formulæ given in Art. 112, but is fairly small, and can often be neglected in actual practice.

After determining the values of resistance, reactance, susceptance and conductance as above, the auxiliary line constants are obtained and the calculation of an underground line is a straightforward transmission problem, no different from that of an overhead line. In the majority of cases the electrical length of underground lines is quite small, and sufficient accuracy is obtained by using the localised-admittance methods of calculation.

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CHAPTER X.

UNDERGROUND CABLES—THERMAL CHARACTERISTICS.

130. Methods of Laying.—Before going on to discuss thermal phenomena it is necessary to consider briefly the chief methods of laying underground cables, *viz.*, the direct, draw-in, and solid systems.

Direct System.—This method of installation being comparatively simple and cheap is much favoured in modern practice, the cables being laid on a bedding of riddled soil at the bottom of an open trench. Where more than one cable is laid in the same trench a horizontal interaxial spacing of at least 12 inches is advisable in order to reduce the effect of mutual heating, and also to ensure that a fault occurring on one cable will not damage an adjacent one. This remark applies, of course, to separate feeders, and not to individual single-core cables forming a three-phase circuit, in which case the three cables are usually laid as close together as possible in triangular formation, being bound together every few feet by means of tarred jute yarn. If more than one tier of cables is necessary, riddled soil is filled in the trench and punned down so as to form a solid bed at a height of 12 to 18 inches above that for the first tier. After the last cables have been laid, further riddled soil is filled in to a height of about 3 inches above their top surfaces, and overall is laid a continuous layer of interlocking tiles, or reinforced-concrete slabs. This precaution is intended to give warning to future excavators of the presence of cables. This method of laying gives the best conditions for dissipating the heat generated in the cables, and almost completely eliminates trouble due to traffic vibration, etc.

Draw-in System.—In congested city areas where excavation is expensive and inconvenient, the draw-in system is often used. Ducts or pipes are laid in the ground, with manholes at suitable

positions and distances, and the cables afterwards drawn therein. The great advantage is thus secured that once the conduits have been laid, repairs, alterations, or additions to the system can be made without re-opening the ground.

Cable ducts are usually of glazed earthenware, either in the form of single pipes or multi-way ducts about 3 ft. in length, and furnished with special spigot and socket ends. They may be buried direct in the ground or preferably set in concrete. Fibre conduits are also in use to a certain extent. These are comparatively thin tubes made of wood-pulp saturated with asphaltic or bituminous compound, and are usually laid in concrete. They can be bent on site by warming, and are useful in situations where room is restricted as they can be installed at closer centres than the corresponding size of earthenware pipe. Steel or cast-iron pipes make extremely good cable ducts but high cost prevents their extensive use.

The first cost of the draw-in system is rather high, and another disadvantage is that the current-carrying capacity of the cables is reduced owing to the close grouping of cables and other factors unfavourable for the dissipation of heat. Furthermore, cables drawn into ducts are more subject to longitudinal expansion and contraction than cables laid direct, and American practice shows that this may lead to severe sheath damage at the joints or at the point of entry from the manhole into the duct.

Cables intended for drawing into ducts are often plain lead covered, as any serving increases the difficulty of installation and subsequent removal. On the other hand, where the relative sizes of cables and duct permit, served cables should be used in order to secure greater protection against the risk of chemical or electrolytic corrosion. With three-core cables it is the best practice to use armoured cable, with the textile covering, if any, underneath the armour.

Solid System.—In this method of laying the cables are laid in wood, earthenware, cast-iron, or asphalt troughs filled up solid with bitumen or pitch, which is poured in after being heated to a fluid state. Cables laid in this manner are usually plain lead covered, as the troughing affords good mechanical protection.

As regards troughing material, creosoted wood has the advantage of cheapness, and can be made conveniently in greater lengths than troughing made from other materials. Glazed

earthenware troughing is also largely employed, and has the advantage over wood that it is not liable to rot. It cannot, however, be conveniently obtained in lengths greater than 3 feet, so that a large number of joints are inevitable. Cast-iron troughing can be supplied in long lengths, but is expensive. Both the latter kind of troughs are usually supplied with socket ends whereby each length of troughing is jointed with the next. Asphalt troughing appears to be very suitable for a solid system as it is easily laid, and can be jointed into one homogeneous length.

Cables laid in troughing are supported every few feet on insulating bridge pieces or saddles, so that the filling-in compound entirely surrounds the cables. Hard asphalt is a very suitable material for these saddles, as under proper conditions of trough filling the surfaces which come into contact with the hot filling compound are melted, and in cooling the whole forms a practically homogeneous mass. Asphalt troughing has the advantage of not requiring any saddles.

After the filling material, which should preferably be pure bitumen, has been run in, a covering, which may be of bricks, tiles, or wood, is applied to the trough. In the case of cast-iron or asphalt troughs, however, the cover is usually of the same material as the trough.

This method of installation is little used nowadays as it is more expensive than the direct-laid system, and more dependent for its success on skilled supervision and favourable weather conditions whilst the job is proceeding. Furthermore, the facilities for heat dissipation from the cables are distinctly poor.

131. Fundamental Thermal Phenomena in Buried Cables.—The current-carrying capacity of high-voltage cables may be governed by economic laws or considerations of temperature rise, but the latter factor is the more important since it establishes a definite physical limit to the current rating. In some cases, therefore, economic current densities cannot be realised in practice owing to the excessive heating of the dielectric which would be caused. On account of their high cost it is most important that cables should be loaded up to their safe limit, and a great deal of research work has been done in recent years to establish safe current-carrying capacities. In Great Britain, this work has been carried out under the *ægis* of the British Electrical and

Allied Industries Research Association (referred to hereafter as E.R.A.). Reference should be made to the reports and tables of current ratings published by the Association by all who are desirous of pursuing the subject further than is possible in the limited confines of the present work.[1, 2]

In general, when an electric cable is carrying current there are three sources of heat, *viz.*, the I^2R losses in the conductors, the dielectric losses and the sheath losses. Both the two latter sources being of secondary importance they will be ignored in the following preliminary discussion of the subject.

In the determination of current-carrying capacity as limited by temperature rise, methods of calculation are based on the fundamental theory of heat transmission in the steady state. A cable carrying no load and laid, for example, direct in the ground assumes the temperature of the soil at the depth at which it is buried. On passing current through the cable, heat is generated in the conductors and flows through the dielectric, lead sheath, protective coverings and soil, raising the temperature of these various elements of the heat path until a steady state has been attained. The heat dissipated is then equal to that generated, and the case is analogous to that of an electric current flowing through resistances in series, heat flow corresponding to current and temperature difference to potential difference. Thermal resistance values can thus be conveniently expressed in thermal ohms, this unit being the resistance of a path through which a temperature difference of 1 degree C. produces a heat flow of one watt (one joule per second). Similarly, the thermal resistivity of any material is the temperature drop in degrees C. produced by the flow of one watt between opposite faces of a centimetre cube of the material, and the thermal resistance of any heat path is the temperature difference across it in degrees C. per watt transmitted.

Applying these principles to the case of a buried multicore cable, the temperature rise may be expressed mathematically as

$$\theta = H(S + G), \quad \ldots \quad (166)$$

where θ = temperature rise of conductors in degrees C.,

H = total heat developed in conductors in watts,

S = thermal resistance between conductors and outer surface of cable in thermal ohms,

and G = thermal resistance of ground path.

The total heat generated in the cable is

$$H = nI^2R_\theta, \quad . \quad . \quad . \quad . \quad (167)$$

where I = current per conductor,

n = number of conductors,

and R_θ = resistance of each conductor in ohms at working temperature corresponding to the temperature rise θ,

both skin and proximity effects being taken into consideration when necessary. Substituting this value of H in (166) gives

$$I = \sqrt{\frac{\theta}{nR_\theta(S + G)}}. \quad . \quad . \quad . \quad (168)$$

Supposing, therefore, that the values of S and G are known, the steady-state current loading of the cable can be calculated for any particular temperature rise, and final temperature.

In case it should be necessary to take dielectric losses into consideration the total heat generated in the cable

$$H = nI^2R_\theta + W_d, \quad . \quad . \quad . \quad (169)$$

where W_d = dielectric loss in watts per phase.

In actuality the dielectric losses are distributed throughout the dielectric, but to be on the safe side they may be assumed to be concentrated at the conductor surface, so by substitution again in (166)

$$I = \sqrt{\frac{\theta - W_d(S + G)}{nR_\theta(S + G)}}. \quad . \quad . \quad (170)$$

The method of taking sheath and armour losses into account will be deferred until the case of single-core cables has been considered.

In the above formulæ it is immaterial to what length of cable the quantities R_θ, W_d, S and G relate, so long as consistency is observed. It is customary, however, to express S and G in terms of one centimetre length of cable, so R_θ and W_d must therefore apply to the same length.

Formulæ (168) and (170) apply to all types of multicore cables, but require modification before they can be employed for the calculation of current loadings for single-core cables owing to the mutual heating effect of the three cables forming a complete transmission circuit, and the heavier sheath losses. A treatment of this case will be given later.

132. Thermal Resistance of Single-core Cables. — The formula for the thermal resistance between conductor and sheath of a single-core cable can easily be obtained as follows :—

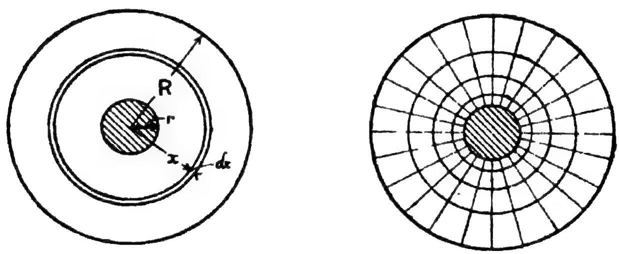
Considering the thermal resistance of a length of cable one centimetre long to be the sum of the thermal resistances of successive layers of insulation, there may be written

$$dS' = \frac{Kdx}{2\pi x},$$

where dx is the thickness, and $2\pi x$ the area, of a cylindrical element of radius x (Fig. 78 (a)). K is the thermal resistivity in thermal ohms, *i.e.* the temperature difference in degrees C. between opposite faces of a centimetre cube to cause transference of one watt of heat. From the above

$$S' = \frac{K}{2\pi}\int_r^R \frac{dx}{x},$$

$$= \frac{K}{2\pi}\log_e \frac{R}{r} \quad \text{thermal ohms per cm.} \qquad (171)$$



(a) (b)

FIG. 78.—Lines of heat flow and isothermal lines in a single-core cable.

As regards the value to be taken for K in the above formula, the figure recommended by the E.R.A. for all types of paper-insulated cables for pressures above 1 500 volts is 550 thermal ohms. Substituting this value, and converting to practical units

$$S' = 202 \log_{10} \frac{R}{r}. \qquad (172)$$

(An analogous formula can be obtained in the case of cables for pressures of 1 500 volts or under, where the standard value of thermal resistivity is taken as 750.)

Fig. 78 (b) shows the lines of heat flow and isothermal lines in a single-core cable.

Where the cable is served with hessian tapes, jute yarn or other textile coverings, the thermal resistance S may be considered as made up of two portions, S' and S'', such that $S = S' + S''$,

where S' = thermal resistance between conductor and sheath,
and S'' = thermal resistance between sheath and external surface of cable.

The thermal resistivity of lead itself is only about 2·9, and so the influence of this portion of the heat path on the temperature rise can be neglected. The thermal resistance of the protective covering is then

$$S'' = \frac{K_1}{2\pi} \log_e \frac{R_2}{R_1} \quad \text{thermal ohms per cm.,} \qquad . \quad (173)$$

where R_2 = external radius of protective covering
and R_1 = external radius of lead sheath.

The value of K_1 can be taken as 500 so that in this case, converting to practical units,

$$S'' = 183 \log_{10} \frac{R_2}{R_1} \quad \text{thermal ohms per cm.,} \qquad . \quad (174)$$

133. Thermal Resistance of Three-core, Belted-type Cables.—The derivation of a satisfactory theoretical formula for the thermal resistance of three-core, belted-type cables is a matter of some difficulty. Both Russell[3] and Mie[4] have given solutions for cables with circular conductors, but owing to various approximations introduced in the mathematical work there is a considerable difference between results obtained by the two methods of calculation. With both formulæ the error is small for the case of extremely small conductors and extremely thick insulation, but over the practical range of cable dimensions there are serious divergences. It is interesting to note that Russell's formula gives values of thermal resistance which are too high, and Mie's formula gives results too low.

Since neither of the theoretical formulæ mentioned are satisfactory it has been necessary to fall back on experimental work for a solution of the problem, and Simons[5] has developed a graphical method of correcting the error in Mie's formula. From results based on this method it is found that the thermal resistance between

conductors and sheath of three-core, circular-conductor, belted-type cable is

$$S' = \frac{K}{6\pi}\left(0{\cdot}85 + \frac{0{\cdot}2t}{T}\right) \log_e \left[\left(8{\cdot}3 - \frac{2{\cdot}2t}{T}\right)\frac{T + t}{d} + 1\right]$$

thermal ohms per cm., (175)

where t = belt-insulation thickness,
T = conductor-insulation thickness,
and d = conductor diameter,

all in the same units.

If K is taken as 550, the above formula may be written in practical form as

$$S' = \left(57{\cdot}1 + \frac{13{\cdot}4t}{T}\right) \log_{10} \left[\left(8{\cdot}3 - \frac{2{\cdot}2t}{T}\right)\frac{T + t}{d} + 1\right]$$

thermal ohms per cm. (176)

A direct check of formula (175) by measuring the thermal ohms per cm. temperature rise of actual cables is impracticable as there are two unknowns: (1) The thermal resistivity K of the dielectric (which is liable to considerable variation in practice), and (2) a factor which depends on the cross-sectional dimensions of the cable and is known as the "geometric factor." Neither K nor the geometric factor can thus be obtained without a knowledge of the other.

Fortunately, other experimental methods are available for determining the geometric factor, these methods being based on the close relationship between the theories of heat, electrostatics and current flow, established in Table 13.

TABLE 13.—*Analogy between Thermal, Dielectric, and Electric Circuits.*

Thermal Circuit.	Dielectric Circuit.	Electric Circuit.
Thermal flux	Dielectric flux	Current
Thermal resistance	Elastance	Resistance
Temperature difference	Potential difference	Potential difference

This analogy is found to be exceedingly useful when dealing with multicore cables. Thus, supposing that the conductors of a multicore, belted-type cable are bunched together and an alternating pressure applied between conductors and sheath, then

the path taken by the dielectric flux is also the path taken by the leakage current and the heat generated in the conductors. Also the equipotential lines and isothermal lines are coincident.

Hence if

$$S' = \text{thermal resistance},$$
$$R' = \text{insulation resistance},$$

and

$$C' = \text{capacitance},$$

all these values being for the arrangement of the three conductors against the sheath and for the same length of cable, it can easily be shown that

$$G = \frac{6\pi S'}{K} = \frac{6\pi R'}{\rho} = \frac{3\epsilon}{2C'}$$

where the geometric factor

$$G = \left(0{\cdot}85 + \frac{0{\cdot}2t}{T}\right) \log_e \left[\left(8{\cdot}3 - \frac{2{\cdot}2t}{T}\right)\frac{T + t}{d} + 1\right],$$

and K, ρ and ϵ are respectively the thermal resistivity, volume resistivity and permittivity of the dielectric.

Although it thus appears possible to use two other methods of obtaining G, *viz.* by measurements of R' or C', the same dilemma is reached if measurements on actual cables are attempted owing to unknown variation in the values of ρ and ϵ for impregnated paper when forming part of a cable structure.

The difficulty is avoided by the use of cable models in which the dielectric is represented by a material whose electrical constants are accurately known. From principles of symmetry the models can be of any convenient size, provided that the cross-sectional dimensions are proportional to those of the actual cable.

In practice, different methods of carrying out measurements have been employed. Thus Atkinson[6] and Sachetto[7] have measured the ohmic resistance between three symmetrically-spaced tubes representing the three conductors, surrounded by a fourth tube representing the sheath, all immersed in electrolyte. The former has also employed other models consisting of sheets of tinfoil on which were soldered three copper rings to represent the conductors, surrounded by a fourth ring representing the sheath. In connection with the E.R.A. research, an electrostatic model was constructed with air as dielectric and measurements of capacitance made. Probably the resistance methods are the most convenient in practice, and have the advantage that the field distribution

between conductors and sheath can be completely mapped out. Fig. 79, for example, shows the lines of heat flow and isothermal lines in a typical three-core cable, this diagram having been obtained by exploring the distribution of electric potential in the electrolytic type of model.

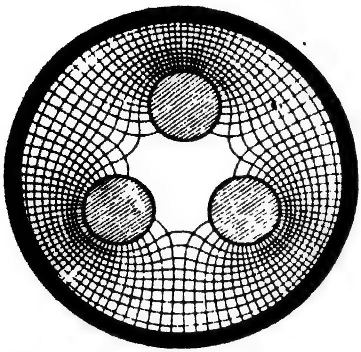


FIG. 79.—Lines of heat flow and isothermal lines in three-core, belted-type cable.

It is satisfactory to note that the values of thermal resistance derived from the various experimental methods described above are in close agreement with those calculated by formula (175), so that full confidence can be placed in the latter.

In the case of three-core, belted cables with shaped conductors, the thermal resistance between conductors and lead is substantially less than for circular conductors of the same area (diameter = d), and having the same thickness of insulation. Approximate correction factors for S' are given in Table 14.

TABLE 14.—*Approximate Correction Factors for Thermal Resistance* (S') *of Three-core, Belted-type Cables with Shaped Conductors.*

$\frac{T+t}{d}$	Correction Factor.	$\frac{T+t}{d}$	Correction Factor.
0·10	0·58	0·50	0·80
0·20	0·67	0·60	0·82
0·30	0·73	0·70	0·84
0·40	0·77	0·80	0·86

However, owing to the fact that the smaller overall diameter of a shaped-conductor cable involves a corresponding increase in the thermal resistance external to the cable, the increase in current-carrying capacity of this type of cable *in situ* is usually negligible.

The thermal resistance S'' of the protective covering of a three-core cable is calculated by an analogous formula to that given in connection with single-core cables (Art. 132). It is only necessary

to take into account the thermal resistance of the textile wrappings—bedding or serving as the case may be—as the temperature drop through the sheath and any metallic armouring is relatively insignificant.

134. Thermal Resistance of Screened and S.L.-type Cables.—Although the metallic layers round the cores of screened cables are extremely thin, their high thermal conductivity—in the case of aluminium 1 100 times, and in the case of copper 2 000 times, that of impregnated paper—improves the heat dissipating properties of the cable very considerably. Heat flowing outwards from the conductors arrives at the screens, and has then two paths by which it may reach the sheath: (1) Circumferentially along the screens, and (2) through the fillers. No satisfactory theoretical method of calculating the thermal resistance for this type of cable has yet been established, although Simons [8] has given a rigid solution on the assumption that all the heat is transmitted along the screens. Where the screens are composed of 3-mil. copper tape Simons' formula gives reasonably accurate results, but with 1·5-mil. aluminium-foil screens neglect of the alternative heat path through the fillers introduces serious errors. An ingenious method of correcting for these errors has been given by Beavis,[9] which is probably the best treatment of the problem from a theoretical point of view.

On the whole, however, it seems preferable to establish thermal resistance values by means of experimental work. In connection with this, it is interesting to note that in screened cables the electric, dielectric and thermal paths between conductors and sheath are not analogous, owing to the fact that the screens, although equipotential, are not isothermal, surfaces. The difficulty is avoided by using electrical resistance networks as cable models, as in this case by suitable design similarity can be secured between current flow in the model and heat flow in the cable. Errors due to the variation of dielectric thermal resistivity in actual cable samples are also avoided. The results of some work of this character, carried out by the author, show that the reduction in thermal resistance between conductors and sheath depends on such factors as the size of conductor, relative thickness of insulation, thickness and material of screens, etc. With practical sizes of cable the reduction with 1·5-mil. aluminium-foil screens is 15 to 30 %, and with 3-mil. copper-tape screens 30 to 45 %.[10] Thermal resistance charts have

been given for practical sizes of cables, but failing these the thermal resistance between conductors and sheath of screened cables (K = 550) can be calculated by the following formula :—

1·5-*mil. Aluminium-foil screens.*

$$S' = \left(57{\cdot}1 - \frac{10{\cdot}2}{(1 + T)(0{\cdot}40 + A)}\right) \log_{10}\left(\frac{8{\cdot}3T}{d} + 1\right)$$

thermal ohms per cm. (177)

3-*mil Copper-tape screens.*

$$S' = \left(57{\cdot}1 - \frac{39{\cdot}8}{(1 + T)(1{\cdot}15 + A)}\right) \log_{10}\left(\frac{8{\cdot}3T}{d} + 1\right)$$

thermal ohms per cm. (178)

where T = conductor-insulation thickness,
and d = conductor diameter

both in the same units,

and A = sectional area of conductor in sq. ins.

These formulæ apply to the case of cables with circular conductors. Corresponding values for screened cables with shaped conductors may be calculated from data published in the latest E.R.A. report,[2] but—as in the case of belted cables—the decrease in internal thermal resistance due to conductor shape is found to be more or less negatived by the increase in external thermal resistance due to smaller overall diameter of the cable. Hence, the current rating of shaped-core screened cables can be determined in practice by calculating the current-carrying capacity of the equivalent circular-conductor cable, *i.e.* a cable having the same sectional area of conductor, and same insulation thickness.

The calculation of the thermal resistance of the protective covering with either type of cable is carried out as described in connection with belted cables.

The S.L.-type of cable is not quite analogous to the screened cable from a thermal point of view, as the metallic sheaths round the individual cores are not in direct contact with the outer isothermal layer formed in this case by the armouring. There are the following elements of the thermal path to be taken into consideration : (*a*) conductors to sheaths, (*b*) sheaths to armouring, and (*c*) protective covering. There is no doubt that owing to the substantial character and isolation of the sheaths they may be considered for all practical purposes as isothermal surfaces, and hence

(*a*) may be considered as the thermal resistance of three single-core cables arranged in parallel. The thermal resistance (*b*) between sheaths and armouring may also be calculated by the following modification of Simon's formula :—

$$S'' = 64{\cdot}2 \log_{10}\left(\frac{6{\cdot}1t'}{d'} + 1\right) \text{ thermal ohms per cm.,} \quad (179)$$

d' now being the external diameter of each lead sheath, and t' the thickness of taping between sheaths and armouring. In this formula the thermal resistivity K_1 of the filling material and tapes over the lead-covered cores is taken as 500. The only remaining part of the path is (*c*), which is formed by the serving, and does not present any difficulties, being calculated by a suitable modification of formula (174).

135. Thermal Resistance of Ground Path.—When a cable is buried direct in the ground, the heat flow external to the cable is subject to the same laws that apply to the flow through the cable itself. It is thus dependent on two factors, the thermal resistivity of the soil and the lines of heat flow from the cable.

The thermal resistivity of the soil depends largely upon the amount of moisture present, and partly upon the type of soil. Both these factors are liable to wide variation from point to point along the cable route, particularly in city areas, and an idea of the range of values that may be found in practice are given in Table 15. The important influence of varying moisture content on

TABLE 15.—*Thermal Resistivity and Moisture Content of Various Types of Soil.*

Type of Soil.	Moisture Content.	Thermal Resistivity.
Clay . . .	20-25 %	80-160 thermal ohms
Sandy loam .	10-15 %	90-130 ,,
Gravel . .	4-8 %	50-100 ,,
Sand . . .	4-12 %	70-110 ,,
Chalk . .	15-20 %	90-140 ,,
Made-up soil .	12-22 %	90-140 ,,

certain types of soil is also shown in Fig. 80, but here again there are considerable variations in the thermal characteristics of soils nominally of the same type, and the curves given are strictly applicable only to the particular samples tested. In general, it is

found that the nature and moisture content of soils likely to be met in cable-laying practice are such as to limit the range of thermal resistivity to values between 70 and 160. In anticipation of later remarks it should be stated that all values mentioned above are those obtained by "box" methods in the laboratory.

As regards the path taken by the heat dissipated from the cable, two theories have been put forward. Kennelly[11] assumes that the surface of the ground above the cable is a plane isothermal, implying that all the heat is ultimately transmitted to the surface of the ground, which, being in contact with free air, remains at a uniform

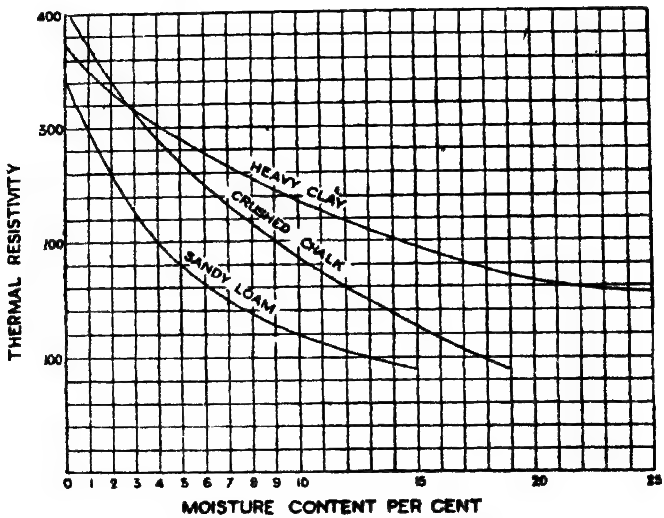


FIG. 80.—Thermal resistivity of soil.

temperature. On this assumption it can be shown that the thermal resistance of the ground path

$$G = \frac{g}{2\pi} \log_e \frac{2l}{R_2} \quad \text{thermal ohms per cm.,} \qquad (180)$$

where g = thermal resistivity of the soil,

l = depth of cable axis below surface of ground,

and R_2 = overall radius of cable (supposed small in comparison with l).

According to Apt[12] the effect of the earth surrounding the cable can be considered to be equivalent to that of a cylinder of

material equal in resistivity to the soil, and having a radius equal to the depth of the cable axis below the surface of the ground. On this assumption the cable is surrounded by a series of concentric isothermals and

$$G = \frac{g}{2\pi} \log_e \frac{l}{R_2} \quad \text{thermal ohms per cm.} \qquad (181)$$

Experimental work with buried cables seems to show that neither of the above theories are completely satisfactory, but that the effect of varying depth is represented much better by formula (180). It is found, however, that soil resistivity values determined by the usual laboratory methods must be multiplied by a correction factor of two-thirds in order to obtain correlation between calculated and measured values of current-carrying capacity. More recently, methods of measuring the thermal characteristics of soil *in situ* have been evolved, and it appears that the necessity for this correction factor is due to errors which are inherent to box methods of testing using disturbed soil. If these improved methods of measurement are adopted there is no need for any correction factor.

In the preparation of current-rating tables it is desirable to allow for the worst conditions likely to be met with in practice, and for this purpose the actual thermal resistivity of the soil is taken as 120 (box value = 180). Substituting in formula (180), therefore, the thermal resistance of the ground path under standard conditions is

$$G = 44 \log_{10} \frac{2l}{R_2} \quad \text{thermal ohms per cm.} \qquad (182)$$

The influence of soil resistivity and depth of laying on the current-carrying capacity of a cable depends on the relative values of the thermal resistances external and internal to the cable. For high-voltage, three-core cables laid under normal conditions, G may vary between 1 to 3 times the value of S. Hence, if the thermal resistivity of the soil is reduced from 120 to 60, the current-carrying capacity can be increased by 15 °/。 to 30 °/。. Variations in the depth of laying within reasonable limits affect the permissible loading to a comparatively small extent. The reduction in rating due to increasing the depth of laying from 3 ft. to 6 ft., for instance, would be about 8 °/。.

136. Grouped Cables Laid Direct in the Ground.—Hitherto, only cables laid singly have been dealt with, but the more important

diameter of the cables tested, varying inversely as the nth root, n having a value between 5 and 10 according to the number of cables involved and their arrangement. This variation in h is, however, only of significance in the case of cables of small diameter. For cables between $1\frac{1}{2}$ and 4 inches overall diameter—which practically covers the entire range of high-tension cables—h may be taken as substantially constant.

The practical formulæ for external thermal resistance of cables given in Table 16 have been derived from the N.P.L. data. In connection with these formulæ it should be noted that values of h for unserved cables—plain lead-sheathed or bare-wire armoured—

TABLE 16.—*External Thermal Resistance of Cables in Air.*

Arrangement of Cables.	Thermal Resistance in Thermal Ohms per cm.	
	Served Cables.	Unserved Cables.
Single cable	$\dfrac{133}{R_2\theta_s^{\frac{1}{4}}}$	$\dfrac{153}{R_2\theta_s^{\frac{1}{4}}}$
Two cables touching	$\dfrac{157}{R_2\theta_s^{\frac{1}{4}}}$	$\dfrac{184}{R_2\theta_s^{\frac{1}{4}}}$
Three cables in flat formation touching	$\dfrac{174}{R_2\theta_s^{\frac{1}{4}}}$	$\dfrac{202}{R_2\theta_s^{\frac{1}{4}}}$
Three cables in trefoil formation touching	$\dfrac{216}{R_2\theta_s^{\frac{1}{4}}}$	$\dfrac{241}{{}_2\theta_s^{\frac{1}{4}}}$

have been taken at the mean between black and bright surface conditions, and in the case of served cables h has a value corresponding to the black condition. In all cases R_2 must be expressed in inch units.

When using the formulæ, θ_s must be taken at an assumed figure, say two-thirds of the permissible temperature rise θ of the conductor, and the approximate value of A calculated. θ_s is then corrected from the relation

$$\theta_s = \frac{\theta A}{S + A}, \qquad . \quad . \quad . \quad . \qquad (187)$$

and a final calculation of A made if necessary.

A cursory examination shows that the approximate rating reduction factor due to grouping is 95 °/. for two cables run together

touching, and 85 °/。 for three cables run together in trefoil formation and touching. Cables run with a minimum spacing of 3 in. between adjacent surfaces suffer no appreciable reduction of rating due to grouping, but where a large group of cables are run together in situations where there is no free ventilation conditions are rather different. The reduction of rating necessary in this latter case is best taken into consideration by fixing a higher ambient air temperature, and thus reducing the permissible temperature rise of the conductors. Cables run outdoors should naturally be screened from the direct rays of the sun or a serious diminution of rating will be involved, but the method of screening should be such that no ventilating restrictions are imposed.

Generally speaking, cables in air have a greater carrying capacity than when laid direct in the ground. An increase of 5 to 15 °/。 for multicore cables, and rather more in the case of three single-core cables run together, is often possible, the comparison being made on the same basis as regards temperature conditions.

138. Cables Drawn into Ducts.—The rating of cables drawn into ducts presents a more difficult problem than that of cables laid direct in the ground owing to the greater complexity of the thermal path. In the simplest case—one cable in an isolated single-way duct—the thermal resistance exterior to the cable can be divided into three parts: (*a*) outer surface of cable to inner wall of duct, (*b*) inner wall of duct to outer wall of duct, and (*c*) outer wall of duct to surface of ground. Both (*b*) and (*c*) can be calculated by means of formulæ of the type previously developed, but the determination of (*a*) is a matter of more uncertainty, depending as it does on heat losses by radiation, convection, and conduction. However, by assuming typical values for the emissivities of cable and duct surfaces, eccentricity of cable in duct, etc., it is possible to calculate from theoretical principles the thermal resistance of this part of the path, and hence the total thermal resistance external to the cable. Reference should be made to the latest E.R.A. report [2] for full details, but it may be stated that the method is rather tedious. In actual practice sufficient accuracy is obtained by taking the current rating of a cable in a single-way duct as 85 °/。 of that applicable to the same cable when laid direct, similar conditions of initial temperature and temperature rise being assumed in each case.

The case of a cable drawn into a multiway duct or group of single-way ducts, either alone or with loaded cables in the adjacent duct ways, is too involved for mathematical treatment. Nevertheless, as a result of experimental work, both with actual duct systems and duct models, it has been possible to derive approximate ratings for a group of cables not exceeding nine in number. It is assumed that standard multiway ducts are employed, or in the case of single-way ducts that they are arranged as compactly as possible, that the diameter of the duct ways is 4 in. or 5 in., and that the cables are similar and equally loaded. The rating of a single cable drawn into such a duct system can be taken as 80 °/₀ of that applicable to the same cable when laid direct under the same temperature conditions. When more than one loaded cable is run together in the duct system, the rating for a cable running alone must be multiplied by the appropriate reduction factor given in Table 17.

TABLE 17.—*Approximate Reduction Factors for Loading of Cables Run Together in Ducts at Close Spacings.*

2 Cables	0·92	6 Cables	0·70
3 ,,	0·84	7 ,,	0·66
4 ,,	0·79	8 ,,	0·63
5 ,,	0·74	9 ,,	0·60

In cases where cables are drawn into special arrangements of ducts having wider spacings there will naturally be some amelioration of rating, and the figures in Table 17 can then only be taken as limiting values.

139. Cables Laid on the Solid System.—There is not much information available on the current-carrying capacity of cables laid solid owing to the comparative infrequency of this method of installation. Generally speaking, the facilities for heat dissipation are not so good owing to the comparatively high thermal resistivity of the bitumen trough-filling compound; pure bitumen having a thermal resistivity of about 500. For all practical purposes the current rating can be taken as 90 °/₀ of the rating for the same cable laid direct in the ground ($g = 120$). Grouping factors are also usually higher, since all the cables run in one trough must be laid quite close together.

140. Submarine Cables.—For such situations no thermal resistance external to the cable need be assumed as the water maintains the outer surface at base temperature. Hence, the current-carrying capacity is much higher than for cables laid direct in the ground, increases of 40 to 100 °/。 in the rating being possible.

141. Current-carrying Capacity of Single-core Cables.—In the case of power transmission by single-core cables mutual heating and sheath losses must be taken into account, and at the higher voltages there is also the question of dielectric losses. In the following discussion it will be assumed that the dielectric losses are concentrated at the conductor surface, so that the heat generated by these losses and the I^2R losses flows through the complete thermal path. The loss generated in the sheath affects, of course, only that portion of the path external thereto. Hence, the fundamental equation for the temperature rise of a circuit of single-core cables laid direct in the ground is

$$\theta = (I^2R_\theta + W_d)S' + \{I^2R_\theta(1 + p) + W_d\}(S'' + G \quad G_x) \quad (188)$$

and the current-carrying capacity

$$= \sqrt{\frac{\theta - W_d(S' + S'' + G + G_x)}{R_\theta\{S' + (1 + p)(S'' + G + G_x)\}}}, \quad (189)$$

where θ = temperature rise of conductor in °C.,
R_θ = resistance of conductor in ohms at working temperature corresponding to temperature rise θ (skin and proximity effects included),
W_d = dielectric loss in watts per phase,
p = ratio between sheath losses and conductor losses,
S' = thermal resistance between conductor and sheath of one cable,
S'' = thermal resistance of protective covering,
G = thermal resistance of ground path,
and G_x = apparent thermal resistance due to mutual heating.

In the case of one feeder laid direct in the ground with the three cables touching each other

$$G_x = \frac{g}{\pi} \log_e \frac{l}{R_2} \quad . \quad . \quad . \quad (190)$$

where R_2 is the overall radius of each cable. Where more than

one feeder is laid in the same trench G_s must be extended to include the effect of all the other individual cables.

For single-core cables run in air

$$I = \sqrt{\frac{\theta - W_d(S' + S'' + A)}{R_\theta\{S' + (1 + p)(S'' + A)\}}}, \qquad (191)$$

the correction for mutual heating in this case being incorporated in the appropriate value of A derived from Table 16. If the cables are plain lead covered, as is often the case for this method of installation, $S'' = 0$.

The current-carrying capacity of single-core cables in ducts can be determined by the application of the methods given in Art. 138.

For cables operating at voltages lower than 66 kV. the effect of dielectric losses on the rating is usually negligible and the term involving W_d can then be eliminated from the above formulæ. Similarly, for cases where open-circuited sheaths may be used, such as short runs of cable in buildings, p may be taken as equal to zero.

142. Maximum Permissible Temperature for Cables.—Methods of calculating the current-carrying capacity of cables installed under various conditions have now been described, and there remains to be settled the question of operating temperatures. A considerable amount of research has been carried out to determine the temperature to which cables can be subjected for long periods without damage. As regards the insulating material it is well known that at sustained temperatures of the order of 100° C. impregnated paper suffers injury to its mechanical and electrical properties, and the dielectric losses which are increased at these temperatures may lead to rapid deterioration. Hence, so far as the properties of the dielectric are concerned, a maximum temperature of 80° to 85° C. should not be exceeded. It is necessary, however, to take into consideration the expansion and contraction effects and consequent mechanical strains on cables subjected to alternate heating and cooling. Taking the following coefficients of expansion,

Copper . . .	$16 \cdot 8 \times 10^{-6}$ per degree C.
Lead	$27 \cdot 5 \times 10^{-6}$,, ,, ,,

the actual expansion per 100-yard length of cable for a 50° C.

rise would be about 3 in. for copper and 5 in. for lead. It should be borne in mind, however, that the lead sheath undergoes less temperature variation than the conductor, the sheath temperature rise in high-voltage cables being from one-third to two-thirds of the temperature rise of the conductor. Hence, the lower temperature rise of the lead compensates to some extent for its higher coefficient of expansion, and the actual differential expansion of the two metals may be smaller than would be indicated by the above figures.

In practice, with cables laid direct in the ground the longitudinal expansion of the cable rarely gives any trouble, but when the cables are installed in ducts the possibility of longitudinal movement is much greater. Hence, the temperature limits must be kept to reasonable figures, and care must be taken with the layout of cables and joints at all jointing positions so as to accommodate this movement without damage. This is particularly important when the cable is unarmoured owing to its greater susceptibility to damage at the point of entry into the ducts.

There is also the effect of temperature rise on the physical and electrical constants of the impregnating compound to be considered. At high temperatures the viscosity of the compound suffers serious reduction, and drainage may occur from the higher sections of cables laid on gradients. Excessive differential expansion effects between compound and sheath will also result in void formation under partial vacua when load is removed, with consequent risk of ionisation troubles. Also, with cables for the highest voltages there is the danger of thermal instability due to steep increases in the dielectric losses.

Consideration of the above phenomena leads to the conclusion that it is necessary to adopt somewhat different temperature limits according to the type, voltage class, and method of installation, of the cable.

It is readily seen that the question of operating temperature is not an easy matter to settle, but a maximum conductor temperature of 70° C. for cables of normal construction has been finally adopted by the Cable Makers' Association. This figure applies to the case of cables for pressures below 11 kV., the cables being laid direct in the ground or in air. At higher voltages or with cables laid in ducts, lower maximum temperatures are adopted, and a certain amount of differentiation is also made between belted

cables and those of other types. The basic values of maximum operating temperature now in use are given in Table 18. With cables having special dielectrics or operated under special conditions, such as oil-filled cables or pressure cables, the maximum conductor temperature can be raised to 75° C. except for the case of unarmoured cables drawn into ducts.

The permissible *temperature rise* of a cable depends on the value taken for the initial or base temperature. In the case of buried cables a standard temperature of 15° C. has been adopted

TABLE 18.—*Maximum Conductor Temperatures for Paper-insulated Cables.*

Voltage.	Cables Laid Direct in the Ground, and in Air.			Cables Laid in Ducts.		
	Belted	Single-core.	Screened and S. L.-type.	Belted.	Single-core.	Screened and S.L.-type.
Below 11 kV.	70° C.	70° C.		70° C. (A) 50° C. (U)	50° C. (U)	
11 kV.	65° C.	70° C.	70° C.	65° C. (A) 50° C. (U)	50° C. (U)	70° C. (A) 50° C. (U)
22 kV.	55° C.	65° C.	65° C.	55° C (A) 50° C. (U)	50° C. (U)	65° C. (A) 50° C. (U)
33 kV.		60° C.	60° C.		50° C. (U)	60° C. (A) 50° C. (U)
66 kV.		*60° C.			*50° C. (U)	
Oil-filled and Pressure Cables. (All voltages.)		75° C.	75° C.		50° C. (U)	75° C. (A) 50° C. (U)

A = Armoured. U = Unarmoured. * 45° C. for the earlier installations.

as the basis of calculations. With cables in air the ambient temperature is liable to vary within wider limits and would, in general, be higher than 15° C. The tables of current ratings published by E.R.A., for example, are based on an ambient temperature of 25° C.

143. Transient Heating Conditions and Intermittent Loading.—So far in dealing with the heating of cables only the ultimate temperatures attained have been considered. The period necessary for the attainment of steady temperature conditions for cables laid underground is measured in days rather

than hours, but varies with different classes of cable, and different methods and depths of laying. The rate at which the temperature rises is also considerably influenced by the specific heat of the soil, as it requires time to heat the layers of soil until they become steady isothermal surfaces. As a rough guide, the time taken for this temperature to attain 90 °/₀ of its final value with a steady load may be taken as 12 to 24 hours with cables laid direct in the ground. With cables in air the period for a similar percentage rise would be about three hours, and cables laid in ducts would generally occupy an intermediate position between cables laid

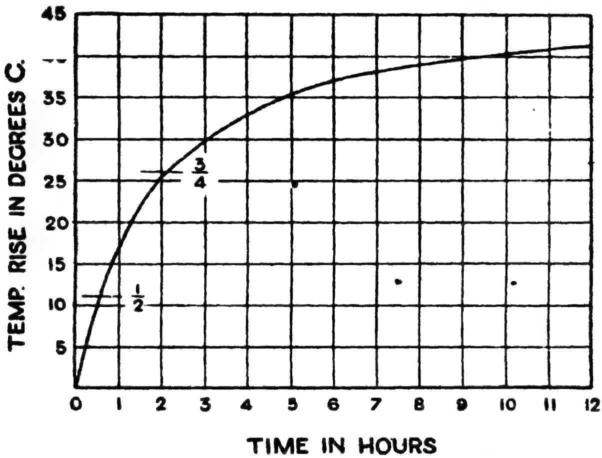


FIG. 81.—Temperature rise of 20 000-volt, 0·35 sq. in., three-core armoured cable laid direct in the ground; ultimate temperature rise = 50 degrees C.

direct in the ground and in air. Fig. 81 shows a typical time/temperature rise curve for a 20 000-volt, 0·35 sq. in., three-core cable laid direct in the ground at a depth of 2 ft., the final temperature rise being 50° C.[1]

In all the cases previously considered the load has been assumed to be steady, but under practical operating conditions this is not so, the maximum load only occurring for a comparatively short period each day. Hence, in the case of cables intermittently loaded, it is reasonable to suppose that a greater load than that obtained from calculations based on steady state theory can be transmitted at times of peak without exceeding the temperature limit of the cable.

There are three chief types of intermittent loading to be considered :—

1. Rapidly fluctuating loads.
2. Emergency loading.
3. Cyclic loading due to daily load factor.

1. Rapidly fluctuating loads may occur in the case of traction feeders or with cables supplying rolling-mill motors, etc., the period of variation being measured in minutes. Their effect may be taken into account by determining the current-carrying capacity of the cable on the basis of the R.M.S. value of the current.[11]

2. The emergency loading of a cable for a period of one or two hours can be readily determined if a time / temperature rise curve of the cable *in situ* is available. To illustrate the method employed take the simplest case where the cable has not previously been carrying any load. The permissible current is deduced by setting off the emergency time on the time / temperature rise curve starting from zero ; then the square root of the ratio between the permissible temperature rise of the cable and the actual temperature rise in the short time selected gives the multiplier for that time, to apply to the continuous permissible current. Thus, referring to Fig. 81, the temperature rise after 2 hours is approximately 25 degrees C., so the multiplying factor is $\sqrt{\frac{50}{25}} = 1{\cdot}41$, which means that the particular cable under consideration will carry for 2 hours starting from cold 41 °/ₒ more current than the continuous rating, without exceeding the specified temperature rise of 50 degrees C.

If the cable has previously been carrying a steady load the relative temperature rise for this load can be calculated from formula (168), and this rise is taken as the starting-point on the time / temperature rise curve. The emergency time is then set off from this point, and the total temperature rise from zero read off and treated as before, thus giving the multiplying factor for the specified conditions.

In the curve of Fig. 81 has been marked the final temperature points 11 degrees C. for one-half continuous, and 26 degrees C. for three-quarters continuous, load respectively. Supposing, then, that the cable has been previously carrying a steady load equal to

one-half its rating, and it is required to find what load can be carried for an emergency period of 2 hours. The temperature rise corresponding to the initial load on the cable is 11 degrees C., and marking off from this point on the curve the scale value of 2 hours horizontally, a temperature rise of 28 degrees C. is obtained. Then the multiplying factor is $\sqrt{\frac{50}{28}} = 1{\cdot}33$, or a permissible overload of 33 % can be carried for the 2 hours, without exceeding the limiting temperature rise of 50 degrees C.

TABLE 19.—*Rating Factors for Emergency Loading of Cables (One-hour Period.)*

Diameter of Cable over Lead Sheath.	Cables Laid Direct in the Ground. Subsequent to Steady Loading of 50 % Continuous Load.	Cables Laid Direct in the Ground. Subsequent to Steady Loading of 75 % Continuous Load.	Cables in Air. Subsequent to Steady Loading of 50 % Continuous Load.		Cables in Air. Subsequent to Steady Loading of 75 % Continuous Load.	
1 in.	1·16	1·09	1·06 (s.c.)	1·04	1·05 (s.c.)	1·02
1½ ,,	1·20	1·11	1·15 (s.c.)	1·09	1·10 (s.c.)	1·06
2 ,,	1·24	1·14	1·17		1·10	
2½ ,,	1·28	1·17	1·24		1·15	
3 ,,	1·33	1·20	1·32		1·20	
3½ ,,	1·40	1·24	1·40		1·25	

s.c. = single-core cables only. All other values are for three-core cables.

In actual practice curves similar to Fig. 81 are rarely available, and Whitehead[2] has made a theoretical investigation of the subject The main difficulty is the estimation of the thermal diffusivity of the soil which varies widely, but taking a low value for this constant in order to furnish a margin of safety it is possible to calculate emergency-loading factors for cables buried in the ground. Similar factors for cables in air may also be determined. Table 19, prepared on this basis, gives emergency-loading factors for a one-hour period after the cable has been carrying a steady load of 50 % or 75 % of its continuous load as the case may be. For

example, the sheath diameter of the cable referred to in Fig. 81 is 3¼ in., so from Table 19 the emergency one-hour loading when buried direct would be 40 % more than continuous load after carrying one-half load, and 24 % more than continuous after three-quarters load. These figures compare with 48 % and 28 % derived from the experimental curve.

3. Whitehead[2] has also derived formulæ for calculating the effect of cyclic loading due to daily load factor, and the basic type of load variation adopted is that in which the cable carries each day a steady current for 8 hours and is then idle for the remaining 16 hours. Investigations show that with cables buried in the ground the magnitude of the effect depends chiefly on the depth of laying, and only to a comparatively small extent on the type, size, and voltage class of the cable. At a depth of 18 in., for instance, the permissible maximum load of any cable is 7 % greater than the continuous loading; at a depth of 3 ft., the corresponding increase is 13 %. In the case of cables in air the transient heating period is so short that no extra allowance can be made for cyclic loading of the type considered.

144. Cable Rating as Affected by Practical Conditions of Installation.—In any scheme of transmission by underground cables, temperature conditions along the route are rarely uniform, and it may happen that the rating of a long length of cable is limited by a comparatively short section where the facilities for heat dissipation are poor. Quite apart from variations in the thermal characteristics of the soil and cable dielectric—which are covered by the conservative values of thermal resistivities adopted as standard—hot spots may be produced by extra depths of laying or closer spacing of cables rendered necessary by obstacles in the run of the trench. Cables, nominally laid on the direct system, are also often drawn through ducts under important thoroughfares in order to avoid disturbance to traffic during, or subsequent to, installation. Naturally the cable rating is determined by the conditions imposed in these sections of the route, and care should be taken to adopt wide spacings for ducts used in these particular circumstances. On the other hand, where cables are taken through foundations or walls of buildings by means of ducts—the length involved being two or three feet only—there is considerable amelioration owing to longitudinal conduction, and the presence of the ducts has no substantial effect on the rating.

Care should always be taken to see that the loading of a feeder is not unduly restricted by terminal lengths of cable run in air. High ambient temperatures are sometimes encountered, and under these conditions it may happen that a larger conductor section is necessary notwithstanding the lower external thermal resistivity of cables in air as compared with cables laid direct in the ground. Adequate ventilation is important in this connection, since the heat dissipation from cables in air is much improved when the air is in movement, however slight this may be.

As regards intermittent loading it should be noted that, in general, cables in air are not so favourably placed as cables buried in the ground. No increase in rating due to the effect of daily load factor can be allowed, so that the cables must be designed for the peak load. With emergency loads there is little difference in the behaviour of cables of medium and large diameter, whether in air or buried in the ground. The case often occurs, however, where a three-core cable in the ground is terminated by single-core tails in air. The continuous rating of the tails may be equal to, or greater than, that of the underground cable, but the smaller diameter of the cables in air, and their correspondingly small emergency rating factor, may again determine the permissible peak loading. The question of conductor sizes on terminal lengths of cables should, therefore, always receive careful consideration.

145. Numerical Examples.—In order to illustrate the methods of calculation previously derived, the following examples will be taken :—

Problem 9.—Find the thermal resistances of 25 000-volt, three-core, 0·25 sq. in., circular-conductor cables of the following types :—

(*a*) *Belted.* Conductor-insulation thickness 0·225 in., belt-insulation thickness 0·175 in., overall diameter 3·60 in.

(*b*) *Screened.* Screens of 3-mil. copper tape, insulation thickness 0·30 in., overall diameter 3·62 in.

The protective coverings have the same thicknesses in each case, *viz.*, lead sheath 0·15 in., armouring 0·128 in., hessian tapes 0·15 in.

(*a*) *Belted-type.*

In this case $\dfrac{t}{T} = \dfrac{0\cdot 175}{0\cdot 225} = 0\cdot 78,$

$$\frac{T + t}{d} = \frac{0\cdot 225 + 0\cdot 175}{0\cdot 651} = 0\cdot 615,$$

and taking $K = 550$, the thermal resistance between conductors and sheath from (176) is

$$S' = 67{\cdot}2(0{\cdot}85 + 0{\cdot}2 \times 0{\cdot}78) \log_{10} [(8{\cdot}3 - 2{\cdot}2 \times 0{\cdot}78)0{\cdot}615 + 1]$$
$$= 47{\cdot}5 \text{ thermal ohms per cm.}$$

The thermal resistance of the protective covering from (174) (after making the necessary modifications) is

$$S'' = 183 \log_{10} \frac{1{\cdot}80}{1{\cdot}65}$$
$$= 6{\cdot}8 \quad \text{thermal ohms per cm.,}$$

so that the thermal resistance of the complete cable

$$S = 47{\cdot}5 + 6{\cdot}8$$
$$= \underline{54{\cdot}3 \quad \text{thermal ohms per cm.}}$$

(*b*) *Screened-type.*

In this case $\dfrac{T}{d} = \dfrac{0{\cdot}30}{0{\cdot}651} = 0{\cdot}461,$

and substituting in (178) the thermal resistance between conductors and sheath

$$S' = \left(57{\cdot}1 - \frac{39{\cdot}8}{1{\cdot}3 \times 1{\cdot}4}\right) \log_{10} (8{\cdot}3 \times 0{\cdot}461 + 1)$$
$$= 24{\cdot}0 \quad \text{thermal ohms per cm.}$$

The thermal resistance of the protective covering is

$$S'' = 183 \log_{10} \frac{1{\cdot}81}{1{\cdot}66}$$
$$= 6{\cdot}8 \quad \text{thermal ohms per cm.,}$$

so that the total thermal resistance of the cable

$$S = 24{\cdot}0 + 6{\cdot}8$$
$$= \underline{30{\cdot}8 \quad \text{thermal ohms per cm.}}$$

Problem 10.—Find the current-carrying capacity of two 25 000-volt, three-core, 0·25 sq. in., screened-type cables of the dimensions given above, when laid under the following conditions:—

(*a*) Direct n the ground at a depth of 4 feet, with a horizontal interaxial spacing of 18 in. ($g = 120$).

(*b*) In air, with an interaxial spacing of 8 in.

(*c*) In 5-in. closely-spaced earthenware ducts at a depth of 4 feet ($g = 120$).

The initial temperature in each case is assumed to be 15° C., and the maximum permissible operating temperature 55° C.

(*a*) *Laid direct.*

The thermal resistance of the ground path from (182) is

$$G = 44{\cdot}0 \log_{10} \frac{96}{1{\cdot}81}$$
$$= 75{\cdot}9 \quad \text{thermal ohms per cm.,}$$

and the apparent thermal resistance due to mutual heating is

$$G_x = \frac{60}{\pi} \log_e \frac{96}{18}$$
$$= 32{\cdot}0 \quad \text{thermal ohms per cm.}$$

Hence, the total thermal resistance of the cable *in situ* is $30{\cdot}8 + 75{\cdot}9 + 32{\cdot}0 = 138{\cdot}7$ thermal ohms per cm.

The resistance of the conductor at 20° C. from tables is 0·108 ohm per kilometre. Hence, allowing 2 °/ₒ for coring, the resistance at the maximum operating temperature 55° C. is

$$R_\theta = 0{\cdot}108\,(1 + 0{\cdot}004 \times 35)\,1{\cdot}02 \times 10^{-5}$$
$$= 1{\cdot}26 \times 10^{-6} \quad \text{ohms per cm.}$$

The current-carrying capacity of each cable from (185) is thus

$$\sqrt{\frac{40}{3{\cdot}78 \times 10^{-6} \times 138{\cdot}7}}$$
$$= \underline{276 \quad \text{amperes.}}$$

(*b*) *In air.*

The distance between adjacent cable surfaces in this case is 4·4 in., so that the mutual heating effect can be taken as negligible and the apparent thermal resistance external to the cable from Table 16, assuming $\theta_s = 35°$ C., is

$$A = \frac{133}{1{\cdot}81 \times 35^{\frac{1}{4}}}$$
$$30{\cdot}2,$$

so more accurately from (187)

$$\theta_s = \frac{55 \times 30{\cdot}2}{30{\cdot}8 + 30{\cdot}2}$$
$$27{\cdot}2° \text{ C.}$$

and recalculating

$$A = \frac{133}{1{\cdot}81 \times 27{\cdot}2^{\frac{1}{4}}}$$
$$= 32{\cdot}3 \quad \text{thermal ohms per cm.,}$$

so that the total thermal resistance of the cable *in situ* is $30{\cdot}8 + 32{\cdot}3 = 63{\cdot}1$ thermal ohms per cm.

The current-carrying capacity of each cable is therefore

$$\sqrt{\frac{40}{3{\cdot}78 \times 10^{-6} \quad 63{\cdot}1}}$$
$$= \underline{410 \quad \text{amperes.}}$$

If the initial temperature had been 25° C. in this case instead of 15° C., the corresponding current rating would have been

$$410 \times \sqrt{\frac{30}{40}} = 355 \quad \text{amperes.}$$

(*c*) *In ducts.*

Following the method given in Art. 138 the rating of one cable laid direct in the ground is

$$\sqrt{\frac{40}{3{\cdot}78 \times 10^{-6} \times 106{\cdot}7}}$$
$$= \underline{315 \quad \text{amperes,}}$$

so the rating of each cable when drawn into ducts is

$$I = 315 \times 0{\cdot}80 \times 0{\cdot}92$$
$$= \underline{232 \quad \text{amperes.}}$$

(In all these cases the skin and proximity effects total 1 %, and sheath and armour losses are about 3 %, so that the combined effect on the cable rating is insignificant. With larger conductor sections, where it is necessary to take these matters into consideration, the conductor resistance must be corrected for skin and proximity effect, and the thermal resistance external to the cable sheath increased to cover the sheath and armour losses as shown in the next example.)

Problem 11.—Find the current-carrying capacity of a 66 000-volt, three-phase circuit formed by three single-core, 0·40 sq. in. served cables (external diameter 2·77 in., lead sheath and tape serving each 0·15 in. thick), laid in close triangular formation under the following conditions:—

(*a*) Direct in the ground at a depth of 4 feet (g = 120).
(*b*) In air.
(*c*) In trefoil ducts at a depth of 4 feet (g = 120).

In each case the initial temperature is 15° C. and the maximum permissible temperature 45° C.

(a) *Laid direct.*

Taking $K = 550$ the thermal resistance of each cable between conductor and sheath from (172) is

$$S' = 202 \log_{10} \frac{1 \cdot 085}{0 \cdot 418}$$
$$= 83 \cdot 5 \quad \text{thermal ohms per cm.,}$$

the thermal resistance of the protective covering with K_1 500 from (174) is

$$S'' = 183 \log_{10} \frac{1 \cdot 385}{1 \cdot 235}$$
$$= 9 \cdot 0 \quad \text{thermal ohms per cm.,}$$

the thermal resistance of the ground path from (182) is

$$G = 44 \cdot 0 \log_{10} \frac{96}{1 \cdot 385}$$
$$= 81 \cdot 0 \quad \text{thermal ohms per cm.,}$$

and the apparent thermal resistance introduced by the heating effect of the other two cables from (190) is

$$G_s = 88 \log_{10} \frac{96}{2 \cdot 77}$$
$$= 135 \cdot 0 \quad \text{thermal ohms per cm.}$$

The d.c. resistance of the conductor at 20° C. from tables is 0·066 ohm per kilometre. Hence, at the maximum operating temperature of 45° C., the resistance—allowing for 1·3 °/ₒ skin effect and 0·3 °/ₒ proximity effect—is

$$R_\theta = 0 \cdot 066(1 + 0 \cdot 004 \times 25) \times 1 \cdot 016 \times 10^{-5}$$
$$= 0 \cdot 74 \times 10^{-6} \quad \text{ohms per cm.}$$

The sheath losses, which have been previously calculated (Art. 112), are about 12 °/ₒ so that $p = 0 \cdot 12$.

The dielectric losses, taking the capacitance of the cable as 0·36 microfarads per mile, and the dielectric power factor as 0·005, are

$$W_d \quad \frac{2\pi \times 50 \times 0 \cdot 36 \times 10^{-6} \times 38 \cdot 1 \times 38 \cdot 1 \times 10^{6} \times 0 \cdot 005}{1760 \times 36 \times 2 \cdot 54}$$
$$0 \cdot 005 \quad \text{watt per cm.}$$

Hence, by substitution in (189), the current-carrying capacity of the circuit is

$$\sqrt{\frac{30 - 0{\cdot}005 \times 308{\cdot}5}{0{\cdot}74 \times 10^{-6}(83{\cdot}5 + 1{\cdot}12 \times 225)}}$$

$$\underline{340 \quad \text{amperes.}}$$

(*b*) *In air.*

From Table 16 the thermal resistance from the surface of each cable, assuming $\theta_s = 30°$ C., is

$$A = \frac{216}{1{\cdot}385 \times 30^{\frac{1}{4}}}$$
$$= 66{\cdot}7,$$

so more accurately from (187)

$$\theta_s = \frac{45 \times (66{\cdot}7 \times 1{\cdot}12)}{92{\cdot}5 + (66{\cdot}7 \times 1{\cdot}12)}$$
$$20{\cdot}1° \text{ C.}$$

and recalculating

$$A = \frac{216}{1{\cdot}385 \times 20{\cdot}1^{\frac{1}{4}}}$$
$$= 74{\cdot}3 \quad \text{thermal ohms per cm.,}$$

and by substitution in (191) the current-carrying capacity is

$$\sqrt{\frac{30 - 0{\cdot}005 \times 166{\cdot}8}{0{\cdot}74 \times 10^{-6}(83{\cdot}5 + 1{\cdot}12 \times 83{\cdot}3)}}$$

$$\underline{472 \quad \text{amperes.}}$$

(*c*) *In ducts.*

The sheath losses at an interaxial spacing of 5½ in. (Table 12) are about 38 °/₀ and following the method given in Art. 138 the rating of one cable laid direct in the ground is

$$\sqrt{\frac{30 - 0{\cdot}005 \times 173{\cdot}5}{0{\cdot}74 \times 10^{-6}(83{\cdot}5 + 1{\cdot}38 \times 90)}}$$
$$= 435,$$

so the rating of each cable when laid together in ducts is

$$I = 435 \times 0{\cdot}80 \times 0{\cdot}84$$
$$= 292 \quad \text{amperes.}$$

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CHAPTER XI.

ECONOMIC PRINCIPLES AND CALCULATIONS.

146. Introductory.—It is proposed to deal in this chapter with one or two of those fundamental economic principles which are of vital importance, and closely influence the electrical design of a transmission line. It should not be forgotten that the justification of any scheme of power transmission must be commercial, as well as technical success. As Still has said: 'It is an easy matter to design a bridge of ample strength for the load it has to carry, or a transmission line with conductors of so large a size, insulators with so large a factor of safety, and supports so closely spaced and so strong, that the electrical losses will be small, and the risk of mechanical failure almost nil; but neither the bridge nor the transmission line will reflect credit on the designing engineer unless he has had before him constantly the commercial aspect of the work entrusted to him, and has so chosen or designed the various parts, and combined these in the completed whole, that all economic requirements are as nearly as possible fulfilled.'

The application of economic factors to the design and layout of a complete power-transmission scheme is outside the scope of the present work, and it is only possible to consider very briefly one or two of the simpler problems.

147. Economic Size of Line Conductor.—The cost of conductor material is generally a very considerable part of the total cost of a transmission line, and hence the determination of the size of conductor to be employed is a very important problem. No matter what particular scheme of transmission is adopted, a little consideration will show that the first cost of the conductors, and the annual expenditure represented by the energy wasted in heating the conductors, follow opposite laws. To reduce the power losses in the line it is necessary to employ conductors of low resistance and therefore large cross-sectional area. On the

other hand, to reduce the first cost, conductors of small cross-section must be installed. Thus, the initial cost and subsequent working expenses are both governed by the size of conductor chosen, but while the former increases with the area the latter decreases as the area increases. It is evident that in any transmission scheme there must be at least one particular size of conductor for which the sum of the annual charges (interest and depreciation on initial cost of conductors plus annual cost of energy dissipated therein) is a minimum.

Taking into consideration first of all merely the initial capital outlay on conductor material, the annual charges on this initial cost for a given line are

$$P_1 = k_1 A a, \qquad . \quad . \quad . \quad . \quad . \quad (192)$$

where A is the cross-sectional area of the conductor, a is the percentage to be taken to cover the annual interest and depreciation, and k_1 is a constant.

The cost of the energy dissipated in the line per annum is

$$P_2 = \frac{k_2}{A}, \qquad . \quad . \quad . \quad . \quad . \quad (193)$$

k_2 again being a constant. The total expenditure will be a minimum when

$$\frac{dP_1}{dA} + \frac{dP_2}{dA} \quad 0.$$

This gives

$$k_1 a \quad \frac{k_2}{A^2},$$

and

$$\sqrt{\frac{k_2}{ak_1}}. \qquad (194)$$

Inserting this value of A in (192) and (193) gives

$$P_1 = \sqrt{k_1 a k_2},$$

and

$$P_2 = \sqrt{k_1 a k_2},$$

hence

$$P_1 = P_2, \qquad (195)$$

or the most economical area of conductor is that which makes the annual cost of the power losses equal to the annual interest and depreciation on the capital cost of the conductor material.

148. Kelvin's Law and its Modifications.—The above statement, known as Kelvin's law, was first enunciated by Lord Kelvin (then Sir William Thomson) at a meeting of the British Association in 1881.[1] To quote his exact words: 'The most economical size of the copper conductor for the electric transmission of energy . . will be found by comparing the annual interest of the money value of the copper with the money value of the energy lost annually in the heat generated in it by the electric current. . . . The gauge to be chosen for the conductor does not depend on the length of it through which the energy is to be transmitted but solely on the strength of the current to be used, supposing the cost of the metal and of a unit of energy to be determined.'

Kelvin's law is based on the assumption that the cost of poles or towers, insulators, and the labour of erection and stringing of wires is either negligible or independent of the actual size of the conductor. In practice, neither of these assumptions is correct. This is owing to the fact that line supports and insulators tend to become more expensive as heavier conductors are used in consequence of the increased mechanical stresses occasioned thereby. Also the cost of stringing the wires will increase somewhat as the size of wire increases. As shown by Kapp,[2] a closer approximation to the truth is obtained by assuming that the initial capital expenditure on the complete line installation can be split up into two parts, one independent of the area of the conductor, and the other directly proportional thereto. It is therefore more correct to write

$$P_1 = (k_0 + k_1 A)a, \quad . \quad . \quad . \quad (196)$$

where k_0 represents that part of the capital outlay which is constant, and independent of the size of conductors.

This modification in the formula for P_1 makes no difference in the differential equation, as

$$\frac{dk_0}{dA} \quad 0,$$

hence, as before,

$$\sqrt{\frac{k_2}{ak_1}},$$

and substituting

$$P_1 = ak_0 + \sqrt{k_1 a k_2}$$

and $$P_2 = \sqrt{k_1 a k_2},$$

or the interest and depreciation on the capital cost of the installation and the annual cost of the dissipated energy are now in the relation

$$P_1 = P_2 + ak_0. \qquad (197)$$

Thus the interest and depreciation on the initial cost of the line must in this case be greater than the annual cost of the energy losses.

Substituting for P_1 in (197) and rewriting, there follows that

$$k_1 A a, \qquad (198)$$

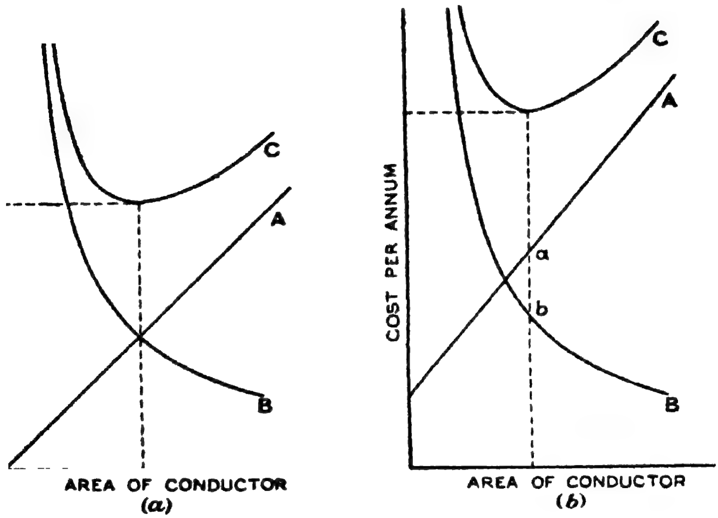

FIG. 82.—Graphical illustration of Kelvin's law.

or the most economical area of conductor is that which makes the annual cost of the dissipated energy equal to the annual interest and depreciation on that portion of the capital outlay on the line which can be considered to be proportional to the weight of conductor material employed.

Fig. 82 (*a*) exhibits the above results very clearly. In the diagram the straight line *A* shows the relation between the annual interest and depreciation of the capital spent on the line conductors, and the size of the conductor. Similarly, the rectangular hyperbola *B* gives the cost per annum of the energy wasted in the conductors. By adding the ordinates of the two

curves A and B, the curve C is obtained, of which the minimum value corresponds with that conductor cross-section which it will be most economical to use. This minimum occurs where the two curves A and B cross, *i.e.* where the values of their ordinates are equal. In Fig. 82 (b) the first diagram is modified so as to deal with the complete line installation by (1) adding to the capital cost of the conductors an amount independent of the conductor area, (2) slightly increasing the slope of A so as to take into account the higher cost of line supports, stringing of wires, etc., with the heavier conductors. In this case, at the points on curves A and B corresponding to the minimum value of C, the ordinate of curve A is greater than that of B by the distance ab, *i.e.* the amount of the independent portion of the total capital expenditure.

A simpler and more general statement which includes both the above results may be called the law of maximum economy and is expressed as follows: *For maximum economy the annual cost of the wasted energy in the transmission line added to the annual charges on the initial capital outlay shall be a minimum.*

149. Kelvin's Law in Practice.—Although the principles previously enunciated are well known, there is often considerable difficulty in applying them to a proposed scheme of power transmission. This is chiefly owing to the difficulty of estimating the power losses, and of fixing a correct money value on them when estimated.

The difficulty of estimating even approximately the probable amount of energy wasted during a year's working is very great. Even if the load to be delivered can be accurately forecasted, the determination of the current value to be used in conductor calculations is uncertain, and as Beard [3] has shown, even a knowledge of the load factor by itself is not sufficient. The losses over a given period of time are proportional to the square of the current flowing, so that the R.M.S. current flowing during the given period must be used in calculating them. Hence it is necessary to take into account the actual shape of the load curve. This is readily seen by considering the two extreme forms of a 50 °/。 load-factor curve given in Fig. 83. In the full line curve the R.M.S. current is equal to the average current, while in the dotted curve it is $\sqrt{2}$ times the average, for this particular load factor. Load curves met with in practice lie between these extremes, and for a typical 50 °/。

load-factor curve the R.M.S. current would be about 1·20 times the average current. In every projected transmission scheme, therefore, probable load curves for different parts of the year should be drawn out, and the mean value of I^2 calculated therefrom. In the lack of other data the curve shown in Fig. 84 can be used to obtain values of the ratio of R.M.S. to average current for typical load curves.

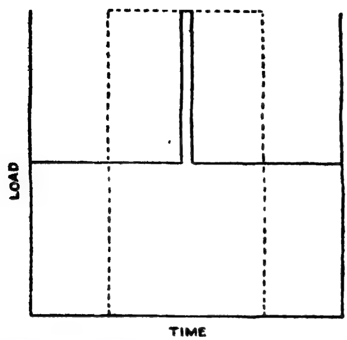

FIG. 83.—Extreme forms of load curve for a given number of units at 50 °/₀ load factor.

The cost per kilowatt-hour of the wasted energy is also somewhat difficult to decide with any accuracy. Generally speaking, the cost of the wasted energy is more than the generating cost because the load factor of the losses is less than that of the load, and in addition to this ought to be added the annual

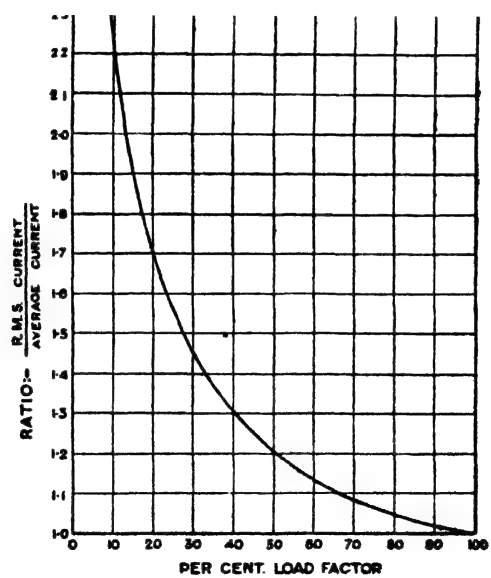

FIG. 84.—Ratio of R.M.S. to average current for typical load-factor curves.

charges on the increased size of conductor necessary to transmit the losses. The station cost per unit might be increased by as much as 30 °/$_{\circ}$ to cover the lower load factor of the losses and these extra annual charges. Furthermore, in cases where the demand for power exceeds the supply, that is if all the power saved in the line could be promptly sold, the price to allow for the dissipated energy is actually the selling price. Every case should be taken on its own merits.

With regard to a reasonable percentage to allow for interest and depreciation on the cost of conductors, this will depend on commercial considerations and the estimated life of the conductors, also on the probable scrap value of the metal when it is replaced. So far as interest is concerned, if cash is to be paid for the conductors the figure to be taken for interest on capital should be on a par with the expected percentage profit on the complete undertaking. In fixing the depreciation it should be remembered that the scrap value of the conductors will probably be small. This is owing to the fact that the transmission line will very likely be a long way from the place where there is a demand for the copper, and a considerable deduction from the price will be necessary to meet the transportation charges and the labour cost of removing the wires from the poles.

150. Economic Current Density in Overhead Conductors.—As previously stated, the annual charges on the initial cost of a transmission line installation can be represented fairly closely by a linear equation

$$P_1 = (k_0 + k_1 A)a. \quad . \quad . \quad . \quad (196)$$

It should be remembered that the second term in the above expression varies at a rather higher rate than it would do if the cost of the line conductors only were being examined. With overhead lines, however, it is generally found sufficient to take into account the variation of cost of conductor material, and ignore other secondary factors. This simplification, in practice, gives results to the degree of accuracy warranted by the state of knowledge of the other data of the problem.

A convenient formula for the economic current density in overhead conductors can now be derived, bearing in mind that the annual charges are: (1) the annual charge for the interest and depreciation on the capital outlay on the conductors, (2) the

annual cost of the energy wasted in the conductors, and that the assumed equality of these two items of cost determines the size of conductor and consequent economic current density.

Let p_1 = cost of conductor in pence per pound.

a = percentage to be taken to cover annual interest and depreciation on conductors.

R_m = resistance of conductor 1·0 sq. in. cross-section, in ohms per mile.

m = weight of conductor 1·0 sq. in. cross-section, in lbs. per mile.

A = section of the conductor in square inches.

p_2 = cost of one kilowatt-hour of wasted energy in pence.

I = R.M.S. current in conductor during the year.

Then the energy dissipated annually (8 760 hours) in one mile of the conductor is

$$\frac{I^2 R_m \, 8\,760}{1\,000\,A} \quad \text{kilowatt-hours,}$$

and the cost of this energy is

$$\frac{p_2 I^2 R_m \, 8\,760}{1\,000\,A} \quad \text{pence.} \qquad (199)$$

The cost of one mile of the conductor is

$$mAp_1 \quad \text{lb.,}$$

and the annual interest and depreciation on this cost is

$$\frac{mAp_1a}{100} \quad \text{pence.} \qquad (200)$$

For equality between (199) and (200),

$$\frac{p_2 I^2 R_m \, 8\,760}{1\,000\,A} = \frac{mAp_1a}{100},$$

or

$$\frac{I}{A} = \sqrt{\frac{mp_1a \times 1\,000}{R_m p_2 \times 100 \times 8\,760}}. \qquad (201)$$

For hard-drawn, stranded copper conductors at 20° C.

$$m = 21\,100 \quad \text{lb.,}$$
$$R_m = 0{\cdot}0445 \quad \text{ohm,}$$

and substituting, the economic current density is found as

$$\frac{I}{A} = 23{\cdot}3\sqrt{\frac{ap_1}{p_2}}. \qquad (202)$$

For hard-drawn, stranded aluminium conductors ($m = 6\,230$ and $R_m = 0{\cdot}0755$) the economic current density is

$$\frac{I}{A} = 9{\cdot}7\sqrt{\frac{ap_1}{p_2}}. \quad . \qquad (203)$$

151. Influence of Line Voltage on Cost of Transmission System.—It is often not fully realised that the application of Kelvin's law by itself is insufficient to enable the size of the line conductors to be calculated. Kelvin's law merely gives the current density at which the conductors should be worked for maximum economy, but the actual conductor cross-section cannot be determined unless the value of the line current is known, and this depends upon the pressure at which the energy is transmitted.

If the cost of the conductors forming the transmission line and the I^2R losses themselves were the only consideration, the highest obtainable line voltage would be the best, on account of the corresponding reduction of current for a given amount of power to be transmitted. But the transmission system must be looked at as a whole, and it will be found that it does not pay to increase the voltage above a certain point. Above this limit the saving in conductor material is more than counterbalanced by the increased cost of the line insulation and terminal apparatus. Furthermore, at extremely high pressures, other sources of losses such as corona and leakage over insulators begin to make their appearance. The choice of line voltage for any particular transmission scheme is thus mainly an economic question, and must be determined by taking into account the cost of the various parts of the complete system which are influenced by alterations in the line voltage.

152. Determination of Economic Voltage and Size of Conductors.—In order to illustrate the application of the above principles, a concrete scheme will be taken. It is assumed that a maximum load of 20 000 kVA. (16 000 kW. at 80 °/$_\circ$ power factor, 50 cycles) is to be supplied over a distance of 50 miles, the load factor being 60 °/$_\circ$. Conductors of hard-drawn, stranded copper are to be used, and the average line span is 600 feet. To guard against interruption to service two sets of conductors are employed carried on one set of steel towers.

Fig. 85 shows diagrammatically the layout of the system, which is designed to allow for considerable flexibility of operation. Two three-phase transformers (or banks of single-phase transformers) each of 10 000 kVA. capacity are provided, both at the

sending and receiving end of the line. A saving in the high-tension switchgear could be effected by treating each transformer group and set of conductors as one unit, in which case isolating switches would only be required on the high-tension side to disconnect the transformers from the line when necessary. While

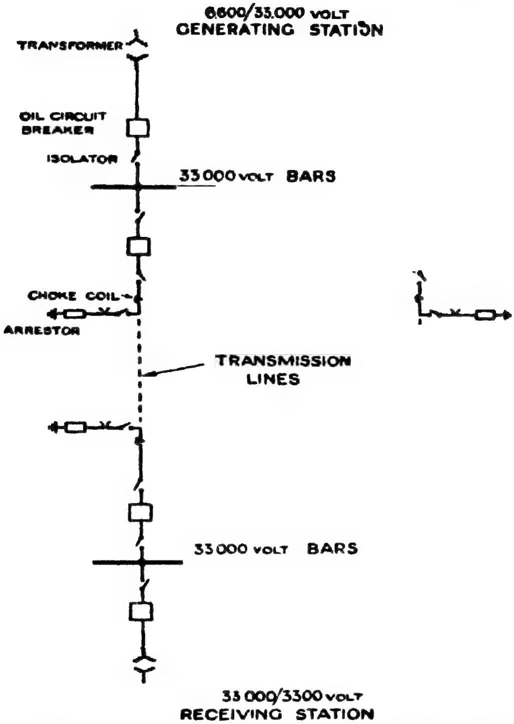

FIG. 85.—Layout of transmission system to illustrate example on economic design.

this arrangement simplifies the switching system it does so at the expense of the flexibility of the system. In the proposed layout the line circuit-breakers are of larger current-carrying capacity than the transformer circuit-breakers, and in case of breakdown to one line the other could be permitted to carry the full load of

20 000 kVA. This is a great advantage, and the additional cost of the line circuit-breakers is comparatively small.

The first step in the solution is to determine the economic current density for the line conductors. Let the cost of hard-drawn, stranded copper conductors on site p_1 be taken as 9d. per lb., the cost of the wasted energy p_2 as 0·5 pence per kilowatt-hour, and the interest and depreciation on the cost of conductors a as 10 °/ₒ per annum.

Then from formula (202) the economic current density is

$$\frac{I}{A} = 23{\cdot}3\sqrt{\frac{9 \times 10}{0{\cdot}5}}$$
$$= 313 \quad \text{amperes per sq. in.}$$

Considering for the moment that the line pressure is 33 000 volts, the current required when the maximum load of 10 000 kVA. per set of conductors is being transmitted is

$$\frac{10\,000 \times 1\,000}{\sqrt{3} \times 33\,000} = 175 \quad \text{amperes.}$$

For a typical 60 °/ₒ load-factor curve the ratio of R.M.S. to average current is 1·13, hence the R.M.S. current flowing in each conductor can be taken as

$$I = 175 \times \frac{60}{100} \times 1{\cdot}13$$
$$= 119 \quad \text{amperes,}$$

and the economic area of conductor is

$$A = \frac{119}{313}$$
$$= 0{\cdot}381 \quad \text{sq. in.}$$

As the weight of a copper conductor 1 mile long, and 1·0 sq. in. cross-section, is approximately 21 100 lb., the total weight of copper required in the line is

$$0{\cdot}381 \times 21\,100 \times 6 \times 50$$
$$= 2{\cdot}41 \times 10^6 \quad \text{lb.,}$$

and thus the total cost of the line conductors on a 33 000-volt system is

$$\frac{2{\cdot}41 \times 10^6 \times 9}{240}$$
$$= £90\,400.$$

The cost of the copper required for other assumed pressures can then be readily calculated, taking advantage of the fact that the cost is proportional to the current and hence inversely proportional to the pressure.

The cost of the terminal apparatus and insulators for operation at 33 000 volts would be approximately as follows:—

Transformers.

Two 10 000-kVA. 3-phase 6 600 / 33 000-volt 50-cycle transformers.

Two 10 000-kVA. 3-phase 33 000 / 3 300-volt 50-cycle transformers.

Price £15 000

Switchgear.

Four 33 000-volt transformer and control equipments complete with oil circuit-breakers, isolators and control panels

Price £2 900

Four 33 000-volt line control equipments similar to above but of larger current-carrying capacity

Price £3 300

Four 33 000-volt electrolytic impulse-gap lightning arrestors complete with isolators and choke coils

Price £2 100

Line Insulators.

Three thousand 33 000-volt pin-type insulators

Price £1 500

The total cost of terminal apparatus and insulators for a 33 000-volt system would thus be about £24 800.

In order to obtain the cost of this apparatus at other pressures use will be made of Table 20, which gives the relative cost of apparatus designed for various standard voltages expressed as a percentage of 6 600-volt costs. These figures are given by Nesbit,[4] and for the sake of argument will be assumed to be correct for the particular problem under consideration. It need hardly be said, however, that these relative costs are liable to fluctuate, and in practice this kind of information would be obtained direct from the manufacturers as and when required.

TABLE 20.—*Approximate Relative Costs of High-tension Apparatus.*

(6 600-volt costs taken as 100 %.)

	Voltage.							
	6 600	11 000	22 000	33 000	44 000	66 000	88 000	110 000
Transformers . .	100	102	108	115	125	150	175	200
Switchgear . .	100	100	100	110	115	155	255	420
Electrolytic lightning arrestors . .	100	151	205	320	430	640	1 600	1 900
Insulators . .	100	135	430	650	1 250	3 500	5 500	6 500

On the basis of Nesbit's figures the result of a comparison between installation costs at the various standard pressures is exhibited in Table 21, and it will be seen that the pressure of 66 000 volts is the most economical for the conditions laid down. In some cases, however, the difference between the economical and the next higher pressure may be quite small, and in such cases it is very probable that the higher voltage would be adopted. One important consideration influencing the choice of the higher pressure is that more trouble is experienced with heavy currents than with high voltages, owing to the more serious effects of transient disturbances due to switching or short-circuits. Hence greater security, and sometimes lower maintenance and operating costs, may be obtained by designing the system for a high pressure. It should be noted also that many mistakes have been made in the past by under-estimating the ultimate demand for electric power, and consequently adopting too low a pressure. When in doubt, the higher voltage should be selected, and will probably justify its adoption by reason of future growth of the system.

Adopting, therefore, the pressure of 66 000 volts for the line voltage of the system, the corresponding theoretical area of conductor required is 0·190 sq. in., and the nearest standard conductor, 7 / 0·193″ stranded copper, having an area of 0·20 sq. in. would be chosen.

The expenditure on line supports, buildings, and wayleaves, is not included in the above comparison. The two latter items are practically constant irrespective of the line voltage, but the

TABLE 21.—*Comparison of Installation Costs at Various Voltages.*
(See Problem—Art. 152.)

	Voltage.							
	6 600	11 000	22 000	33 000	44 000	66 000	88 000	110 000
	Sq. In.	Sq. In.	Sq. In.	Sq. In.	Sq. In.	Sq. In.	Sq. In.	Sq. In.
Theoretical conductor area . . .	1·905	1·143	0·571	0·381	0·286	0·190	0·143	0·114
	£	£	£	£	£	£	£	£
Conductors . .	452 000	271 200	135 600	90 400	67 800	45 200	33 900	27 100
Transformers . .	—	—	14 100	15 000	16 300	19 600	22 800	26 100
Switchgear . .	3 100	3 100	5 600	6 200	6 500	8 700	14 400	23 700
Lightning arrestors .	700	1 000	1 300	2 100	2 800	4 200	10 500	12 500
Insulators . . .	200	300	1 000	1 500	2 900	8 100	12 700	15 000
Total cost	456 000	275 600	157 600	115 200	96 300	85 800	94 300	104 400

cost of the line supports does increase slightly at the higher pressures owing to the greater spacing required for the conductors, and this difference should be taken into consideration if sufficient information is available. In a rigorous comparison it would also be necessary to take into account the probable variation of operating and maintenance costs as influenced by line voltage.

In getting out preliminary estimates for a transmission scheme the following empirical formula due to Still,[5] and based on modern American practice, may be taken as a guide:—

$$5{\cdot}5\sqrt{l + \frac{3P}{100}}, \tag{204}$$

where V is the line pressure in kilovolts (between conductors), l is the distance of transmission in miles, and P is the estimated maximum number of kilowatts per phase to be delivered over one pole- or tower-line.

That both the distance and amount of power to be transmitted do influence the economic voltage of a transmission line can readily be seen. For as the distance of transmission is increased the influence of the terminal-apparatus costs upon the line pressure becomes of less and less importance, leading to a higher economic pressure. Also if a large amount of power is to be supplied, larger generating and transforming units can be employed, with

consequent reduction in the cost per kilowatt of terminal-station equipment.

153. Other Factors Influencing Electrical Design of System.—After working out the size of conductor required from considerations of economy, the matter should always be further investigated to ascertain if the conductor is also technically suitable. Occasionally the economic size of conductor cannot be adopted in practice for one or more of the following reasons:—

1. The regulation of the line may be greater than can be conveniently dealt with.

2. The section of the conductor may be too small, or too large, from a mechanical point of view.

3. The diameter of the conductor may be so small as to lead to excessive corona losses.

4. The current density may be so high as to raise the temperature of the conductor above the safe limit.

1. In the case of excessive line regulation there are two remedies. The first method is to adopt a higher line pressure, and in consequence a smaller current and smaller conductor. The reactance pressure drop ωLI, which chiefly determines the regulation of the system, is thereby decreased, not exactly in proportion to I (as the reactance is increased slightly by the larger value of $\frac{d}{r}$), but nearly to the same extent. Thus the decrease in the percentage regulation is brought about both by the decrease in the actual pressure drop along the line, and the increase in the line voltage itself. The other method of improving the regulation is to split a single circuit into two or more parallel circuits, having the same total area of conductor per phase as the economic conductor. Thus, assuming two sets of conductors in parallel each set taking one-half of the total load, the reduction in the reactive pressure drop ωLI due to the variation in I, is of far more importance than the slight increase of L (due to the higher value of $\frac{d}{r}$), and results in the regulation being reduced very considerably. For instance, assuming the transmission of 40 000 kVA. $\left(\cos\phi_r = 80\ \%\text{ lagging, } E_r = \frac{66}{\sqrt{3}}\text{ kV.}\right)$ over 19/0·166″ strand, 0·40 sq. in. conductors spaced at 90 inches, the full-load regulation would be 19·8 %. Taking into consideration

the voltage drops in the terminal transformers also, this value would be too great to deal with, but by employing two sets of conductors (each 7 / 0·193″ strand, 0·20 sq. in., same spacing) the regulation is brought down to the manageable figure of 12·4 %. It is true that the duplicate set of conductors entails an increase in cost of line supports and insulators, but this is more than compensated for by the valuable insurance provided against total stoppage of supply.

2. In the case of the conductor cross-section coming out so small that the line would be mechanically weak, the obvious remedy is to increase the sectional area somewhat, without regard to the question of economy, or it may be found that aluminium conductors can be adopted with advantage. Generally speaking, it is not advisable to employ line conductors having less than the following sectional areas:—

No. 8 S.W.G. (0·02 sq. in.) for solid copper conductors.

0·025 sq. in. for stranded copper conductors.

0·04 sq. in. for stranded aluminium conductors.

If, on the other hand, the conductor diameter works out too large, there may be difficulty in handling or taking the strain at the individual insulators. The circuit should therefore be subdivided into two sets of conductors in parallel.

3. Excessive corona losses on account of the small diameter of conductor leads to the employment of specially made-up conductors (*see* Art. 93), or here again, aluminium or steel-cored aluminium may be found more advantageous as a conductor material.

4. It is very rare that trouble is experienced through overheating of conductors on an overhead line owing to the fairly low current densities in use and the good facilities for cooling. In short-distance heavy-current transmissions, however, the combined effect of solar heat and current may be sufficient to injure the conductor as copper starts to anneal when the temperature is above 100° C. Trouble may also be caused by excessive sag, and current densities should be such that this figure is not exceeded. As a result of carefully made tests upon various types and sizes of conductors in still air, Luke[6] gives the following formula for determining the temperature rise:—

$$= \sqrt{\frac{\pi d l K_d \theta}{R_\theta}}, \qquad (205)$$

where I = continuous current-carrying capacity,

d = diameter of conductor in inches,

l = length of conductor in inches,

K_d = dissipation constant obtained from Fig. 86,

θ = temp. rise permissible in deg. C.,

R_θ = resistance of conductor at final temperature corresponding to θ.

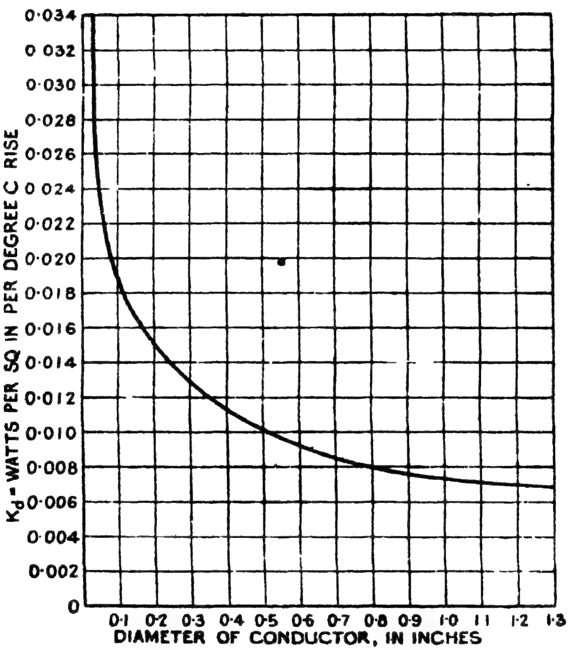

Fig. 86.—Luke's dissipation constants.

This formula gives very safe results as in practice the outside air is always in movement, and this greatly reduces the temperature rise of a conductor suspended therein. It should be noted that a stranded aluminium conductor will carry about 14 °/₀ more current than a stranded copper conductor of equal conductivity.

Before finally deciding upon the transmission voltage careful consideration should be given to the voltage of adjacent systems. There is an increasing tendency to combine neighbouring

generating and transmission systems, and should such an amalgamation take place it would be advantageous if the equipment—transformers, switchgear, insulators, etc.—of the two systems were interchangeable for service on either system.

At the present time a large number of different pressures are in use for transmission schemes, and the design and construction of transformers, circuit-breakers, lightning apparatus, etc., for all these various pressures is expensive. An endeavour is now being made to standardise transmission voltages so as to allow manufacturers to concentrate on fewer voltage ratings with benefit both to themselves and the consumers. In this country the line pressures recommended by the British Standards Institution are 3·3, 11, 33, 66 and 132 kilovolts.[7]

154. Application of Economic Principles to Underground Lines.—It is found that the variation in cost of high-voltage cable with respect to the size of conductor follows very closely a linear law. In other words, the cost per mile of cable is

$$P_1 = k_0 + k_1 A,$$

the actual value of the constants k_0 and k_1 depending on the voltage class of the cable. Unfortunately, the Kapp-Kelvin law cannot be applied direct to the calculation of the economic current density, as beside the I^2R losses there are also the dielectric and sheath losses to be considered.

The best method of dealing with the problem is to tabulate the cost of the line losses and the capital charges corresponding to the standard sizes of cable for various assumed transmission voltages. The final choice can then be made by bringing into the reckoning the cost of the terminal apparatus. A numerical example will serve best to illustrate the method.

Consider the following transmission scheme, *viz.*, the transmission of 15 000 kVA. (12 000 kW. at 80 % power factor lagging, 50 cycles, load factor 50 %), a distance of 10 miles, by means of a three-core cable.

At a pressure of 22 000 volts the line current corresponding to maximum load is

$$\frac{15\,000 \times 1\,000}{\sqrt{3} \times 22\,000}$$
$$= 394 \quad \text{amperes,}$$

and for a typical 50 % load curve the R.M.S. current flowing in each conductor is

$$I = 394 \times \frac{50}{100} \times 1{\cdot}20$$
$$= 236 \quad \text{amperes.}$$

Hence in a 0·20 sq. in. cable (resistance 0·244 ohm per mile at 50° C., capacitance 0·50 microfarad per mile), the total I^2R losses per annum are

$$\frac{236 \times 236 \times 0{\cdot}244 \times 10 \times 3 \times 8\,760}{1\,000}$$
$$= 3{\cdot}57 \times 10^6 \quad \text{kilowatt-hours.}$$

Assuming that the average value of the dielectric power factor is 0·005, the total annual dielectric losses in the cable are

$$\frac{2\pi \times 50 \times 0{\cdot}50 \times 10^{-6} \times 22\,000 \times 22\,000 \times 10 \times 3 \times 8\,760}{1\,000 \times 200 \times \sqrt{3} \times \sqrt{3}}$$
$$= 0{\cdot}03 \times 10^6 \text{ kilowatt-hours.}$$

In a similar manner the losses and therefrom the cost of the losses can be determined for other cable sizes and voltages.

In connection with the calculated results given in Table 22 it should be noted that:—

1. The cost figures for the cables have been obtained from the curves of Fig. 87 which give approximate prices per mile for three-core cables, these being single-wire armoured and served. The prices include the supply and laying of the cables and also the jointing, but are exclusive of the cost of any excavations. The cost of the latter item will naturally be about the same whatever size or class of cable is employed so that this does not enter into the comparison

2. The percentage for annual interest and depreciation on the capital outlay has been taken as 15 % for terminal apparatus and 10 % for cables, and the cost of the wasted energy as 0·5 pence per kilowatt-hour.

3. The conductor temperature has been assumed to be 50° C., and the dielectric power factor 0·005 in all cases.

4. No allowance has been made for the influence of capacitance current on the line losses, the magnitude of the effect over a 24-hour period being comparatively small. It is interesting

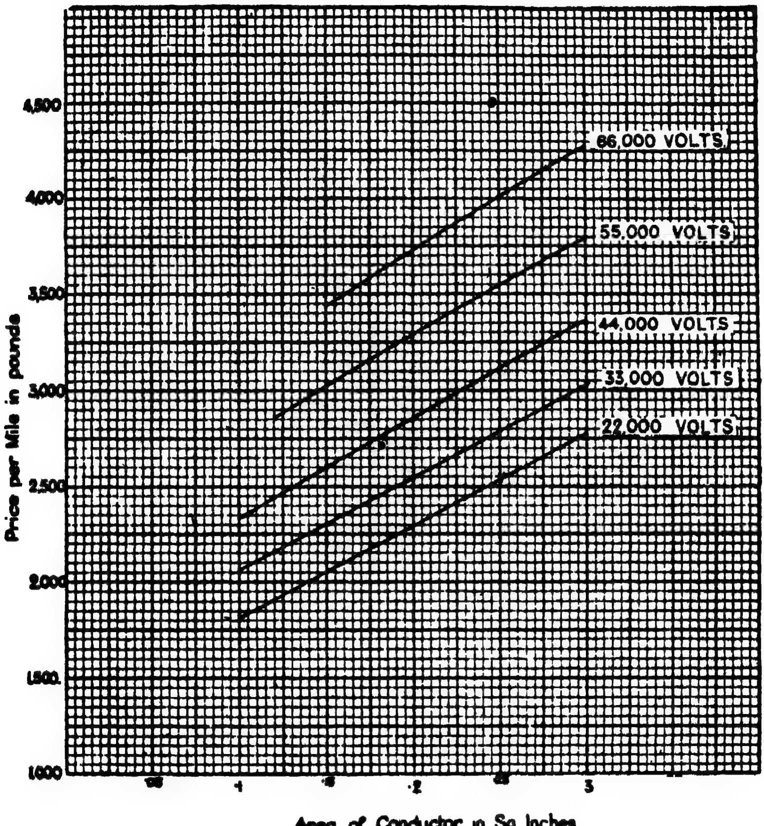

FIG. 87.—Approximate cost of three-core cables laid direct in the ground.

to note that the proportionate change in losses due to capacitance current increases as the line voltage increases, whilst the actual losses decrease with voltage, so that the change is substantially self-compensatory and can be neglected for the present purpose.

TABLE 22.—*Form of Tabulation for Determining Voltages and Size of Conductors.* (See Problem—Art. 154.)

Voltage = 22 kV. R.M.S. current = 236 amperes.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. × 10^6 . .	7·13	4·75	3·56	2·85	2·38
Annual dielectric losses, kWH. × 10^6 .	0·03	0·03	0·03	0·04	0·04
Total losses, kWH. × 10^6	7·16	4·78	3·59	2·89	2·42
	£	£	£	£	£
Cost of losses per annum	14 900	9 980	7 480	6 030	5 050
Interest and depreciation on cost of cable per annum	1 810	2 050	2 290	2 530	2 770
Total cost per annum	16 710	12 030	9 770	8 560	7 820

Voltage = 33 kV. R.M.S. current = 157 amperes.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. × 10^6 . .	3·15	2·09	1·57	1·25	1·05
Annual dielectric losses, kWH. × 10^6 .	0·06	0·07	0·07	0·08	0·08
Total losses, kWH. × 10^6	3·21	2·16	1·64	1·33	1·13
	£	£	£	£	£
Cost of losses per annum	6 690	4 500	3 420	2 770	2 360
Interest and depreciation on cost of cable per annum	2 060	2 300	2 540	2 780	3 030
Total cost per annum	8 750	6 800	5 960	5 550	5 390

TABLE 22.—*Continued.*

Voltage = 44 kV. R.M.S. current = 118 amperes.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. × 10^6 . .	1·79	1·19	0·90	0·72	0·60
Annual dielectric losses, kWH. × 10^6 .	0·10	0·11	0·12	0·13	0·14
Total losses, kWH. × 10^6	1·89	1·30	1·02	0·85	0·74
	£	£	£	£	£
Cost of losses per annum	3 940	2 710	2 130	1 770	1 540
Interest and depreciation on cost of cable per annum	2 340	2 590	2 850	3 100	3 360
Total cost per annum	6 280	5 300	4 980	4 870	4 900

Voltage = 55 kV. R.M.S. current = 94 amperes.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. × 10^6 . .	1·14	0·76	0·58	0·46	0·39
Annual dielectric losses, kWH. × 10^6 .	0·16	0·17	0·18	0·19	0·20
Total losses, kWH. × 10^6	1·30	0·93	0·76	0·65	0·59
	£	£	£	£	£
Cost of losses per annum	2 710	1 940	1 580	1 350	1 230
Interest and depreciation on cost of cable per annum	2 760	3 010	3 270	3 530	3 790
Total cost per annum	5 470	4 950	4 850	4 880	5 020

TABLE 22.—*Continued.*

Voltage = 66 kV. R.M.S. current = 79 amperes.

	Conductor Area in Sq. In.				
	0·10.	0·15.	0·20.	0·25.	0·30.
Annual I^2R losses, kWH. × 10^6	0·79	0·53	0·40	0·32	0·26
Annual dielectric losses, kWH. × 10^6	0·22	0·23	0·25	0·27	0·28
Total losses, kWH. × 10^6	1·01	0·76	0·65	0·59	0·54
	£	£	£	£	£
Cost of losses per annum	2 110	1 580	1 350	1 230	1 120
Interest and depreciation on cost of cable per annum	3 160	3 440	3 720	3 990	4 270
Total cost per annum	5 270	5 020	5 070	5 220	5 390

It will be seen from the table that at 22 000 and 33 000 volts the most economical size of conductor to employ is 0·30 sq. in., at 44 000 volts the best conductor is 0·25 sq. in., at 55 000 volts 0·20 sq. in., and at 66 000 volts 0·15 sq. in., assuming, of course, in each case that these sizes are also suitable from a thermal point of view.

The influence of terminal-apparatus costs on the problem must now be considered. For this purpose the cost of the 33 000-volt apparatus required will be assumed as follows :—

Transformers	£11 000
Switchgear	£3 000

and the figures given in Table 20 again made use of in order to find the approximate costs at other voltages.

The final comparison may then be made as shown in Table 23, from which it will be seen that for the problem under consideration a 44 000-volt, 0·25 sq. in. cable is the best from an economic point of view. Nevertheless, transmission at 33 000 or 55 000 volts costs very little more if these pressures are advantageous for other reasons.

It is very rare that excessive pressure drop gives any trouble in the case of underground lines thus necessitating a lower current

TABLE 23.—*Comparison of Installation Costs at Various Voltages.* (See Problem—Art. 154.)

	Voltage.				
	22 000.	33 000.	44 000.	55 000.	66 000.
	£	£	£	£	£
Annual interest and depreciation on cable + cost of losses per annum	7 820	5 390	4 870	4 850	5 020
Annual interest and depreciation on cost of terminal apparatus	1 950	2 100	2 280	2 505	2 775
Total annual costs	9 770	7 490	7 150	7 355	7 795

density than that derived from economic considerations. This is on account of the shorter distances of transmission, and also on account of the smaller reactance of cables as compared with the reactance of an overhead line using the same size of conductors. It is necessary, however, to ascertain that the current density is not so high as to raise the temperature of the dielectric above the safe limit. This is particularly important in cases where the load factor of the system is small.

In the particular problem considered, the current flowing when maximum load of 15 000 kVA. is being transmitted is

$$\frac{15\,000 \times 1\,000}{44\,000 \times \sqrt{3}}$$
$$= 197 \quad \text{amperes,}$$

and comes well below the value which could be permitted if heating of the cable was the only factor to be taken into consideration. The voltage drop in the line at times of maximum load is about 2 %.

155. Choice of Frequency.—There are several different supply frequencies in existence at the present time, but for new installations the standard frequency is 50 cycles with 25 cycles as a secondary standard. In America both 25- and 60-cycle supplies are used extensively, but the higher frequency is likely to predominate in the future. Many of the 25-cycle systems in operation were installed at a time when the design of converting apparatus for the higher frequencies had not reached a satisfactory stage. Now, however, by improved methods of design and

construction—notably the use of higher peripheral speeds and commutating poles—rotary convertors for 50 or 60 cycles have been made practically equal to those for 25-cycle supply. As regards other apparatus, 50-cycle turbo-alternators are about 10 % cheaper than 25-cycle turbo-alternators, and 50-cycle transformers are about 25 to 30 % lighter and 25 % cheaper than those of the same output at the lower frequency.

One feature that should be considered is the influence of the frequency on the reactance and charging current of the circuit, both these quantities at 50 cycles being twice their value at 25 cycles. The high reactance of line and apparatus at 50 cycles tends to produce poor voltage regulation of the circuit, but has one advantage for the larger systems in that it limits short-circuit currents, and thus assists the circuit-breakers to function properly. The 50-cycle charging current also, in the case of long lines, and particularly where two or more sets of conductors in parallel are employed, may be so great as to limit the choice in transmission voltage. On the other hand, large charging currents can be allowed if synchronous phase modifiers are installed to maintain the line voltage constant.

In the absence, therefore, of special circumstances it appears desirable to choose the higher frequency of 50 or 60 cycles as the case may be. The advantage of unification is to reduce the cost and simplify the manufacture of terminal apparatus due to the use of fewer designs. Furthermore, the adoption of one common supply frequency throughout the country facilitates the interchange of energy between different undertakings by removing the necessity of employing frequency changers as the link between interconnected systems.

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CHAPTER XII.

APPARATUS FOR THE PREVENTION OF DANGEROUS CURRENTS.

156. Circuit-breakers.—The earliest methods of guarding against the effects of dangerous currents was by the employment of fuses in series with the circuit to be protected. This simple form of overload protection worked satisfactorily on the low-voltage small-power transmissions laid down in the early days of the electric supply industry, but it is manifestly unsuitable for present-day systems. High-tension fuses or fuse-switches, however, are often used for small consumers supplied through tappings from the main transmission line, owing to their low cost compared with the cost of circuit-breakers.

Generally speaking, however, high-voltage alternating-current circuits are broken by means of oil circuit-breakers. These breakers possess the valuable characteristic of rupturing the circuit near to the zero point on the current wave. An arc is drawn out between the separating contacts of the breaker under the oil, and this arc is rapidly cooled and compressed by the surrounding oil. Naturally, the capability of resistance to rupture of the hot vapour column forming the arc is at a minimum when the current flowing is at zero, at which instant if the breaker be equal to its duty the circuit is finally opened. It will thus be seen that the circuit is broken at the most favourable moment, in strong contra-distinction to the case of the air-break circuit-breaker with which, on an alternating-current circuit, the tendency to rupture is strongest when the current is at its maximum.

Circuit-breakers are almost always of the automatic type, that is they are equipped with trip coils, which on the occurrence of abnormal conditions in the main circuit are energised by means of relays. The mechanical design of the breaker is such

that when it is closed considerable energy is stored in strong springs and the parts held together by a system of toggles, the latter being so constructed that only a slight pressure on a trigger is needed to open the breaker. When actuated by the protective relay, the trigger is tripped, thereby releasing the potential energy of the springs which causes the contacts to open in a small fraction of a second.

157. **Protective Relays.**—The protective relay may be defined as an electrical instrument interposed between the main circuit and the circuit-breaker in such a manner that any abnormality in the circuit acts on the relay, which in turn, if the abnormality is of a dangerous character, causes the breaker to function and relieve or protect the circuit and apparatus. In recent years great advances have been made in the design and construction of these instruments, and present-day types can be depended upon in practically all conceivable cases of electrical distress. The importance of automatic protective devices has grown with the size of transmission systems and their systematic employment is now almost a necessity. These devices protect the plant from damage in the event of heavy fault currents occurring, but their chief function is to preserve the continuity of the general supply by rapidly removing from the system faulty lines or apparatus. By the employment of selectively-acting protective relays, defective lines can be automatically located and cut out of circuit with the minimum amount of disturbance to the rest of the system, and in many cases while the fault is in its early stages.

A current flowing in an electric circuit becomes dangerous either when it increases in magnitude beyond the capacity of the conductor, or when the electric power corresponding to the current in question flows in the wrong direction, or when due to failure of the insulation the current follows a path other than the prescribed one. Hence automatic protective devices may be roughly classified as follows :—

1. Overload relays or devices which operate when the current exceeds a certain predetermined limit.

2. Reverse-power relays which operate when the flow of power at any instant is in the reverse direction to that which is normal at that instant.

3. Leakage relays which operate when current flows through other than the proper channels.

Generally speaking, protective devices used for the purpose of opening circuit-breakers have their connections divided into three circuits. The first is the primary winding of a current transformer which is connected in series with the main circuit to be protected. This primary winding very often consists of a single turn, or even of the main conductor merely linked through an iron core. The second circuit consists of the secondary winding of the current transformer and the relay-operating winding. The third, or tripping circuit, may be either D.C. or A.C. and consists of a source of supply, the trip coil on the circuit-breaker, and the relay stationary contacts. The operation of the relay is effected by another contact which is attached to the moving element of the relay. When the abnormality on the protected circuit reaches a certain predetermined magnitude, the moving element of the relay causes this latter contact to bridge the stationary contacts, thus completing the tripping circuit, energising the trip coil of the breaker, and causing the breaker to open.

One of the most important points in connection with the installation of protective devices is the source of supply for the tripping circuit. This supply should be absolutely reliable, and if possible independent of the system being protected. A separate storage battery fulfils the requirements best, and is always used for tripping purposes in systems of any magnitude.

158. Constructional Details of Relay.—The majority of protective relays depend on the magnetic effect of the current for their operation and fall roughly into three classes; (1) the electromagnet type, utilising the attractive force of an electromagnet on an armature, (2) the solenoid-and-plunger type, making use of the suction effect of an energised solenoid on an iron plunger, and (3) the induction type, employing the basic principles of the induction motor and watt-hour meter. Relays are made in a large variety of forms and a detailed description of the various makes in use is outside the scope of this work. However, for the sake of completeness, a short description of the design and construction of a typical induction-type relay will be given. It is generally acknowledged that relays based on the induction principle are the most accurate and reliable for overload or reverse-power protection. The calibration has the valuable quality of permanency, and independent adjustment can be made within wide limits both as regards current and time.

The form of relay made by the Westinghouse Co. is illustrated in Fig. 88. The conductor to be protected forms the primary of a current transformer, and the secondary winding of this transformer is connected in series with the relay winding *E*. On the same core as *E* is another winding *A*, which supplies the pole-piece windings *B* and *D* by transformer action; the path of the magnetic flux being indicated by the dotted lines in the figure. The value of overload at which the relay will operate depends on the number of turns of *E* connected in series with the current transformer, and several tappings are taken from one end of *E* thus providing for an adjustable current setting. When current

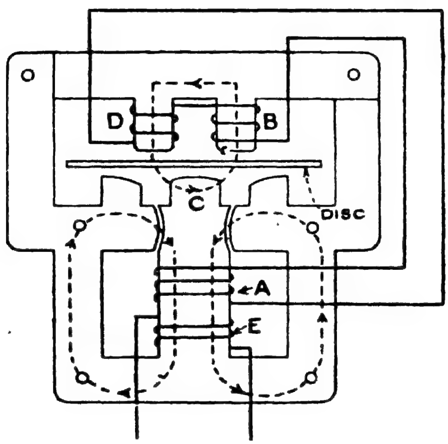

FIG. 88.—Magnetic and electric circuits of Westinghouse induction-type relay.

passes through *E* the disc *K* (shown in plan in Fig. 89) tends to rotate, but this movement is opposed by the spiral spring *I* fastened with its outer end to the permanent support *M* and its inner end to the shaft *L* which also carries the moving contact *F*. The rotation of the disc is damped by the permanent magnets *O*.

When sufficient current flows through the coil *E* to develop in the disc the necessary torque, it rotates until the contact *F* touches contact *G*, thereby completing the trip circuit. It is essential that the time of action of the relay for a given overload can be adjusted, and to accomplish this the angle through which the contact *F* must travel is varied. Thus, if the contact must travel one-half revolution it may take two seconds, but the stop *H* can

be arranged so that the contact need only make one-fourth of a revolution and then the time will be halved. Other positions of *H*, set by a small lever, enable any time from instantaneous to a maximum to be set.

Owing to the accuracy required in a relay the parts must be small, and delicately constructed. Consequently the contacts of the trip circuit cannot be of a heavy character and are not designed to open the trip circuit, either intentionally or unintentionally, once it has been established. This is an important point as the tripping circuits are as a rule highly inductive, and the D.C. arc which would naturally follow the opening of the circuit might persist for a considerable length of time, and result

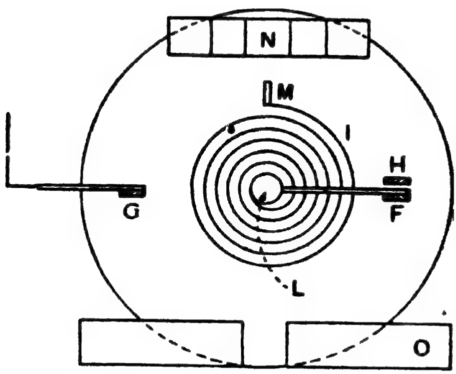

FIG. 89.—Top view showing position of contacts, etc., in Westinghouse relay.

in serious damage to the contacts. For this reason it is necessary that the tripping circuit be opened by a small auxiliary switch fastened to the moving parts of the circuit-breaker, in such a manner that the opening of the breaker automatically opens the tripping circuit also.

A contactor switch connected in parallel with the relay contacts is usually provided for two reasons. In the first place, a circuit-breaker takes a definite period of time, say, 0·2 to 0·3 seconds to open after the relay contacts have closed and the trip coil of the breaker has been energised. Now if during this small interval of time the overload disappears, the relay would instantly attempt to reset, and in so doing would open the tripping circuit before the auxiliary switch attached to the breaker could operate,

thus resulting in severe burning of the relay contacts. In the second place, the overload might just be great enough to barely close the relay contacts. This weak closing might not allow sufficient current to pass through to operate the trip coil of the breaker, and the contacts would chatter and burn badly. By employing the contactor switch the weak fluttering contact is changed into a good positive contact, which ensures that the tripping circuit is kept closed until opened externally by the auxiliary switch on the breaker. A diagrammatic scheme of the contactor switch is shown in Fig. 90. The relay contacts *A*, when they close the tripping circuit of the relay, energise the small coil *B* which attracts an iron plunger *C* and closes the contacts *D* which are in parallel with *A*. Thus, it will be seen

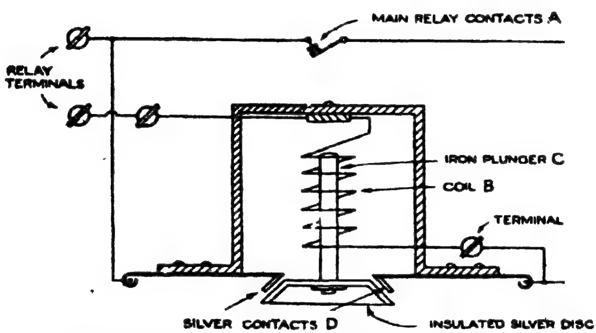

Fig. 90.—Contactor switch for use with relays.

that even though the contacts *A* should open, the current will pass through *B*, and contacts *D* will stay closed until the auxiliary switch on the circuit-breaker opens the circuit; when this occurs coil *B* loses its pull and allows contacts *D* to open.

159. Overload Relays.—The application of plain overload relays for the protection of a three-phase line is illustrated in Fig. 91. The current transformers together with their respective relay-operating coils are connected in star, and on the occurrence of an excessive current in the line, either due to a short-circuit between conductors or between a conductor and earth, the circuit-breaker is immediately tripped out. On an unearthed system only two current transformers and two relays are necessary. In this case it is obvious that an earth on one phase only cannot produce an overload, hence two or three phases must be affected, and at

least one relay will function when the predetermined magnitude of current is exceeded.

160. Overload Relays with Series Tripping.—Occasionally it is desired to have some form of overload protection without incurring the expense of installing and maintaining a storage battery supply for tripping purposes. In this case the secondary current of the current transformer can be used as the emergency source, and a series tripping relay employed. The secondary of the transformer is connected in series with the trip coil on the circuit-breaker, but this coil is normally short-circuited by the

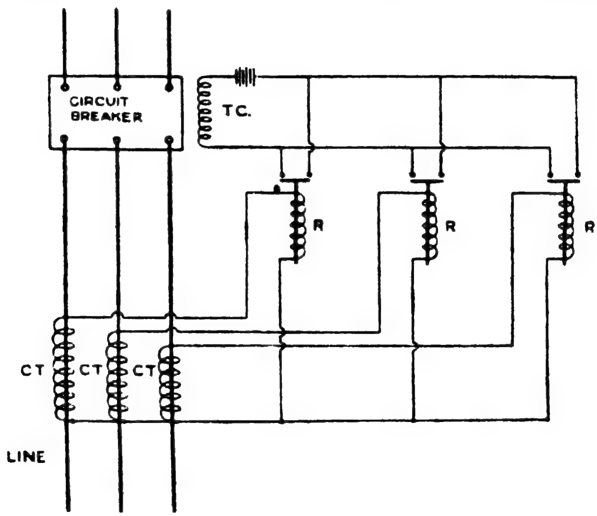

FIG. 91.—Plain overload protection for three-phase line.

relay. On the occurrence of an abnormal current in the line the contacts of the relay open, and the secondary current flows through the alternative path formed by the trip coil thereby energising it and opening the breaker. The diagram in Fig. 92 shows the connections for a single-phase line. With an unearthed three-phase system two current transformers and relays, and with an earthed three-phase system three transformers and relays, are required for complete overload protection. The drawback of this method is that if the relay contacts become dirty, or make poor contact due to vibration, they may shunt current through the

trip coil under normal load conditions and trip the circuit-breaker without cause. Hence it would be difficult to justify the omission of a storage battery or other separate source of energy on installations of any importance.

161. Time Delays.—For many purposes an instantaneously-acting, overload relay is undesirable, and a time lag is introduced therein. In the event of a momentary fault or temporary overload the circuit will then not be disturbed. The method of lagging or damping the moving element of a relay depends largely on the principle of operation. In the solenoid-and-plunger type some manufacturers employ a leather bellows with a small adjustable needle valve to allow the air to escape slowly. As the plunger attempts to rise the air is compressed in the bellows,

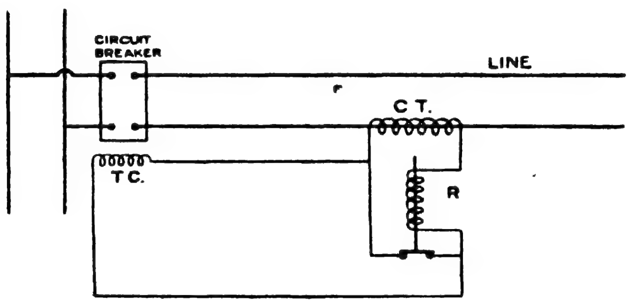

Fig. 92.—Overload protection with series tripping.

thus retarding the movement. Other manufacturers use a dashpot with oil to retard the action. In the induction type of relay the time lag is obtained by setting back the fixed stop, so that the disc has to rotate through a greater angle before making contact. The permanent magnets also exert a damping effect.

In the event of series tripping being employed, a time lag can be imparted to the relay by connecting a fuse in parallel with the relay contacts. On the occurrence of an overload the contacts open, thus causing the excess current to pass through the fuse until the latter blows and the current is diverted through the trip coil. A cheaper arrangement still is to do away with the relay and simply employ a fuse connected in parallel with the trip coil of the breaker, the trip coil being across the secondary of the series transformer.

The above methods of introducing a time lag into the action of the relay result in the production of relays having an inverse time limit. By this is meant that the time required for the relay to operate becomes shorter the greater the overload, the time lag varying approximately inversely as the current. An ordinary fuse gives protection of this character. Fig. 93 gives typical curves of an inverse-time-lag overload relay.

Another form of time lag used with relays is the definite time limit. As the name implies, in this type of protection there is a definite time delay between the instant of disturbance and the

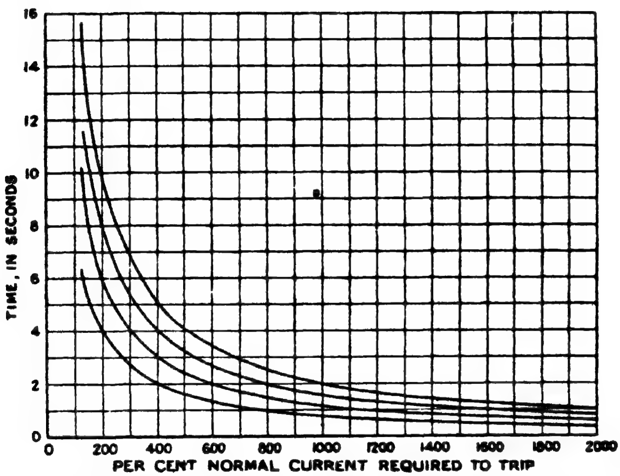

FIG. 93.—Typical time-load curves for inverse-time-lag relays.

closing of the relay contacts, and this delay in the action is in no way affected by the magnitude of the abnormality.

Both inverse-time-lag and definite-time-lag overload relays have their field of application, but on transmission systems they have been almost superseded by the compensated overload relay which will be described below.

162. Application of Overload Relays to a Radial Transmission System.—In any system of power transmission there will be found several circuit-breakers in series such as the generator breaker *A*, the main line breaker *B*, the receiving-station feeder breaker *C*, and the substation feeder breaker *D*. This type of layout, shown diagrammatically in Fig. 94, is known

as a radial system of transmission. Supposing now that each of these circuit-breakers are fitted with instantaneous overload relays it is exceedingly probable that a heavy short-circuit occurring on the consumer's premises would open the whole series right up to and including the sending-end station breakers, thus involving a general stoppage of supply. This would also be the case if the relays were of the inverse-time-lag type since at heavy loads, as shown by the curves, their action is almost instantaneous. Both these relays are unsuitable for a transmission system as they have no discriminative properties. The object to be aimed at is to open the circuit-breaker nearest to the fault, and all the others should remain unaffected so as to limit the area of the disturbance as much as possible.

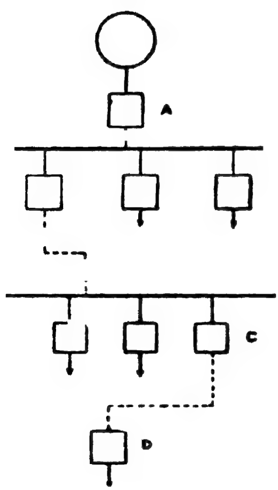

Fig. 94.—Radial system of transmission.

The first reliable method of ensuring the correct sequence of operation was to employ definite-time-lag relays. Thus *D* would be set to trip instantaneously, *C* would trip after 1 second, *B* after 2 seconds, and *A* after 3 seconds, no matter what the magnitude of the overload. This method has the undesirable feature that a 100 °/。 overload is tripped as quickly as a short-circuit. Furthermore, on a large system having, say, four or five subdivisions of the radial layout, the time delay close in to the generating station would become dangerously long under short-circuit conditions.

To meet these difficulties the compensated overload relay has been developed and now enjoys a wide application. The relay is of the induction type but is supplied from a small transformer called a 'torque compensator' which is so designed that it becomes saturated with heavy currents and thus limits the disc speed to a predetermined maximum. By this means an inverse time-lag can be obtained at moderate overloads, but with heavy overloads or short-circuits there is a definite minimum time of operation, as shown by the curves of Fig. 95.

Modern circuit-breakers can be depended upon to open a circuit within about 0·25 second of the relay contacts closing, and allowing the same interval of time to compensate for any inaccuracies which may exist in individual breakers or relays, a reliable selective action can be obtained with differences of 0·5 second between consecutive relays. Generally speaking, it is not safe to sustain a short-circuit on the generators for longer than 2 seconds, and the last relay may be instantaneous. Hence, about five compensated overload relays may be used in series on a radial system,

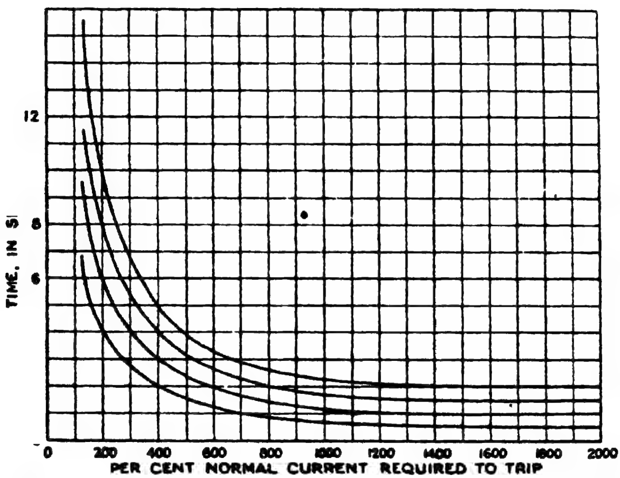

FIG. 95.—Ideal time-load curves for compensated-time-lag relays.

and each division will not only be reasonably well protected, but will not cause interruption to any good feeder between the busbars supplying the defective feeder and the generators.

163. Protection of Parallel Lines by Overload and Reverse-power Relays.—In any important system of power transmission reliance is not placed on one single line or feeder. At least two lines are employed and are connected normally in parallel so as to share the load. These lines may or may not run on the same towers or the same right of way. Special means of protecting such parallel lines are available which, in the event of a fault occurring, will select and isolate the defective line while the other instantly assumes the increased load.

The simplest method is to install time-graded overload relays at the sending end of the lines and instantaneous reverse-power relays at the receiving end, as shown in Fig. 96. If, now, a heavy fault or short-circuit occurs on one line at the point X, current will flow into it through B, and also through A, C, and D. The direction of power flow will be reversed through the relay on D, which will open. The excess current is then confined to B until its overload relay operates and trips the breaker, thus completely isolating the defective line.

This method of protection until recently suffered from faults due to the employment of reverse-power relays. These relays have not always been of a reliable character and the develop-

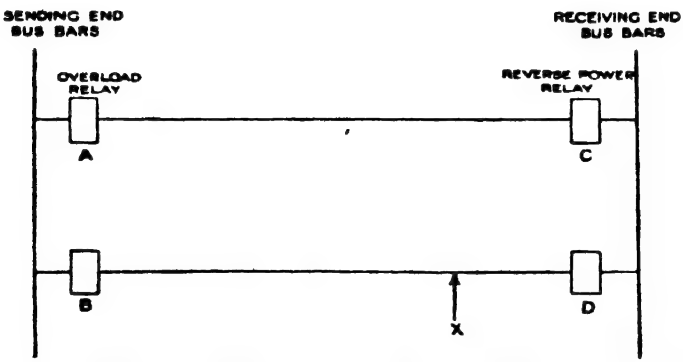

FIG. 96.—Protection of parallel lines by means of overload and reverse-power relays.

ment of a satisfactory design has not been a simple proposition. It might appear at first sight an easy matter to place contacts on a wattmeter, the contacts being normally open, and closing on reversal of power. In the case of a short-circuit near the generating end of the line, however, the voltage will be very low even if the current is high. In fact, actual tests have shown that under such circumstances the voltage may drop down to 1 or 2 % of normal. Furthermore, in the case of three-phase systems, phase distortions are introduced on the occurrence of short-circuits on one phase only or between one conductor and earth. These asymmetrical short-circuits may so distort the relation of current and voltage as to cause the angle between them to be almost 90 degrees. Consequently the ordinary

wattmeter movement has been found to be entirely inadequate, on account of there being insufficient torque to operate the mechanism when short-circuits have caused the voltage and power factor to drop to low values. Another disadvantage has been that sudden momentary surges, due to synchronising or other switching operations, would trip out the breakers unnecessarily.

The reverse-power relay brought out by the Westinghouse Co. is claimed to possess neither of the above disadvantages. Fig. 97 gives a diagram of the electrical connections. In the relay two

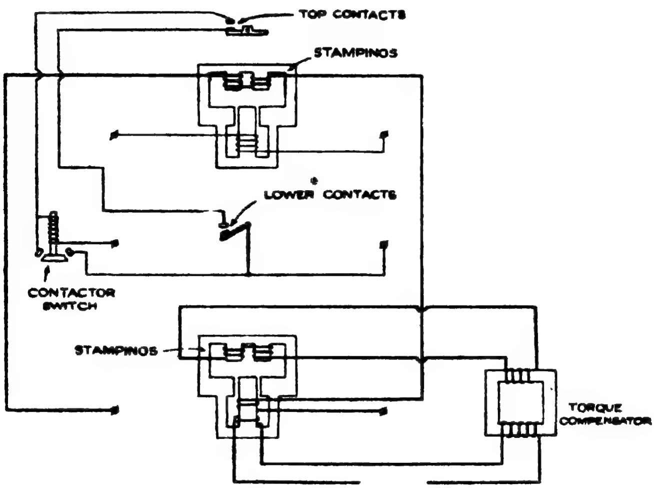

FIG. 97.—Magnetic and electric-circuits of Westinghouse reverse-power relay.

entirely separate moving elements are employed, one being a simple overload element, and the other a wattmeter or directional element. The relay contacts of both these elements are in series. The directional element is made very quick-acting and sensitive, the least flow of power in the reverse direction being sufficient to close its contacts. The circuit-breaker, however, cannot be tripped unless the excess current is of such a magnitude as to close the overload-element contacts also. This prevents the breaker being tripped by switching surges or temporary reversals of energy due to hunting of substation machinery. On the other hand, with current in the normal direction, the directional-element

contacts are always open, so that if the overload-element contacts were closed by an excessive current the breaker would not be tripped. It is stated that this type of relay selects between overload in the normal and reverse directions even though the pressure drops to 2 °/° of normal, and the power factor to 10 °/°. In connection with the diagram of Fig. 97 there should be noted the employment of the torque compensator previously mentioned in order to obtain a definite minimum time of operation for the overload element.[1]

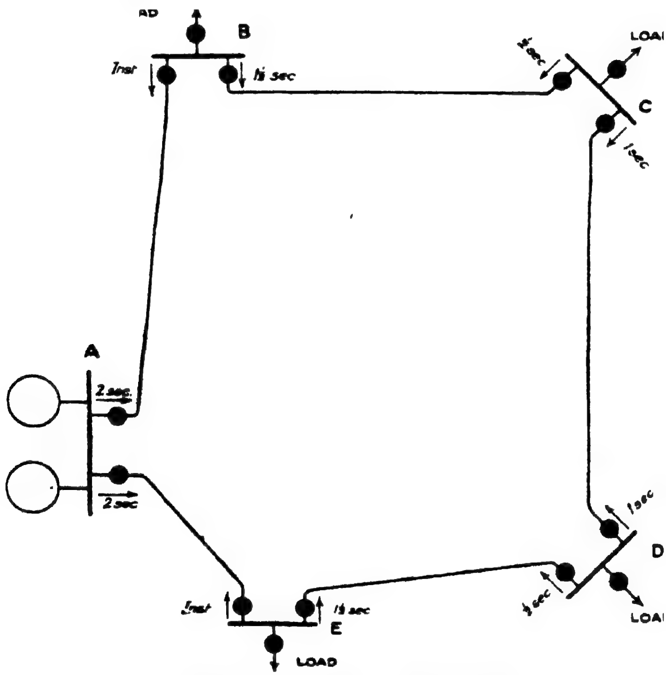

FIG. 98.—Protection of ring system.

164. Protection of Ring System.—When several substations are fed from the receiving-end station and their geographical situation is favourable, the ring system is one of the best means of securing uninterrupted supply with a minimum expenditure on transmission lines or feeders. An elementary diagram of such a system is illustrated in Fig. 98, where *A* is the receiving station, and *B*, *C*, *D*, and *E* are the substations. The lines running

between any two of the substations are known as interconnectors. In the lines AB and AE the flow of power must be always in the same direction, that is from the source outwards, but in the interconnectors BC, CD, and DE the flow may be in either direction varying with the load conditions. It will be readily seen that if a fault occurs on any of the lines, and the defective line can be promptly isolated, there will be no interruption of the supply to any of the substations.

The protective arrangements for such a system comprise time-lag overload relays at the main station, and time-lag reverse-power relays at both ends of the substations. The reverse-power relays are set so that they will only trip when an overload flows away from the substation which they protect. Going round the ring in the direction $ABCDE$, the relays on the further side of each station are set with decreasing time lags; for instance, A = 2 seconds, B = 1·5 seconds, C = 1 second, D = 0·5 second, and E is instantaneous. Going round the ring in the opposite direction, the relays on the outgoing sides would be set as follows: A = 2 seconds, E = 1·5 seconds, D = 1 second, C = 0·5 second, and B is instantaneous. Now remembering that the reverse-power relays are set so as to function only when power flows away from the substation at which they are installed, a study of the diagram will make it clear that a short-circuit occurring at any point on the system will cause to operate only the two circuit-breakers immediately adjacent.

When parallel lines are used between substations on a ring system the reverse-power relays will maintain the ring intact by isolating only the faulty line. In ordinary parallel line protection the reverse-power relays are set to act almost instantaneously in order to cut out the defective line as quickly as possible, but when applied to a ring system the relays must be set with a decreasing time-lag as described.

165. Protection of Parallel Lines by Interlocked Relays.—The use of overload and reverse-power relays does not afford a complete solution of the problem of parallel-line protection owing to the fact that the reversal of current alone is not a criterion of failure of the line in which the reversal occurs. Such a reversal may be produced by surges of power fed back from rotary converting apparatus or other synchronous plant installed on the distribution system. In case of a fault occurring near the

generating station the operating voltage is suddenly reduced and, the synchronous machinery then feeds back energy into the fault, the current reversing in perfectly healthy lines. On the other hand, a high-resistance fault on a line would not cause any reversal of current, so that in this case the defective line would not be isolated.

An advance in the art of parallel-line protection takes advantage of the fact that under normal conditions lines of the same length and cross-section will take practically equal shares of the load current. A break-down on one line, however, will alter the relative current values, the faulty line carrying the heavier current. By interlocking the relays of the two lines either electrically, electromagnetically, or mechanically, the isolation of the faulty line can be secured.

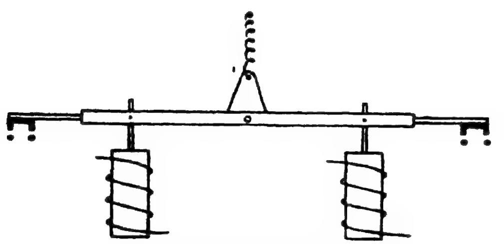

FIG. 99.—Balanced overload relay.

Consider first of all the sending-end protection. To discriminate between two parallel lines, interlocked overload relays of the type illustrated diagrammatically in Fig. 99 may be used. The two coils are operated respectively from series transformers on the two lines, and the balance arm of the relay is pivoted in its middle position. So long as the currents are equal the balance arm will be unaffected, but with an increase of current on one side the arm will be drawn down on that side against the action of the spring, and will close the corresponding pair of contacts to trip the circuit-breaker in the more heavily loaded line.

On the faulty line being disconnected the other takes the whole of the load, and some device is required to prevent its being tripped out. A satisfactory method of accomplishing this end is by means of small auxiliary switches on the circuit-breaker. This automatically converts the protection into time-lag overload protection on the line left in circuit.

In practice, a perfect balance of current between the lines is not obtainable, due not only to the line impedances varying, but also to dissimilarities in the characteristics of the current transformers. Hence, on a severe overload, there will be an appreciable excess current on one side or the other of the relay. For example, if the maximum short-circuit current or 'through' current was 50 times the normal-load current in each line, and the relay coils balance under normal load conditions within 2 %, then on dead short-circuit there will be a difference current equal to normal-load current, and the relays of course must not operate under these conditions. Hence, under these circumstances, a fault setting more than 100 % of the normal load of the line must be used.

This difficulty of balancing is completely eliminated by making use of the principle of biasing first suggested by Wedmore.[2]

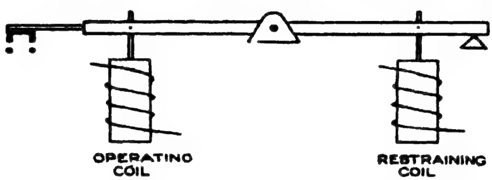

FIG. 100.—Biased overload relay.

Each line is provided with a relay having two coils, one an operating coil, and the other a restraining coil, as shown in Fig. 100. The operating coil is excited from the line that is being protected, and the restraining coil from the line in parallel with it. The arrangement is biased in favour of the restraining coil to the extent of, say, 10 % by placing extra turns on the latter coil, or altering the position of the fulcrum of the balance arm, so as to make its effect more powerful than the operating coil by 10 %. The relay will then operate when the fault current amounts to 10 % of the current flowing in the main circuit whether this is normal load or overload. The devices may be set to operate instantaneously and a gradually developing fault removed while the leakage current is fairly small, but are uninfluenced by heavy 'through' currents due to a short-circuit on some more remote part of the system.

Fig. 101 shows the application of biased overload relays to the protection of two parallel lines. The current transformers are

connected for circulating current and so the points P and P' are normally at the same potential. Thus the transformer current normally circulates through the parallel path formed by the operating and restraining coils of the relays. If one line carries current in excess of that carried by the other, the major portion of this excess current flows through the local circuit formed by the operating coil of the relay protecting the faulty line and the balancing lead.

At the receiving end of parallel lines, protection can be secured by means of interlocked reverse-power relays. The application of the interlocking idea here renders the relays independent of

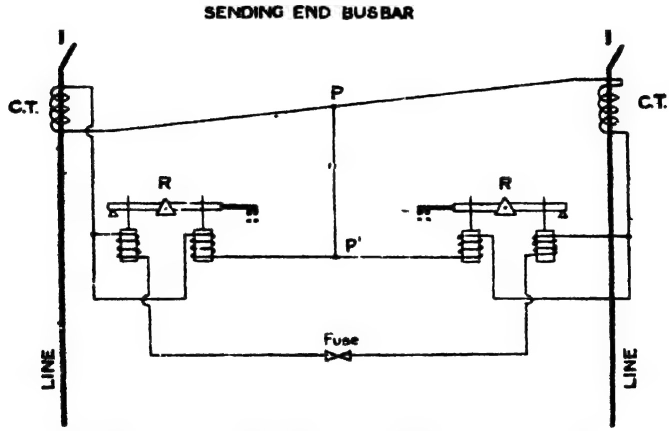

Fig. 101.—Protection of parallel lines at sending end by interlocked overload relays.

surges of power fed back from synchronous machinery on the system. Fig. 102 illustrates the employment of interlocked reverse-power relays arranged for series tripping. The current transformers in the two lines are so connected that current normally circulates between them and the relays, and trip coils are connected in shunt to them. No current, however, flows through the relays, which are connected in series between equipotential points. In case of unbalancing, the difference current flows through the relays in such a direction as to operate the relay on the faulty line and to restrain the other, so that the faulty line only is isolated.

The principle of interlocked overload and reverse-power relays

may be extended to several lines in parallel, and can also be applied direct to the various elements of a radial transmission system. Ring systems with parallel lines can be protected by installing interlocked overload relays at the main station and interlocked reverse-power relays at both sides of the substations.

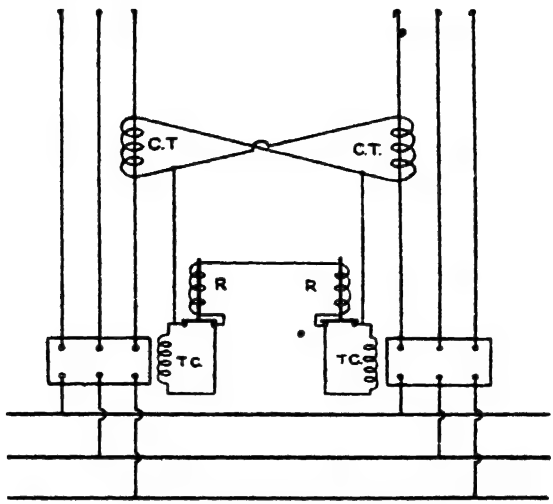

FIG. 102.—Protection of parallel lines at receiving end by interlocked reverse-power relays.

166. Core-balance Leakage Protection.—A number of methods are now in use whereby the leakage of current from an electric circuit is caused to automatically disconnect the faulty circuit from the system. Such leakage protective gear can only be used on earthed systems, as it is essential that any fault current from a conductor to earth should have an adequate return path to the system. On large systems, however, there may be no necessity to earth the neutral point direct, since the path provided by the capacitance of the conductors may have a sufficiently low impedance to ensure operation of the protective gear.

On a transmission line protected only by overload relays, a fault may not be of sufficient magnitude to operate the relays. Thus the leak may continue for a long time and do considerable damage before it ultimately develops into a short-circuit and is removed from the system. The use of leakage protective devices

under such circumstances ensures disconnection of the fault in its early stages

Fig. 103 shows the use of a core-balance leakage relay to protect a line against faults to earth. The three conductors of the three-phase system are surrounded by the iron circuit of a current transformer. This has a single secondary winding which is connected to the relay. If there is no earth fault on the line the instantaneous sum of the three currents is always zero, and there is no resultant flux in the core of the current transformer no matter how much the load is out of balance. The sum of the three main currents, however, is no longer zero if an earth fault appears on the system, in which case a current is induced in the

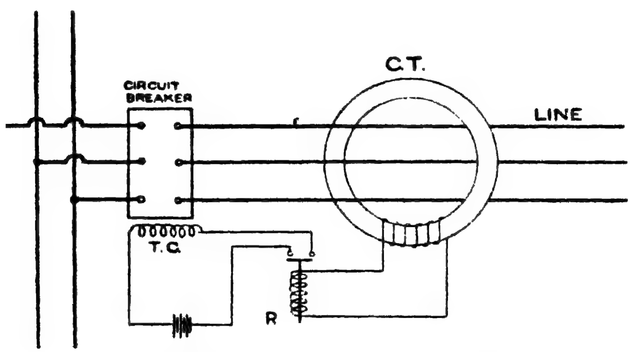

FIG. 103.—Core-balance leakage protection.

transformer secondary causing the relay to operate. By this means it is possible to secure definite and instantaneous operation with a leakage current corresponding to 5 °/ₒ of the full-load current in the circuit protected.

In its application to a multi-core cable a special form of core-balance transformer, the Ferranti-Field current transformer, can be used. Instead of having primary windings, the core of this transformer is in two halves and is clamped over the outside of the cable, first of all removing the armouring of the cable. The transformer is exactly the same in principle as illustrated in Fig. 103. It is remarkably easy to install and test, but cannot be made to respond to such small leakage currents as a transformer of the usual type having a wound primary.

Leakage protection is one of the simplest and probably the

cheapest form of protection against fault conditions, but cannot be regarded as adequate for a radial system of transmission requiring many relays in series owing to its lack of discrimination. It could, however, be made suitable for use on such a system if the neutral was earthed through a high resistance, so as to obtain smaller currents and powers on dead short-circuits to earth. Fault currents would then be definitely limited in value, and could be allowed to flow in the system for several seconds without much fear of damage. Under these circumstances definite time-lags could be fitted to the leakage relays, and discrimination between relays in series with each other could be obtained.

167. Combined Leakage and Overload Protection.—

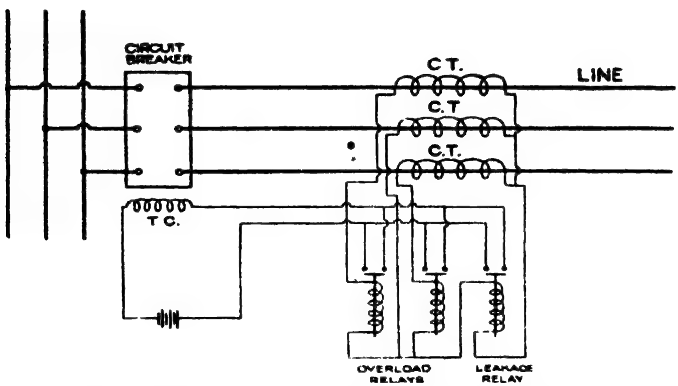

FIG. 104.—Combined leakage and overload protection.

The core-balance protection outlined above suffers from the disadvantage that if the fault or leakage occurs between phases the relay is inoperative. This disadvantage is overcome by the method of using three separate current transformers, due to Wedmore.[2] In this system of protection two overload relays and one leakage relay are connected as shown in Fig. 104. The two overload relays are sufficient to protect all three phases, while the leakage relay receives the resultant current from all three transformers, this being zero unless there is a leakage fault on the system. Hence protection is obtained against faults and short-circuits either to earth or between phases.

A simpler but less sensitive arrangement is to dispense with relays and employ three trip coils on the circuit-breaker, these

coils being connected in the same position as the three relay-operating coils in Fig. 104.

168. Core-balance Protection of Parallel Lines.—Simple leakage protective gear is unsuitable for use with parallel lines. This is seen from a consideration of Fig. 105. If an earth fault occurs on line *A* it is clear that the leakage current will flow not only along *A* but also in part along *B*. If both lines are equipped with leakage protective gear both lines will be disconnected—*i.e.* both the sound and the faulty one which is not desirable.

Possibly, as suggested by Garrard, this could be overcome by interlocking the leakage relays in a similar manner to that described in connection with overload relays, as the faulty line

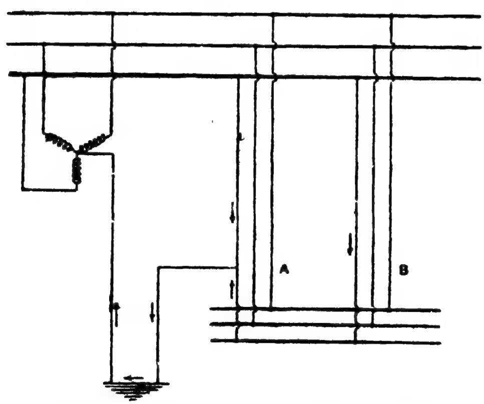

FIG. 105.—Effect of leakage on parallel lines.

would have the heavier leakage current. A biased relay could then be used if necessary, the leakage current through the protected line going through the operating coil, and that of the parallel line through the restraining coil.

169. Merz-Price Protection.—The fundamental principle on which this system of protection is based is that under healthy conditions the current entering one end of a line is equal to that leaving the other end. As soon as a fault occurs, however, this condition no longer holds, and the difference between the incoming and outgoing currents is arranged to operate a relay and isolate the defective line.

The application of this principle can be easily understood from the diagram given in Fig. 106, where the connections are shown for

the sake of simplicity as relating to a single conductor. Two similar current transformers are connected together by means of pilot wires, so that under healthy conditions the voltages of the two transformers are balanced against each other, with the result that no current flows through the relays connected in series with the secondaries of the transformers at each end. Should, however, the currents at either end become unequal due to a fault somewhere on the line, then the secondary voltages no longer balance and a current flows through the relays tripping the circuit-breakers at both ends of the line.

In actual practice, on a three-phase line each conductor has its own pair of current transformers and relays, these being connected by a three-wire pilot circuit. The other ends of the current transformers are connected in star, the neutral point being earthed at one end of the pilot circuit only, so as to prevent

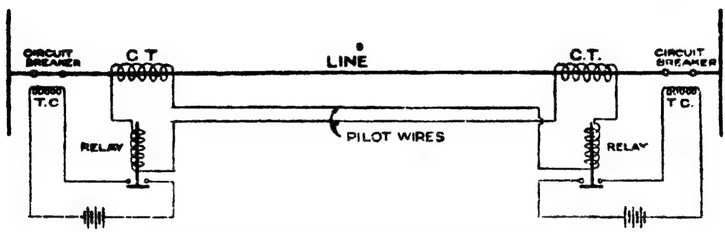

FIG. 106.—Principle of Merz-Price protection.

any current flowing between the two star points as this might operate the relays. This system has been widely employed for the protection of underground lines, and the pilot circuit then consists of a three-core, 7 / 0·029″, low-pressure cable, usually, but not necessarily, laid alongside the protected cable.

In the early arrangements of this gear, high-fault settings were necessary to secure stability when lines were carrying momentarily enormous currents due to a short-circuit outside the protected zone. These high-settings were caused by :—

1. The difficulty of obtaining current transformers to balance accurately at very high loads.

2. The effect of capacitance currents flowing in the pilot circuit.

The out-of-balance effects associated with the use of ordinary solid-core current transformers have now been eliminated by the development of the distributed-air-gap (D.A.G.) type of transformer

which has a straight-line voltage/current characteristic over the whole range of operation.

The trouble due to capacitance currents in the pilot circuit arises from the fact that, under through-fault conditions, voltages of the order of 1 000 volts or more are impressed on this circuit so that capacitance currents are comparatively heavy, and the relays must naturally be set sufficiently high to prevent operation under these circumstances. These capacitance currents have now been

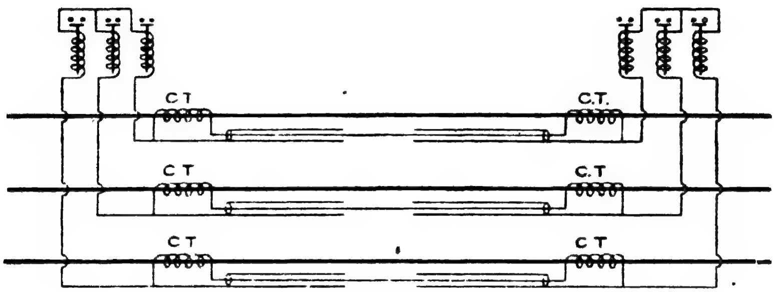

FIG. 107.—Merz-Price protection using compensated pilot cables.

rendered inocuous by the introduction of the Beard-Hunter compensated pilot cable in which means are provided of shunting the currents from the relays. This action is obtained by surrounding each conductor of the pilot cable with a metallic screen or sheath which is divided at the centre of its length so as to form two conductors of equal length. When a heavy overload comes on, a high potential is induced in the transformer secondaries, but the resulting capacitance current, instead of flowing in the relays, flows in the local circuit formed by the sheath, transformer, and pilot wire. This circuit can easily be traced in Fig. 107 and a section of the shielded pilot cable is shown in Fig. 108. As a consequence, much more sensitive relay settings can be employed than with the ordinary pilot cable. Furthermore, owing to the screening effect of the metallic sheaths, it is possible to embody telephone wires in the pilot cable, thus enabling one large strong cable to be used instead of two relatively weak ones.

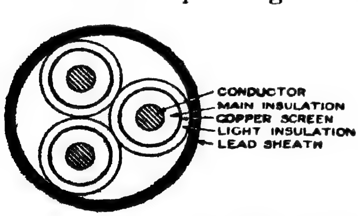

FIG. 108.—Section of Beard-Hunter compensated pilot cable.

170. Ferranti-Hawkins Protection.—This form of protection is very similar to the Merz-Price system, the difference being that instead of three separate current transformers being employed at each end of the line one core-balance transformer is used to link with all three conductors, as illustrated in Fig. 103. The current transformers are of the Ferranti-Field type and are clamped over the three-core cable to be protected, one at each end. The secondary windings of the transformers are connected by means of a two-core pilot cable, and under normal conditions no potential difference exists between the various parts of the pilot-wire circuit. With a fault current to earth the vector sum of the currents in the main circuit at one end is no longer zero, and this causes a current to be induced in the secondary windings of the transformers thus operating the relays.

This gear is very simple and is also discriminative in action. It is, however, sensitive to earth faults only, and will not operate if faults occur between the cores of the cable. In practice these inter-core faults rapidly destroy the insulation to earth and as soon as this happens the gear will operate.

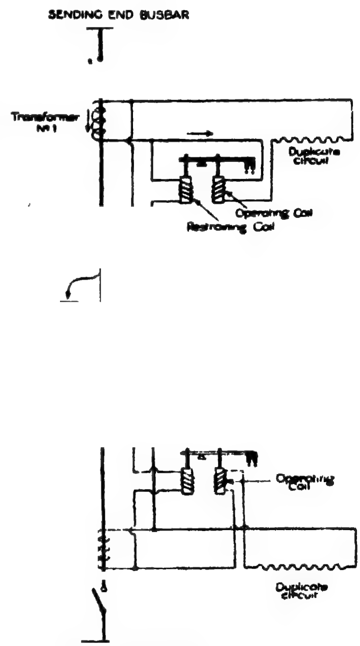

FIG. 109.—McColl biased line protection.

171. McColl Protection.—This system of protection in its application to one phase of a line is illustrated in Fig. 109. The protected conductor forms the primary of two current transformers, one at each end of the line, the secondaries being connected in series by means of two pilot wires. At each end of the line a beam relay is employed, the two restraining coils being connected in series with the transformer secondaries. Each secondary also feeds the operating coil of the beam relay through a duplicate circuit which is adjusted to

have the same resistance as one of the pilot wires. The relay is mechanically biased by having the fulcrum situated somewhat nearer the operating coil than the restraining coil, giving a greater leverage to the latter.

When the line is healthy the secondaries of the two transformers are circulating the same current, the two pilot wires and the two secondaries forming a closed circuit through which the current flows. Thus transformer No. 1 is responsible for the task of passing the current down one-half of the total system, that is down one pilot wire. Now this transformer has also an alternative path for its current in the duplicate circuit connected to it which, as has been stated, is of the same resistance as the pilot wire. Hence equal currents flow round the two coils of each relay and the latter is quite stable.

If, however, a fault develops in the line, more current will flow through transformer No. 1 (this being the sending end of the line) than No. 2, and consequently its secondary will deliver a greater current than the other. There is thus an excess current that does not circulate through the distant transformer. There are two paths open to this additional current; one through its own duplicate circuit, and one through the duplicate circuit at the other end in series with the two lengths of pilot wire. It will be seen that the latter path has three times the resistance of the former, so that three-quarters of the extra current will pass through the operating coil of the nearer relay, and a quarter through the restraining coil of this relay and the two coils of the distant relay.

The result is that 75 °/。 of the extra current affects the operating coil, and thus the relay tends to operate but is restrained by the bias of, say, 10 °/。 which it has been given. If the total currents flowing through the two coils were calculated it would be seen that the effect of a 10 °/。 leak is to increase by almost exactly 10 °/。 the pull of the operating coil of the relay at the sending end of the line relatively to its restraining coil. The result of the 10 °/。 leak is therefore that the relay just balances, and when this percentage increases, operates, completing the local circuit and tripping the circuit-breaker.

The above holds good whatever the load on the line, as the relay deals in percentages only. Hence a small fault setting down to 5 or 10 °/。 of normal full load can be employed, while

there is no risk of the relays tripping owing to heavy 'through' currents.

One advantage of this method of protection is that ordinary standard current transformers can be used. Also there is no trouble due to capacitance currents in the pilot wires as these currents flow through the restraining coils only, and actually produce a stabilising effect. Unlike the two pilot-wire systems described above, a current is constantly circulating through the pilot circuit under normal conditions, so that if by any mischance this should be open-circuited, both relays will at once operate. Frequent inspection and testing of the pilot circuit is therefore unnecessary.

Also, due to the fact that the secondary or pilot circuit carries a current which is a function of the load current in the line, the

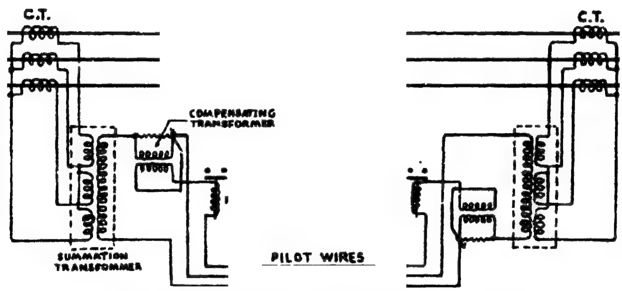

FIG. 110.—Self-compensating pilot-wire protection.

system can be made to give additional overload protection. This feature may be obtained by the simple expedient of inserting fuses in circuit with the pilot wires. In the event of a relay or circuit-breaker failing to operate after a short-circuit on some remote part of the system, the large sustained current flowing in the pilot wires will blow the fuses and divert all the current of each transformer through the operating coil of its relay, thus putting an end to the destructive effects.

172. Self-compensating Pilot Wire Protection. — This method of protection operates on the circulating-current system using plain three-core pilot cables, the general arrangement being illustrated in Fig. 110. The three current transformers at each end of the line energise a summation transformer connected so that two of the primary windings are responsive to phase-fault

current, and the remaining winding to earth-fault current (cp. Art. 167). The secondaries of the summation transformers at the near and remote ends of the line are connected in series through two of the pilot wires, which thus carry a single-phase circulating current varying in magnitude according to the load being transmitted. The relays themselves are connected by means of the remaining pilot wire between the two points X and Y, which under normal operating conditions are at the same potential, so that the resultant current through the relays is zero. On the occurrence of a fault in the protected zone, the balance of the circulating currents in the pilot wires is disturbed, thus causing a current proportional to the fault current to flow in the relay pilot wire and trip the circuit-breakers at each end.

In order to prevent false operation by capacitance currents in the pilot cable, a compensating transformer is used in conjunction with a fixed resistance inserted in each of the two outer pilot wires carrying the circulating current. The voltage drop across these resistances (which is proportional to the voltage drop along the pilot wires themselves) is stepped up by means of this compensating transformer, so as to be numerically equal to one-half of the voltage drop along each of the outer pilot wires. The two ends of the relay pilot wire are thus held at the potential of the centre points of the outer pilot wires, and the capacitance currents flowing to and from the relay pilot wire neutralise each other.

The current transformers used on this system are of the ordinary solid-core type, but designed so as to have low values of magnetising current, so that false tripping with through currents will not take place. Hence, no special pairing of protective equipment is necessary.[3]

173. Translay Protection.—This method of protection operates on the voltage-balance principle, and utilises relays of the induction type. Its application to a single-phase line is shown in Fig. 111, all the tripping circuits being omitted for the sake of clearness. The line current transformers, which are of the ordinary solid-core type, energise the primary windings P of the relays, and thus set up corresponding e.m.fs. in the secondary windings S. The latter windings, which are in series with the relay-operating coils, are connected in opposition by means of the pilot wires, so that no current can flow in this circuit under normal conditions. In the case of a fault in the protected zone,

the resultant difference between the voltages in the secondary windings produces a circulating current, and the consequent interaction between the fluxes produced by the operating and primary windings respectively causes the operation of the relays.

Although the actual current in the pilot wires is small, ample power is available for relay operation as the bulk of this energy is derived from the other co-operating current. The pilot-circuit voltage under the worst conditions is limited by saturation effects to about 130 volts, so that the capacitance current is also small and in any case the latter produces little effect on the relays as it leads the main current by nearly 90°. Any tendency of the relays to operate due to the current transformers having dissimilar characteristics is prevented by a biasing device which sets

FIG. 111.—Translay protection.

up a backward torque in the relay disc, this torque increasing in magnitude as the fault current increases.

In the application of this method of protection to a three-phase system, separate relays for phase and earth faults are generally installed at each end of the line, the setting of the earth-fault relays being adjustable to comparatively low values. In this case a plain three-core pilot is required. A simpler scheme utilises only one relay for both types of fault, a two-wire pilot circuit being then sufficient.[3]

174. Split-pilot Protection.—This system is generally arranged to function on current balance using a plain three-core pilot cable, two cores of which function on the split-conductor principle (cp. Art. 177). Fig. 112 illustrates the application of the method to the protection of a single-phase line. The current transformers at the near and remote ends of the line are connected

so as to produce a circulating current, and from Fig. 112 (*a*) it will be seen that this current divides equally in the split pilots (2) and (3), the resultant flux in the split-pilot transformers being zero. At the centre point of the pilot cable a connection *XY* is established between the common pilot and one of the split pilots. Under normal operating conditions the centre point of each pilot wire is at the same potential so that this circulating current does not affect the balance of the system. In the event of a fault fed, say, from

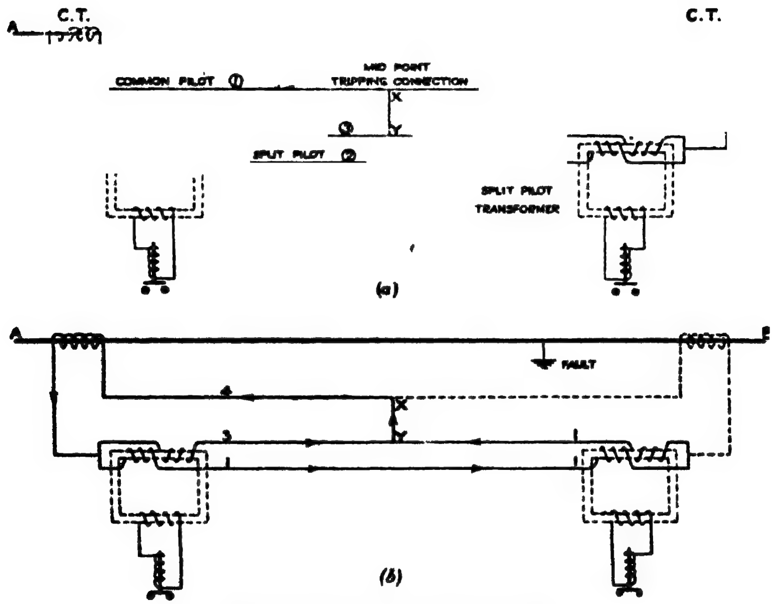

FIG. 112.—Split-pilot protection.

A, the out-of-balance current in the pilot wires distributes itself as shown in Fig. 112 (*b*). Neglecting the effect of that portion of the pilot circuit shown dotted, the pilot current divides in the split pilots in the ratio of three to one. Hence the tripping effect on each relay is the same, *viz.* proportional to three minus one at one end, and one plus one at the other. Although the shunting effect of the dotted path slightly alters the proportionality of the currents just described, the general principle still holds good.

In order to eliminate false operation due to current transformer out-of-balance the distributed-air-gap type of transformer is

generally used, but solid-core type transformers can also be employed if suitable precautions are taken. Capacitance currents in the pilot cable are the same in each split, and thus neutralise each other.

When applied to a three-phase system the current transformers at each end of the line are arranged in delta formation, the transformers in each phase having a different turns ratio, say, in the proportion of 6, 5, and 4. This method enables discrimination for phase and earth faults to be obtained, and has the advantage of subjecting the pilots to a minimum resultant voltage for a given through current.[3]

175. Interlock Protection.—On overhead transmission lines, protection systems involving the use of pilot cables are usually ruled out owing to the comparatively high cost of laying these cables, a special trench generally having to be excavated and reinstated for this purpose. The interlock system is a discriminative method of protection which, although requiring a pilot circuit, only utilises this circuit for a second or so on the occurrence of a fault, and even then only for the transmission of a signal at a low voltage. It can thus be arranged for a couple of telephone cores to be borrowed temporarily for protective purposes under fault conditions, these cores being then restored to normal use again.

This method of protection uses overload and earth-leakage relays, and directional elements are also fitted. On the occurrence of a fault in an interconnected system the fault current must flow out of each healthy section at one end, and by means of the directional elements this outgoing fault current is made to lock out the protective relays at that end, and by means of the temporarily borrowed telephone cores the protective relays at the other end. On the other hand, no fault current flows out of the faulty section, and therefore the protective relays at each end are free to operate. The relays must have a time-lag in order to allow prior operation of the locking-out arrangements, but this lag is quite small and also constant for any number of interconnected sections, so that the long time-lags associated with the use of time-graded relays on an extensive network are avoided.[3]

176. Distance Protection.—Distance relays have been developed to provide a cheap and yet discriminative form of protection for overhead lines where no pilot circuits of any kind are available. These relays function in accordance with the distribution of current

and voltage in a transmission system under fault conditions. For instance, on the occurrence of a fault on a simple radial system of transmission, the ratio of voltage to current at the various substations is least at the substation nearest to the fault on the generating station side, and increases towards the generating station. The ratio at each substation is, therefore, a measure of the impedance, and consequently a measure of the distance between that particular substation and the fault. In protective schemes of the distance type the relays are so designed that their times of operation are proportional to the ratio of voltage to current at the point of installation, and consequently those nearer to the fault will operate quicker than those more remote. On the occurrence of a fault therefore, the only circuit-breakers to be tripped will be those on the faulty section. The superiority of this method over that using ordinary time-graded overload or reverse-power relays arises from the fact that in the former case the time settings are automatically graded by the fault itself. This enables a fault close up to the generating station to be cleared as expeditiously as a fault at the most remote point on the network. The only time-lag required is that sufficient to allow the next relay to operate in case the fault is just beyond on the next section. In practice, however, instantaneous protection can be given to about 75 °/ₒ of each line, and the small time lag (of the order of one second) applies only to the case of faults on the remainder of the line. The principle can, of course, be extended to interconnectors and ring mains by incorporating a directional element in the relays.

In order to eliminate possible inaccuracies arising from the resistance of an arcing fault or, in the case of an earth fault, due to the resistance of the earth path, some methods of distance protection such as the ratio-balance system operate in accordance with the reactance of the fault path rather than the impedance.[3, 4]

177. Merz-Hunter Split-conductor Protection.—This system is another method of securing the benefits of a balanced method of protection without the necessity of employing pilot wires. The principle of operation depends on the fact that two conductors of equal length and impedance when connected in parallel will share the load equally, provided that the insulation of the system is sound. When a fault develops on one conductor it will carry more current than the other, and this inequality of currents is arranged to operate a relay and thus isolate the defective line.

In applying this principle each conductor of the line is divided into two sections or 'splits' which need be only lightly insulated from each other. For an overhead line each split may be carried by separate insulators, in which case the system really becomes a form of parallel-line protection. When applied to underground lines specially constructed cables are used, generally of one of the three types illustrated in Fig. 113. Great care must be taken to ensure equality of impedance between the splits, and if necessary this is obtained by arranging for transpositions at certain of the joints.

The application of the protective gear to a three-phase line is illustrated in Fig. 114. At both ends of the line the two splits forming one phase are connected in series with a differentially-

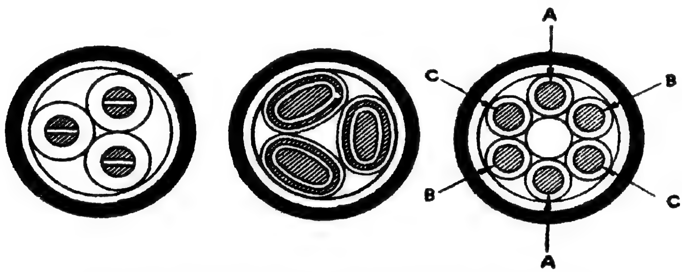

FIG. 113.—Types of three-core, split-conductor cables.

wound current transformer. The single secondary winding of the transformer is connected to the relay-operating coil. It is readily seen that while the currents in each of the splits are equal there is no resultant flux in the core of the transformer, but as soon as dissimilar currents flow (due to a fault developing), a secondary current flows through the relay, and if sufficiently great will trip the circuit-breaker. In the best arrangement of the system special circuit-breakers are installed at the ends of the line so that the splits are not connected until they reach the busbar side of the breaker. The reason for thus carrying the splits right through the circuit-breaker will be clear from the diagram of Fig. 115 where one phase of the line is shown as having a fault very near the receiving end. It will be seen that the current in the one half winding of the receiving-end current transformer has reversed, so that its core will become magnetised and the receiving-end circuit-breaker will trip. The currents in neither

of the windings of the sending-end transformer will have reversed, and supposing that the splits have not been taken right through the receiving-end circuit-breaker, will continue to flow through both

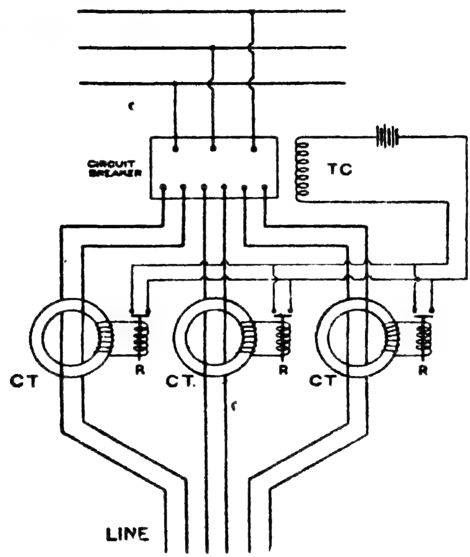

FIG. 114.—Merz-Hunter split-conductor protection.

splits in parallel into the fault. Now on a long line the impedances of the two splits up to the fault would be practically equal, and the inequality of current at the sending end would not be

FIG. 115.—Use of split-contact circuit-breaker with Merz-Hunter system of protection.

sufficient to trip the circuit-breaker at this end. If, however, the splits are carried right through the circuit-breaker this condition cannot arise, for as soon as the receiving-end circuit-breaker is tripped the two splits are isolated from one another at this end, and then the current flowing in the one split to feed the fault will

operate the relay and trip the sending-end breaker, thus completely isolating the defective line.

An alternative arrangement of the system, in cases where it is desired to use the ordinary type of circuit-breaker, is to install special high-reactance current transformers which accentuate the inequality of currents in the sound and faulty splits. It is found, however, that better results are obtained by using the split-contact circuit-breaker, and this is now to be considered the standard practice.

In many cases the cost of the special cables and terminal equipment exceeds that of an ordinary cable plus a separate pilot cable, so that this method of protection has not been used very widely.

178. Callender-Hunter Four-conductor Protection.—This system is a combination of the Merz-Price and Split-conductor

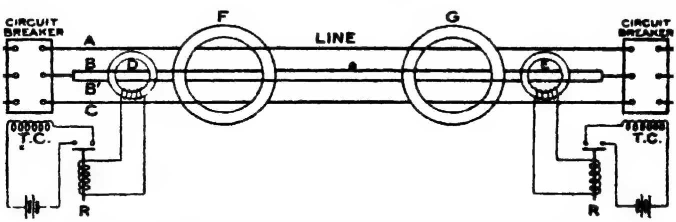

FIG. 116.—Callender-Hunter four-conductor protection.

systems. One phase (*B* in Fig. 116) is protected on the split-conductor system, whilst the other phases (*A* and *C*) are protected on the Merz-Price system, using the splits of phase *B* instead of pilot wires. In the diagram, *D* and *E* are the split-conductor transformers for phase *B*, and *F* and *G* are balancing transformers with which all three phases are interlinked. The relays are actuated by secondary windings on *D* and *E*.

The action of the arrangement for various kinds of faults may be explained as follows:—

Faults from Unsplit Phases to Earth or to Each Other.—Take the case of phase *A*—while the same current leaves *A* at one end as enters it at the other equal fluxes are produced by this current in the cores of *B* and *B'*. These fluxes produce e.m.fs. at each end of *B* and *B'* but the connections are such that these e.m.fs. are in opposition. As a consequence the normal load currents in *B* and *B'* are undisturbed.

In the event of a leakage from A, either F or G will produce the greater e.m.f. and tend to circulate a current in the closed circuit formed by B and B', with the result that the equality of currents in the primaries of transformers D and E will be upset and the relays R operated. A similar sequence of operations would occur in the event of a leakage from C, and also in the event of a leakage from A to C.

Faults from Unsplit Phases to Split Phase.—In this case the fault current produces a direct unbalancing of current in B and B', and although the transformers F and G may accentuate the inequality of currents in B and B' this action is not necessary to ensure operation of the relays on the occurrence of such a fault.

Faults from Splits to Earth.—In the event of leakage from either B or B' to earth a direct unbalancing of the currents is produced and the relays are operated.

Faults between Splits.—The balancing transformers F and G have the effect of tripping the relays in the event of a fault between B and B'. When A or C or both of them are carrying load a flux exists in the cores of F and G. These fluxes produce e.m.fs. in B and B' which are normally opposed and equal, and therefore do not disturb the currents carried by B and B'. In other words, the effect of balancing transformer F is to induce an e.m.f. in B and B' at one end, and the effect of balancing transformer G is to induce a precisely similar but opposite e.m.f. at the other end.

If now a fault occurs between B and B' at any point intermediate of the two ends the two e.m.fs. are no longer opposed but can circulate current each in its own local circuit, with the result that the relays are caused to operate.

It is clear from the above that besides being discriminative the protective arrangement operates for any form of leakage which may occur within the line, whether it be from phase to earth, phase to phase, or between splits.

As in the case of split-conductor protection, the field of application of this system is limited by the cost of the special cables required.

179. Ideal Characteristics of Protective Gear.—The requirements of an ideal system of line protection can be summarised as follows:—

1. The protection should discriminate with absolute certainty *i.e.* it should be certain of action on faulty cables and overhead lines, and inoperative with equal certainty on healthy sections.

2. The protection should be universally applicable, *i.e.* it should be suitable for use on all possible arrangements of lines whether these comprise separate lines, or form interconnectors on complicated networks, or consist of branched lines without switchgear at the points of junction.

3. The protection should be as instantaneous in isolating a faulty line as it is physically possible to be.

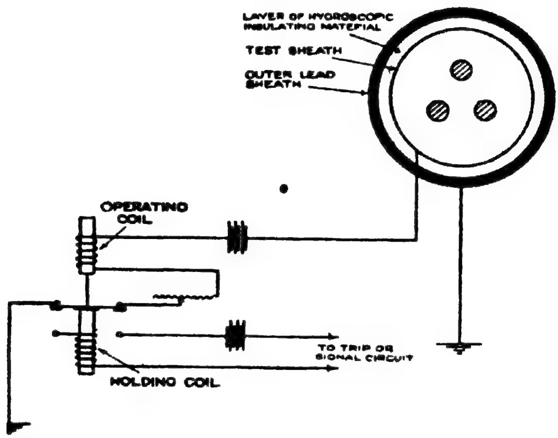

FIG. 117.—Beaver's cable protective system.

4. The protection should not interfere with complete freedom in laying-out, extending and operating the transmission and distribution system.

The only forms of protective gear which have shown in practice that they can make any claim to fulfil these requirements are those based on the balanced principle of protection, and exemplified in the systems described in Arts. 169 to 174. Within certain limitations, however, protective systems necessitating the use of special designs of cables (Arts. 177-8) offer a good standard of performance.

180. Beaver Test-sheath Protection.—If a cable is furnished with an auxiliary sheath, lightly insulated from the outer lead sheath, it is clear that any fault penetrating the cable from with-

out will reach the auxiliary sheath before it attacks the main insulation. This feature is utilised in the protective system described below, due to Beaver.

The system makes use of a special three-core cable wherein a test sheath is provided, surrounding the cores but insulated therefrom, and also lightly insulated by a hygroscopic material from the lead sheath which in turn surrounds it (*see* Fig. 117). A battery is connected between the two sheaths, and an incipient breakdown of the insulation can be detected by an alarm device, or may cause a tripping relay to operate and disconnect the cable. There is thus a possibility of locating a fault in its early stages, the cable being then removed from service and repaired before complete breakdown occurs.

Many other systems have also been developed in which auxiliary conductors in the form of sheaths are inserted in the cables.[5]

181. Short-circuit of Alternators.—In applying any protective scheme the calculation of short-circuit current values is necessary, as a basis of relay settings. In this connection it is unfortunate that the term 'overload' has come into use in reference to sectionalising transmission systems, since it implies that the relays should be set to operate at a value determined by the normal load on the line. A short-circuit on a system, however, produces an abnormal condition which has no relation to normal loads and overloads, and in order to obtain selective action from overload, reverse-power, and certain other types of relays, it is necessary to set the relays for the currents flowing under this condition.

Before describing methods of calculating these abnormal currents it appears desirable to discuss briefly the phenomena occurring when an alternator is short-circuited.

If the impedance of the load circuit of an alternator, with normal excitation, is gradually reduced to zero, the short-circuit current is limited only by the synchronous impedance of the armature. Neglecting the armature resistance, which is very small in comparison with the synchronous reactance, the short-circuit current is

$$I_{sc} = \frac{E}{x_s}. \qquad \qquad (206)$$

x_s is made up of two components: x_l which represents the effect

of the armature leakage flux, and x_r which represents the effect of armature reaction.

The value of I_{sc} for older designs of machines is generally about two or three times full-load current, but turbo-alternators of recent construction have a permanent short-circuit current approximating in value to full-load current, the synchronous reactance x_s varying from about 80 to 150 %.

In the case of a sudden short-circuit the initial current will be much larger than the permanent value I_{sc}, because the component x_r of the synchronous reactance, which represents the

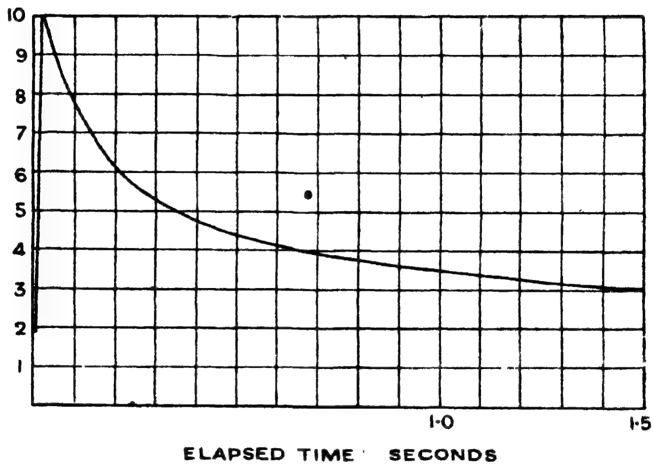

FIG. 118.—Typical curve showing R.M.S. short-circuit current of alternator in terms of full-load current.

effect of armature reaction, does not act instantaneously to limit the current. It represents a change in the flux which interlinks with the field circuit of the machine, and on account of the inductance of the field winding with its large number of turns, this change of flux cannot take place instantaneously but may take several seconds to complete. Again neglecting the armature resistance, the initial short-circuit current is limited only by the inherent or leakage reactance x_l and is

$$I'_{sc} = \frac{E}{x_l}. \tag{207}$$

The amplitude of the current gradually decreases as the armature reaction reduces the flux in the field and the generated

e.m.f. E, until the permanent value I_{sc} is reached. Fig. 118 is a typical curve for an alternator having a leakage reactance of 10 %.

It will be seen that the initial short-circuit current bears to the permanent short-circuit current the ratio

$$\frac{I'_{sc}}{I_{sc}} = \frac{x_s}{x_l}. \qquad (208)$$

With machines of high self-inductance and low armature reaction this ratio will be moderate in value, but large turbo-alternators with low self-inductance and high armature reaction may have a much greater ratio. In extreme cases the initial short-circuit current may reach twenty or thirty times full-load current. Generally speaking, however, the inherent reactance x_l of turbo-alternators varies from 5 to 15 %, so that values of I'_{sc} may be anything between 7 and 20 times full-load current.

It should be noted that I'_{sc} will represent the initial short-circuit current only when the short-circuit occurs at the maximum point of the generated e.m.f. wave as illustrated in Fig. 119 (a). Neglecting the armature resistance, the generated e.m.f. is consumed by the back e.m.f. of armature inductance $= -L\frac{di}{dt}$. The cross hatched area under the e.m.f. wave is

$$\int_0^{\sqrt{2}E} e\,dt = \int_{\sqrt{2}I'_{sc}}^{0} - L\frac{di}{dt}dt$$
$$= \sqrt{2}\, I'_{sc}L,$$

and it causes the current to increase from zero to the peak value of $\sqrt{2}I'_{sc}$. The succeeding quarter-wave of e.m.f. is negative and reduces the current to zero again; it then reverses and grows in the opposite direction to a value slightly less than before, since the armature reaction will have caused a slight reduction in the flux and the generated e.m.f. The current is symmetrical about the zero line, and its amplitude decreases until the reduction of flux by armature reaction is complete, after which it remains constant. The transition from the initial to the permanent value will take from a few hundred to thousands of cycles, depending on the inductance of the field winding. In Fig. 119 (a) it is represented as covering only a few cycles.

Fig. 119 (b) shows the short-circuit current in the other phase of the machine, which is assumed to be two-phase. The short-circuit occurs at the zero point of the generated e.m.f. wave, and the positive area under the wave is twice as great as before and builds up the current to a peak value of $2\sqrt{2}I'_{sc}$, or double the value in (a). The current then decreases, and it alternates, not about the line of zero current as before, but about a line starting above the zero line by an amount $\sqrt{2}I'_{sc}$ on the current scale and sloping down to coincide with the zero line after a few cycles.

The armature current may be considered as being composed of a transient direct current of maximum value $\sqrt{2}I'_{sc}$, shown as

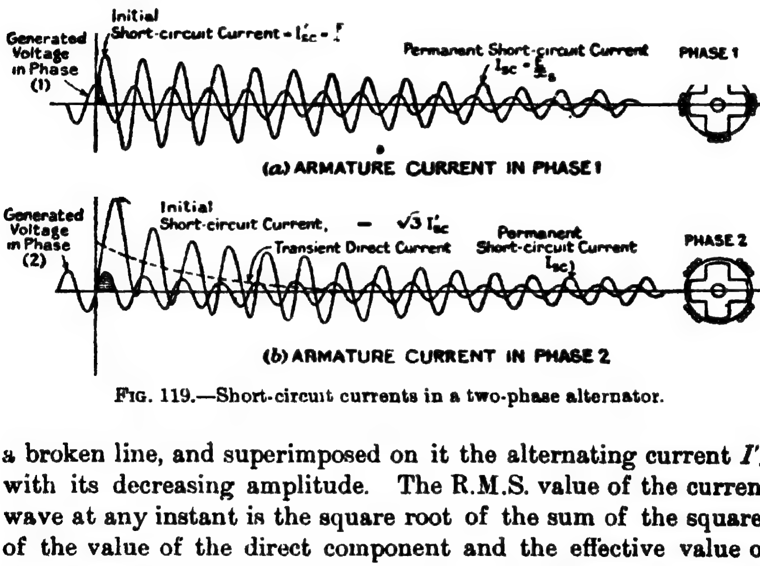

FIG. 119.—Short-circuit currents in a two-phase alternator.

a broken line, and superimposed on it the alternating current I'_{sc} with its decreasing amplitude. The R.M.S. value of the current wave at any instant is the square root of the sum of the squares of the value of the direct component and the effective value of the alternating component at that instant. Hence, under the worst conditions (Fig. 119 (b)), the initial short-circuit current may have a R.M.S. value of $\sqrt{(I'_{sc})^2 + (\sqrt{2}I'_{sc})^2} = \sqrt{3}I'_{sc}$, and in practice the initial value of the current may be anything between I'_{sc} and $\sqrt{3}I'_{sc}$, depending on the phase of the voltage wave at the instant of short-circuit.

So far, the short-circuits have been assumed to take place at the alternator terminals, and in this case the ratio between the initial and permanent short-circuit currents has its maximum

value. In most cases, however, the fault occurs on the line, and considerable reactance is introduced by the apparatus—transformers, reactors, and part of the line itself—connected between the machine terminals and the point of short-circuit. The reactance of this connected apparatus does not vary with time, and hence the ratio between the initial and permanent values of short-circuit current depends to some extent on the proportion of the total reactance external to the alternator.

182. Calculation of Short-circuit Currents.—In determining the current produced by a short-circuit on a transmission system it is customary to neglect the resistances of lines and apparatus and make all calculations with reactances alone. This approximation is justifiable as the reactances involved are usually large compared to the respective resistances, so that the impedance is only slightly greater than the reactance numerically. Furthermore, this method gives values of short-circuit current rather larger than actually obtain, and consequently the results are on the safe side. In the case of underground lines, however, the resistance of the cable is often larger than the reactance and should be taken into account.

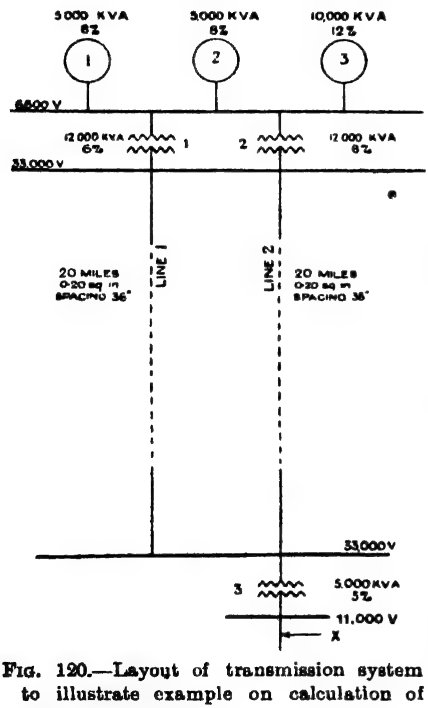

FIG. 120.—Layout of transmission system to illustrate example on calculation of short-circuit currents.

The ordinary method of calculation is to the reactance of all generators, transformers, and lines in ohms to neutral, and by adding up the reactance from the point of short-circuit to and including the generators, and dividing the sum into the voltage to neutral, the initial value of the short-circuit current is found. In practice, however, the reactance of most

apparatus is expressed on a percentage basis, and furthermore, several different voltages are usually found on the same power system, so that it is found more convenient to use the percentage method of calculation described by Wilson.[6]

In this method all reactance of generators, transformers, lines, or other apparatus is expressed as per cent. reactance at a common kVA. base arbitrarily selected. The various reactances are converted up or down as the case may be to this base. The combined reactance from the generator neutrals to the point of short-circuit is determined, and it is assumed that the entire voltage of the generators is used between these two points. The initial short-circuit kVA. is then

$$\frac{\text{kVA. base}}{\text{percentage reactance to point of fault}}. \qquad (209)$$

To illustrate the application of this method the layout given in Fig. 120 will be assumed and the short-circuit current at the point X calculated.

Generator No. 1 rated at 5 000 kVA. inherent reactance 8 %.
Generator No. 2 rated at 5 000 kVA. inherent reactance 8 %.
Generator No. 3 rated at 10 000 kVA. inherent reactance 12 %.

Taking as base 10 000 kVA.

$$\text{Reactance of No. 1} = \frac{10\ 000}{5\ 000} \times 8\ \% = 16\ \%,$$

$$\text{Reactance of No. 2} = \frac{10\ 000}{5\ 000} \times 8\ \% = 16\ \%,$$

$$\text{Reactance of No. 3} = \frac{10\ 000}{10\ 000} \times 12\ \% = 12\ \%,$$

and the combined reactance of the three generators

$$= \frac{1}{\frac{1}{16}+\frac{1}{16}+\frac{1}{12}}\ \%$$

$$= 4{\cdot}8\ \%.$$

Transformer No. 1 rated at 12 000 kVA. reactance 6 %.
Transformer No. 2 rated at 12 000 kVA. reactance 6 %.

At a base of 10 000 kVA.

$$\text{Reactance of No. 1 or No. 2} = \frac{10\ 000}{12\ 000} \times 6\ \% = 5\ \%.$$

and the combined reactance of the two transformers

$$= \frac{1}{\frac{1}{5} + \frac{1}{5}} \ \%$$

$$= 2\cdot5\ \%.$$

Line No. 1, 20 miles long, 0·20 sq. in. stranded copper, diameter 0· 579 inch, spacing 36-inches delta.

Reactance of conductor

$$= 1\cdot654 \times 2\pi \times 50 \times 20$$

$$= 10\cdot39 \quad \text{ohms.}$$

At 10 000 kVA. base (3 333 kVA. per phase) the current is

$$\frac{3\ 333 \times 1\ 000}{\frac{33\ 000}{\sqrt{3}}} \quad \text{amperes,}$$

the reactance pressure drop is

$$\frac{3\ 333 \times 1\ 000 \times 10\cdot39}{\frac{33\ 000}{\sqrt{3}}} \quad \text{volts,}$$

and the percentage reactance is

$$\frac{3\ 333 \times 1\ 000 \times 10\cdot39 \times 100}{\frac{33\ 000}{\sqrt{3}} \times \frac{33\ 000}{\sqrt{3}}}$$

$$= 9\cdot54\ \%.$$

Line No. 2 has a similar percentage reactance and the combined reactance of the two lines is

$$\frac{1}{\frac{1}{9\cdot54} + \frac{1}{9\cdot54}} \ \%$$

$$= 4\cdot8\ \%.$$

Transformer No. 3 rated at 5 000 kVA. reactance 5 %. On a base of 10 000 kVA.,

$$\text{Reactance} = \frac{10\ 000}{5\ 000} \times 5\ \% = 10\ \%.$$

The total reactance up to point of short-circuit

$$= 4\cdot8 + 2\cdot5 + 4\cdot8 + 10\cdot0$$

$$= 22\cdot1\ \%.$$

The kVA. at the point of short-circuit

$$= \frac{10\,000 \times 100}{22 \cdot 1}$$
$$= 45\,200 \quad \text{kVA.}$$

This is the initial value of the short-circuit and the current corresponding thereto is 2 380 amperes. The current to be ruptured by the circuit-breaker will be lower than this, being intermediate in value between the initial and permanent short-circuit currents. Its actual value will depend on the time-lag setting of the protective relay, and the time-lag introduced by the operation of the circuit-breaker, *i.e.* between the energising of the trip coil and the separation of the breaker contacts.

The determination of the actual short-circuit current at any given instant during the transient period is a matter of some difficulty, but by making certain assumptions which take into account typical operating conditions and machine constants it is possible to draw up data which give approximate results in a very convenient form. In America, for instance, standard decrement curves have been in use for a considerable time. Table 24 gives a selection of values from the latest revision of these curves published in an article by Hahn and Wagner.[7]

The values given in the table are factors by which the normal full-load current must be multiplied to get the short-circuit current at any given instant. The percentage reactance figures represent the total reactance in the circuit, including that of the generator, up to the point at which the short-circuit takes place. At the shorter time intervals allowance has been made for possible doubling-effect, but after 0·3 second the effect of asymmetry is negligible.

In calculating the possible short-circuit current all synchronous machinery connected to the system must be taken into account, and not merely the total capacity of the generating plant. A rotary convertor or synchronous phase modifier possesses sufficient flywheel-effect to enable it to supply current as a generator during the time taken by the circuit-breaker to operate.

Table 24 is based on the assumption that at the time of short-circuit the generators are carrying full-load current at 80 % power factor, and that no automatic voltage regulators are installed. Furthermore, all values of reactance up to 15 % are

TABLE 24.—*Current Decrement Factors for Three-phase Short-circuits.*

Reactance.	10 °/₀.	20 °/₀.	30 °/₀.	40 °/₀.	50 °/₀.	100 °/₀.
Elapsed Time in Seconds from Start of Short-circuit.	Decrement Factors expressed as number of times Full-load Current.					
0·05	13·6	7·0	4·6	3·5	2·8	1·4
0·10	10·2	5·5	3·8	2·9	2·4	1·2
0·15	8·3	4·6	3·3	2·5	2·1	1·1
0·25	6·0	3·8	2·8	2·2	1·8	1·0
0·50	4·4	3·2	2·5	2·0	1·7	0·9
1·00	3·0	2·7	2·2	1·8	1·6	0·9
2·00	2·2	2·2	1·9	1·6	1·4	0·9
3·00	2·0	2·0	1·8	1·5	1·4	0·9

considered to be inherent to the generators. No serious error will be introduced, however, if the figures are applied to cases where only part of this reactance is inherent, and part external.

When automatic voltage regulators are used they will increase the excitation on the occurrence of a short-circuit in the endeavour to keep up the voltage. The effect of this increased excitation is not noticeable during the first 0·5 second but thereafter the short-circuit current will be higher, and the permanent short-circuit current after, say, 2 seconds may be about 50 °/₀ higher than the values given by Table 24. This condition only holds, however, when the fault is close to the generator terminals. If the short-circuit occurs at a remote point in the system, so that the normal voltage can be maintained at the busbars, the value of short-circuit current is not increased since the limiting factor is the reactance of the line and the connected apparatus.

In Great Britain very little standardisation of decrement factors has taken place. It is generally assumed, however, that the interval between the occurrence of the short-circuit and its rupture will not be less than 0·2 second, and under these

circumstances the current to be broken is taken at 0·6 of its initial value when the fault occurs close up to the alternator terminals. In determining this initial value no allowance is made for doubling-effect. Assuming, for example, that the inherent reactance of the alternator is 10 °/₀, the conditions would be represented by the curve shown in Fig. 118. In the case where the external reactance is of the same magnitude as that of the alternator reactance the current to be broken after 0·2 second is taken at 0·75 times its initial value, and where the external reactance is much greater than the machine reactance no falling off with time is assumed.[8]

As an illustration of the use of Table 24 for the determination of relay settings, suppose that the low-tension circuit-breaker of Transformer No. 3 (Fig. 120) is furnished with a relay intended to operate under short-circuit conditions after an interval of 1 second. The relay must then be set for the current value at the end of this interval. First of all it is necessary to find the reactance at the point of short-circuit, based on the total generator capacity. Converting the initial short-circuit kVA. to a percentage reactance based on the total generator capacity of the system (20 000 kVA.) gives a percentage reactance of

$$\frac{20\,000}{45\,200} \times 100 = 44{\cdot}2\ \%.$$

Interpolating the values given in Table 24 for 44·2 %, reactance and 1 second time interval gives the number of times full-load current as approximately 1·7. Therefore the short-circuit current after a time-lag of 1 second can be taken as

$$\frac{1{\cdot}7 \times \text{gen. kVA. per phase} \times 1\,000}{\text{voltage to neutral}}$$

$$= \frac{1{\cdot}7 \times 6\,667 \times 1\,000}{\dfrac{11\,000}{\sqrt{3}}}$$

$$= 1\,800 \quad \text{amperes.}$$

183. Unbalanced Faults and Short-circuits.—So far in the treatment of the subject it has been assumed that all faults or short-circuits occur simultaneously on each phase of the system. In practice, unbalanced faults, such as those occurring from one conductor to the ground or between two conductors are commonest,

and the calculation of short-circuit currents under these conditions can only be carried out by the method of symmetrical components developed by Fortescue.[9] This extremely valuable method of analysis is based on the fact that any unsymmetrical system of n vectors (n being a prime number) may be resolved into $n - 1$ different symmetrical systems each of n phases, and a remaining system of n equal coincident vectors. A three-phase unbalanced system can therefore be split up into three balanced systems known as the positive-phase sequence, negative-phase sequence, and zero-phase sequence components respectively. In general, the impedances of station apparatus and transmission lines vary according to the particular phase sequence involved, but once these values have been determined, by means of calculations or tests, the current distribution for unbalanced fault conditions can be readily solved. The subject is too vast to be pursued further here, but the application of phase-sequence components to the calculation of abnormal currents and voltages occurring under fault conditions is of great importance as in many cases the worst conditions are only obtained with unbalanced faults.[10]

184. Effects of Short-circuit Currents.—With the great increase in size of generating stations, the power liberated on the occurrence of a short-circuit is enormous, and it is increasingly difficult and expensive to provide circuit-breakers capable of rupturing the current. Thus, besides the installation of protective relays for tripping the breakers on the occurrence of a fault, it has become necessary to provide means for positively limiting the magnitude of the current flowing.

For this purpose current-limiting reactors are often employed. These reactors consist of large reactance or choking coils connected permanently in circuit, and under normal conditions only entail a small loss of power. When a short-circuit occurs, however, they limit the current to a safe value, and tend to localise the effect of the fault by restricting the amount of power that can flow into the defective section from other parts of the system. They also protect the generating, transforming, and transmitting apparatus from damage due to the thermal and electro-mechanical effects of very heavy currents.

When a short-circuit takes place on a transmission line heat is generated in the conductors very rapidly, and there is little time for radiation, convection, or conduction.

Hence if I = R.M.S. current flowing in amperes,
r = Resistance per centimetre of conductor in ohms,
G = Weight per centimetre of conductor in grammes,
S = Specific heat of conductor material = 0·095 in the case of copper,
and g = Specific gravity of conductor material = 8·9 in the case of copper,

the temperature rise per second in degrees C. is

$$\frac{0{\cdot}24I^2r}{GS},$$

or since

$$G = gA \times (2{\cdot}54)^2,$$

and

$$r = \frac{\rho}{A \times (2{\cdot}54)^2},$$

where ρ = resistivity of the conductor material,
and A = cross-sectional area in square inches,
this may be written as

$$\frac{0{\cdot}24I^2\rho}{(2{\cdot}54)^2 \times (2{\cdot}54)^2 \times A^2Sg}. \qquad (210)$$

In the case of copper the temperature rise is

$$\frac{0{\cdot}012I^2}{A^2 \times 10^6} \text{ deg. C. per second.} \qquad (211)$$

Besides this heating effect there are mechanical forces set up between the conductors which have been short-circuited. These forces assume special prominence in the case of underground lines owing to the close interaxial spacing of the conductors, and cables have been known to burst their paper and lead sheathing on the occurrence of a short-circuit.

Consider first the case of two parallel conductors carrying a current I amperes in opposite directions, the interaxial spacing being x centimetres. The repulsive force between the conductors is

$$F = \frac{0{\cdot}2I^2}{x} \text{ dynes per centimetre length,}$$

or changing into more practical units

$$F = \frac{5{\cdot}4I^2}{d \times 10^7} \text{ lbs. per foot,}$$

where d is the distance between centres of conductors in inches.

If the current instead of being continuous is alternating, the R.M.S. value being I, the mechanical stress exerted between the two conductors pulsates between zero and the maximum value—

$$F = \frac{1 \cdot 08 I^2}{d \times 10^6} \text{ lbs. per foot.} \qquad (212)$$

In a three-phase circuit the actual currents that will develop at a short-circuit, and the consequent mechanical stresses between conductors, depend upon the nature of the fault and the arrangement of the conductors. Exact formulæ for various cases will be found in a paper by Robinson.[11]

185. Current-limiting Reactors.—The reactance of these pieces of apparatus, like that of generators and transformers, is usually expressed as a percentage. Thus a 5 °/ₒ reactor is one in which the voltage drop when carrying full rated current is 5 °/ₒ of the voltage to neutral of the circuit in which it is placed. In case of short-circuit the current flowing through a reactor of this value will be high, but (neglecting any initial doubling effect) will always be limited to a magnitude not exceeding 20 times full-load current.

Supposing that an iron core was used in the construction of a reactor the normal magnetic flux density would have to be kept very low so as to avoid saturation with a m.m.f. of many times normal. Otherwise the protective effect of the reactor would compare badly with the unavoidable pressure drop at normal load. This low flux density makes the use of iron uneconomical in reactors, and even if its use were electrically permissible an iron magnetic circuit would be much larger than an air circuit.

There is considerable variation in details of construction of these reactors. One of the most successful types utilises bare conductors embedded in concrete supports which are cast round the coil, and ensure a very rigid construction. This is necessary on account of the large forces which are suddenly called into play when the first rush of current occurs.

Oil-immersed reactors are also employed, particularly for the higher voltages, and have the advantage of being suitable for outdoor use. With this type of apparatus precautions must be taken to prevent the external magnetic field of the rector from cutting the tank and thereby causing high losses and heating. To achieve this end a low reluctance magnetic circuit of iron laminations

may be built round the reactor coil to carry the external field, or alternatively short-circuited copper rings may be fitted to the inside of the tank.

The losses in reactors are due to the I^2R and eddy-current losses in the conductors and to reduce the latter losses the conductors are stranded. The total loss usually varies from 1 °/₀ to 3 °/₀ of the kVA. rating of the reactor.

As previously pointed out, an ordinary iron-core reactor cannot be economically employed owing to the iron becoming saturated at increasing values of load and thus having a much lower reactance at short-circuit than at normal load. A special type of reactor known as the saturated-core reactor has precisely opposite characteristics.[12] In this apparatus the iron core is excited by an auxiliary D.C. winding, and by suitable design the ratio of the short-circuit reactance to the normal-load reactance may be made as high as three to one. In other words, if the reactance under normal conditions was 10 °/₀, the reactance under short-circuit would be about 30 °/₀. This highly desirable rising-reactance characteristic of the saturated-core reactor is obtained at the expense of higher losses and costs, and so this type of reactor will only be used in special cases.

186. Location of Reactors in System.—Reactors may be used in series with generators or transmission lines, or inserted between various busbar sections.

If reactors are put in series with each generator, as in Fig. 121 (*a*), the current is limited on short-circuit no matter whether the fault occurs on the busbars or on the line. The protection afforded, however, in case of a line short-circuit is less than with a separate line reactor, as the whole system is affected by any drop in voltage. When new machines are being built the usual practice is to incorporate the necessary reactance in the design of the machine itself, so that it can stand repeated short-circuiting without damage. As a rule, generator reactance is only used with high-voltage machines, or in series with old machines with the object of protecting the windings from excessive mechanical shock. In such cases the reactor may have a value of 5 to 10 °/₀.

Since the vast majority of faults occur on the transmission system, reactors in series with the line, as shown in Fig. 121 (*b*), are most effective, and localise the trouble to the single line on which the fault occurs. To obtain a given degree of protection, *i.e.* to

limit the current on short-circuit to a definite figure, less reactance is required in the line than elsewhere. When used, line reactors are usually of the order of from 3 °/。 to 5 °/。 based on the normal load. As in the case of generators, transformers for high-voltage transmission systems are built whenever possible with a comparatively high internal reactance, so that external line reactors are not required.

In both these cases the current normally passes through the reactor, involving not only additional losses but also a varying voltage drop according to the load. With reactors installed in the

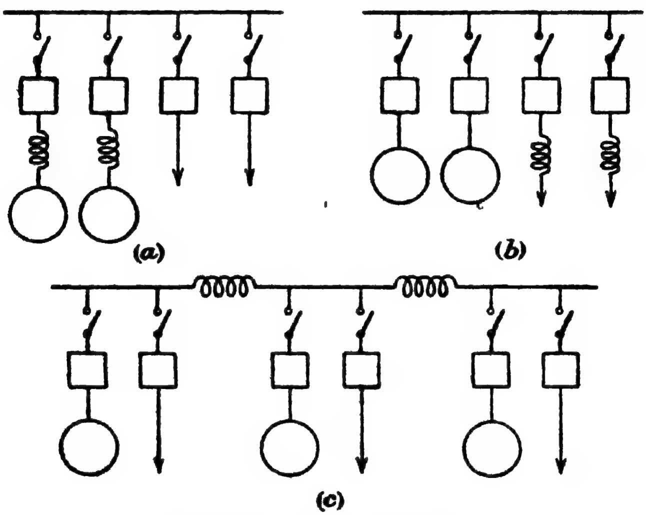

FIG. 121.—Location of reactors.

run of busbars between groups of generators, as shown in Fig. 121 (*c*), only as much current passes from one section to another as is necessary to equalise the loading of the generators. Thus losses are reduced and voltage regulation improved. The effects of a short-circuit are also largely confined to those circuits supplied from the bus section to which is connected the line in trouble. As busbar reactors do not normally carry much current they are frequently as great as 25 °/。 based on the capacity of one group of generators. The busbar reactor arrangement is probably more extensively used than any other, often by itself but sometimes in conjunction with individual or group line reactors.

187. Experimental Methods of Determining Short-circuit Currents.—A knowledge of the magnitude of the short-circuit current at various points on a power system is essential for many purposes. For example, this information is required in dealing with the following problems :—

1. Selection of circuit-breakers of the required rupturing capacity.
2. Determination of the size of current-limiting reactors.
3. Determination of relay settings in relay systems depending on selective action from overload and directional relays.
4. Calculation of mechanical stresses in the structural elements of apparatus subject to heavy electromagnetic forces.

The size and complexity of present-day systems, however, have reached such a point that the determination of short-circuit values by mathematical methods is extremely lengthy and laborious. Hence experimental methods of solution have been evolved which give results quicker, and of sufficient degree of accuracy.

The most extensively used of these experimental methods is the short-circuit calculating table, which is an apparatus in which the actual A.C. system is represented by an approximately equivalent D.C. network, reactances being represented by resistances, etc. In the later forms of tables any particular problem can be set up rapidly by an arrangement of telephone jacks and cord circuits. The magnitude of the currents in the various circuit branches of this equivalent D.C. network can then be read off, and converted, by the use of the proper factor of proportionality, into the corresponding A.C. currents of the original system. According to Schurig[13] the errors are usually under 20 %, and often under 10 % in error.

Miniature A.C. power systems have also been built up using three-phase generators and transformers for the station apparatus, and conveniently-arranged units of inductance, capacitance and resistance for the lines and loads. More accurate results can be obtained with this type of system than with the D.C. short-circuit calculating table, the chief errors being introduced by switching in measuring instruments, and by the difficulty of making the large number of simultaneous readings required.[14]

The latest development is the single-phase A.C. network analyser which is of the static type, thus eliminating the practical difficulties associated with the use of rotating machinery. In

this apparatus, generators are represented by phase-shifting transformers, and by careful selection of the voltage and current scales errors introduced by instruments have been considerably reduced. The results obtained are in most cases considered to be less than 3 % in error.

Both D.C. and A.C. types of apparatus can be used to determine unbalanced fault conditions by setting up separate single-phase networks for each phase sequence and interconnecting these according to the type of fault. The cost of an A.C. network analyser is, of course, much greater than that of the D.C. short-circuit calculating table, and the extra expense would hardly be justified if its only function was the determination of short-circuit currents. The A.C. apparatus has a far wider field of application, however, as it may be used in addition for the determination of voltage, current and power data under normal operating conditions, and also for the solution of problems associated with the stability and power limits of transmission systems.[15]

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CHAPTER XIII.

PRESSURE RISES.

188. Natural Impedance of a Circuit.—Before discussing the origin of pressure rises in transmission systems, it is necessary to examine briefly the fundamental properties of an oscillating circuit. Consider the case of an electric circuit in which an oscillation of energy is occurring, the power losses in the circuit being negligible. It is obvious that at the instant when the current wave passes through zero value the whole of the energy must be stored in the electrostatic field, and similarly, at the instant when the pressure wave passes through zero value the whole of the oscillating energy must be stored in the electromagnetic field. As it is the same stored energy appearing alternately as dielectric and magnetic energy, it follows that

$$\frac{Ce^2}{2} = \frac{Li^2}{2},$$

or

$$\frac{e}{i} = \sqrt{\frac{L}{C}} = Z_0. \qquad (213)$$

The quantity Z_0 is thus seen to be of the nature of an impedance, and is expressed in ohms. It is generally known as the natural impedance, or surge impedance, of the circuit and its reciprocal Y_0 is the natural or surge admittance.

In the case of an overhead transmission line the inductance is

$$L_0 = \left(0{\cdot}080 + 0{\cdot}741 \log_{10}\frac{d}{r}\right)10^{-3} \text{ henries per mile,} \qquad (9)$$

and the corresponding value of capacitance is

$$C_0 = \frac{0{\cdot}0388}{\log_{10}\frac{d}{r}} \times 10^{-6} \text{ farads per mile.} \qquad (15)$$

Neglecting the first term in the inductance formula, which is usually quite small in comparison with the second term, the natural impedance of an overhead line is closely given by

$$Z_0 = \sqrt{\frac{L_0}{C_0}} = 138 \log_{10} \frac{d}{r} \quad \text{ohms.} \qquad (214)$$

For commercial lines Z_0 usually lies between 400 and 600 ohms.

In underground cables the capacitance is much larger relatively to the inductance, and the natural impedance has a smaller value which may be about one-tenth of the value for overhead lines.

A knowledge of Z_0 is exceedingly useful as it enables a calculation of the transient voltages and currents which may occur in a circuit. For instance, if a line carrying a current i has this current suddenly interrupted, the maximum value of the oscillating voltage produced thereby is iZ_0.

If one conductor of an unearthed system is earthed the maximum transient current flowing to earth is eY_0, where e is the voltage to earth.

If lightning strikes a line and the high-frequency breakdown voltage of the insulators is e, the maximum discharge current in the conductors is limited to eY_0.

Power-transmission systems are always complex in character, that is, they consist of sections or elements—such as generator, transformer, transmission line, and load—having different electrical constants. In such a compound circuit, oscillations harmless in one part of the circuit may reach a dangerous condition in other parts, due to the variation in Z_0. For example, consider the case of a long transmission line one section of which is laid underground. An oscillating current in the cable, of little importance in itself, may give rise to dangerously high oscillating voltages when it enters into the overhead part of the line, owing to the far higher natural impedance of the latter.

Similarly, if a transformer is connected across the end of an overhead line, the natural impedance of such a transformer may be between 2 000 and 4 000 ohms, which is very much higher than that of the line itself. Consequently an oscillating current, which only gives rise to moderate oscillating voltages in the line, may produce destructive voltages in the windings of the transformer.

A consideration of the relative values of Z_0 in the different parts of a transmission system thus gives considerable information on the relative danger and protective action of the parts on each other. It also explains to a certain extent why reactive apparatus, such as transformers, are so much more liable to suffer from the disruptive effects of transient voltages than the lines to which they are connected.

189. Oscillation Frequency of a Line.—The rate at which the oscillating energy will pass back and forth between the magnetic and the dielectric fields of a transmission line is entirely independent of the frequency of the power current. From formula (213) we have

$$e = i\sqrt{\frac{L}{C}} \qquad . \quad . \quad . \quad . \quad (215)$$

but since i may be considered as the charging current of a condenser of capacitance C with a voltage e across its terminals, we can also write

$$e = \frac{i}{2\pi f C}. \qquad . \quad . \quad . \quad . \quad (216)$$

Equating (215) and (216) gives

$$f = \frac{1}{2\pi\sqrt{LC}}, \qquad . \quad . \quad . \quad . \quad (217)$$

the usual formula for the period of oscillation of a circuit having massed capacitance and inductance. This formula requires modification when applied to a transmission line where the inductance and capacitance are distributed, as may be seen from the following considerations.

The velocity of propagation of an electric wave along an overhead conductor is approximately the same as the velocity of light or, say, 186 000 miles per second. Hence, if an oscillating current of frequency 1 000 occurs in the conductor, the wave length of the oscillation will be

$$\frac{186\,000}{1\,000} = 186 \quad \text{miles.}$$

In Fig. 122 the curve shows how the instantaneous value of the current varies along a line at one particular instant, *viz.* when the current at P has its maximum value. Q is the nearest point at which the current has exactly the same value as at P,

and PQ is the wave length of the oscillation. In exactly the same way the instantaneous value of the voltage varies along the line, repeating itself every 186 miles. Hence the inductance voltage and capacitance current in each little element of the conductor due to the oscillation also varies progressively along the line, and the total inductance voltage and capacitance current are obtained by adding vectorially the values in each little element. The net result is to reduce the values of conductor inductance voltage and capacitance current in the ratio of the diameter of a semi-circle to the length of the arc. This is exactly analogous to the calculation of the magneto-motive force of a distributed armature winding (n turns, i amperes) which has a value of $\frac{2}{\pi}ni$, not ni, ampere turns.

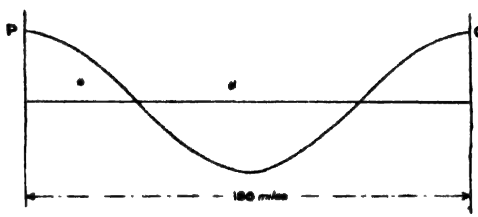

FIG. 122.—Instantaneous current at different points on a long transmission line.

In the case, therefore, of a transmission line, formula (216) must be modified and written

$$e = \frac{i}{4fC}.$$

The surge impedance is unaffected by the distributed character of the inductance and capacitance, so as before

$$e = i\sqrt{\frac{L}{C}}. \qquad (215)$$

Combining the two equations gives for the frequency of oscillation of the line

$$f = \frac{1}{4\sqrt{LC}}. \quad . \quad . \quad . \quad . \qquad (218)$$

This is the lowest or fundamental frequency of oscillation, and the corresponding wave length of the oscillation is four times the length of the line.

Besides this fundamental frequency, higher harmonics may exist in a line oscillation. This can be seen as follows:—

At any point in an oscillating line the effective power is always zero, since voltage e and current i are in quadrature. The instantaneous power ei, on the other hand, is not zero, as equal amounts of power flow first one way and then the other way. Across the ends of the line, however, no energy can flow, otherwise the oscillation would not be limited to the line. This means that at both ends of the line the instantaneous power must be zero.

Thus two cases are possible:—

1. The current is zero at one end, and the voltage zero at the other end.

2. Either the current is zero at both ends, or the voltage is zero at both ends.

In the first case $i = 0$ at one end, $e = 0$ at the other end, so that the line must be open-circuited at one end and short-circuited at the other. The potential and current distribution in the line must then be as shown in Fig. 123. That is, the length of the line represents a quarter wave or an odd multiple thereof.

The frequency of oscillation under these conditions is

$$f = \frac{(2n - 1)}{4\sqrt{LC}}, \qquad (219)$$

where n is any integer.

In the second case $i = 0$, or $e = 0$, at both ends, the line must be open-circuited or short-circuited at both ends, and the current and voltage distribution are as sketched in Fig. 124. The line now represents a half-wave length or a multiple thereof, and the frequency of oscillation is

$$f = \frac{n}{2\sqrt{LC}}. \qquad (220)$$

The velocity of propagation of the oscillation is readily found by multiplying the frequency of oscillation by the wave length corresponding thereto.

Thus the fundamental frequency of oscillation is

$$f = \frac{1}{4\sqrt{LC}}$$
$$= \frac{1}{4l\sqrt{L_0 C_0}}. \qquad (221)$$

where l = length of line in miles,

L_0 = inductance of conductor per mile,

and C_0 = capacitance of conductor per mile,

and the corresponding wave length is $4l$, so that the velocity of propagation is

$$v = \frac{1}{4l\sqrt{L_0C_0}} \times 4l$$

$$= \frac{1}{\sqrt{L_0C_0}}. \qquad (222)$$

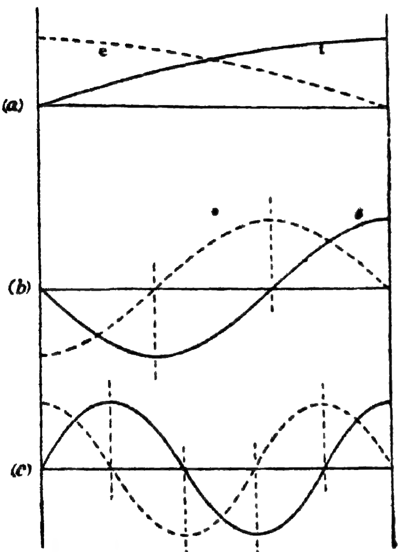

FIG. 123. — Instantaneous voltage and current in oscillating line. Line open-circuited at one end and short-circuited at the other.

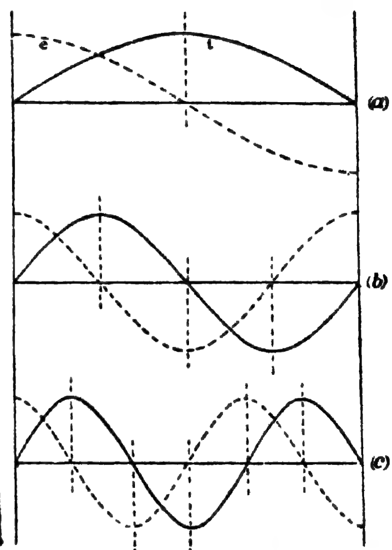

FIG. 124. — Instantaneous voltage and current in oscillating line. Line open-circuited or short-circuited at both ends.

The velocity of propagation v is reduced slightly below the velocity of light owing chiefly to the loss of power in the conductors, and the increase of inductance by the magnetic field inside of the conductor (which would not exist in a conductor of perfect conductivity). If, however, the value of the external inductance per mile of conductor is employed in formula (222), the quantity $\frac{1}{\sqrt{L_0C_0}}$ = 186 000, and the relationship can be used

for the calculation of the capacitance from the inductance and *vice versa*.

190. Travelling Waves.—The effect of a *lightning discharge or similar phenomenon* is to impress suddenly on a part of a transmission line a quantity of electric energy, which moves along the line at approximately the speed of light as a travelling wave or impulse, until it has been diverted from the line or its energy dissipated. The amplitude of the wave may not be hig nor yet of an oscillating character, yet by reason of the steepne of the wave front it may have a very destructive effect by piling up the voltage locally in the windings of reactive apparatus. It is essential to obtain a clear idea of the nature of such steep-fronted travelling waves as they are very common in transmission systems, being produced by other causes besides atmospheric disturbances.

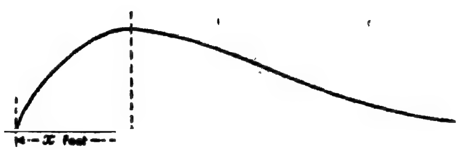

FIG. 125.—Travelling wave or impulse.

In travelling waves the source of danger is the high rate of change of voltage with the time, and the correspondingly high voltage gradient along the conductor, and it is convenient to compare oscillatory and non-oscillatory effects in this respect by speaking of the 'equivalent frequency' of non-oscillatory transients. Fig. 125 represents a non-oscillatory voltage wave or impulse travelling along a conductor from right to left. At the point *a* the voltage of the wave is zero at the instant figured, while the voltage at the point *b* is the crest value of the wave. As the wave moves along, the voltage at *a* will rise from zero to *b*, the time taken being equal to the time taken by the wave to travel a distance equal to *ab*. If the distance *ab* is *x* feet, the time taken for the voltage at any point of the line to rise from zero to *b* is

$$\frac{x}{186\,000 \times 5\,280} \text{ seconds.}$$

A representative value for x for waves due to lightning disturbances is 1 000 feet, for which the time is

$$\frac{1\ 000}{186\ 000 \times 5\ 280} \quad 1{\cdot}02 \times 10^{-6} \quad \text{seconds.}$$

Now, in an oscillating wave, the time taken for the voltage to rise from zero to a maximum corresponds to one-quarter of the periodic time. Hence the effect of the impulse for which the distance ab is equal to 1 000 feet will be somewhat similar to that of an oscillating wave of periodic time = $4{\cdot}08 \times 10^{-6}$ seconds, which corresponds to a frequency of 245 kilocycles per second.

191. Reflection of Travelling Waves.—Electric travelling waves, like waves in water or air, are reflected back whenever they come up against an obstacle. More generally, they are partially reflected and partially transmitted. Such a phenomenon occurs if the nature of the circuit suddenly changes, as for example at the junction of an overhead and underground line, or at terminal stations where apparatus is connected across the line.

FIG. 126.—Voltage and current waves travelling along a transmission line.

In order to fix ideas, consider the arrangement depicted in Fig. 126. Here G is a three-phase generator from which a voltage to neutral e is suddenly impressed on a transmission line by closing, and then immediately opening, a three-pole switch S. Assuming that the resistance of the conductors is negligible, and that the operation of closing and opening the switch is performed very quickly, a rectangular impulse will be produced in the line, the length ab of the impulse depending upon the time during which the switch was closed. The current i flows in the conductors between the points a and b, but neither forward of b nor backward of a. It carries with it the system of magnetic and dielectric flux lines in the space comprised between the planes ac

and bd; the complete energy impulse of voltage e and current i travelling away from G with the velocity of light. The remote end of the line is closed by a circuit having a different natural impedance Z'_0 to that of the line Z_0. On arrival at the transition point, partial reflection and partial transmission of the wave will occur.

It is evident that at the transition point the potential will be the sum of the incoming and reflected waves, while the total current will be given by the difference of the incoming and reflected current waves. Calling e_0 and i_0 the amplitudes of the transmitted voltage and current waves, and e_1 and i_1 those of the reflected waves, the following relations hold at the transition point:—

$$e_0 = e + e_1, \quad . \quad . \quad . \quad . \quad . \quad (223)$$

and

$$i_0 = i - i_1. \quad . \quad . \quad . \quad . \quad . \quad (224)$$

Since

$$Z'_0$$

$$e_0$$

$$Z'_0$$

and

$$Z'_0$$

the amplitude of the transmitted voltage wave is

$$e_0 = \frac{2Z'_0}{Z_0 + Z'_0}e, \qquad (225)$$

and of the reflected voltage wave

$$e_1 = \frac{Z'_0 - Z_0}{Z_0 + Z'_0}e. \quad . \quad . \quad . \quad . \quad . \quad (226)$$

Similarly, the amplitude of the transmitted current wave is

$$i_0 = \frac{2Z_0}{Z_0 + Z'_0}i, \quad . \quad . \quad . \quad . \quad . \quad (227)$$

and the reflected current wave

$$\frac{Z'_0 - Z_0}{Z_0 + Z'_0}i. \qquad (228)$$

The following three cases are of special interest:—

1. Line open-circuited ($Z'_0 = \infty$).
2. Line short-circuited ($Z'_0 = 0$).
3. Line closed through a non-inductive resistance of value R_1 Z_0.

Case 1.—If the line is open-circuited at the remote end, it is obvious that when the wave reaches this point it cannot flow any further but is reflected, the voltage and current of the reflected wave being of the same values as in the original waves because the energy remains constant. The total current of the incoming and reflected wave must, however, be zero, on account of the open-circuited line, and the whole energy is therefore stored at this point in the electrostatic field. The reflected current wave must therefore be reversed and its value is i, while the value of the voltage at the end of the line, where the incoming and reflected

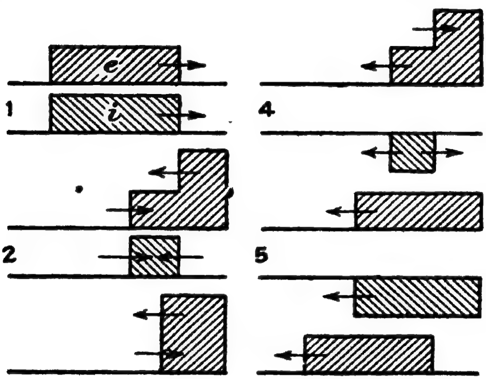

Fig. 127.—Reflection of waves at end of open-circuited line.

waves overlap, is equal to $2e$ as shown in Fig. 127. On arrival at the point where the reversal is complete the voltage will again have its original value of e volts.

Case 2.—With a short-circuited line, complete reflection of the wave will also occur, but it will now be the voltage wave instead of the current wave that is reversed. Due to the short-circuit the total voltage of the incoming and reflected voltage wave must be zero, and the whole energy is therefore stored at this point in the electro-magnetic field. The total current at the end of the line, where overlapping occurs, must be $2i$, and on arrival at the point where reversal is complete is i as before. This state of affairs is represented by Fig. 128.

Case 3.—Comparing the two previous extreme cases, it is evident that there must be a particular intermediate value of Z'_0 which will absorb the wave pulse and prevent reflection. Examination of formulæ (226) and (228) will show that when the end of the line is closed by a non-inductive resistance of magnitude equal to Z_0, reflection cannot occur, because both voltage and current waves will enter the resistance unchanged. If $R_1 > Z_0$ there is a partial reflection with transient rise of pressure and if $R_1 < Z_0$ there is a partial reflection with transient rise of current.

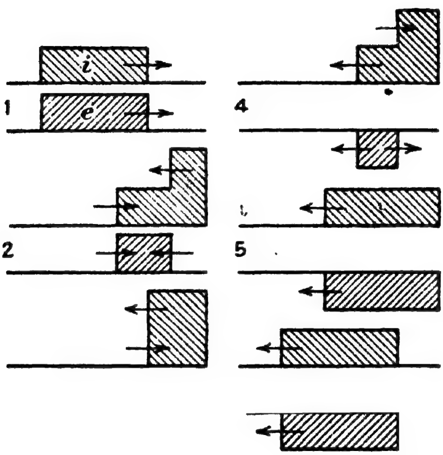

FIG. 128.—Reflection of waves at end of short-circuited line.

192. Origin of Pressure Rises.—There will now be considered in more detail a few of the ways in which dangerous pressure rises may be set up in transmission systems. These abnormal transient voltages are of comparatively frequent occurrence, particularly in overhead systems, and the problem of preventing damage to transmission lines and connected apparatus demands serious consideration.

In general, pressure rises may be caused by conditions external, or internal, to the system. A convenient classification is as follows:—

External Causes.—These are mainly due to lightning and only give trouble on overhead or partially-overhead systems.

1. Direct lightning stroke.

2. Electromagnetically-induced currents due to a lightning discharge in the immediate vicinity of the line. Such a discharge is called a 'side stroke.'

3. Electrostatically-induced charges on the conductors due to the presence of thunderclouds.

4. Electrostatic charges imparted to the line by the friction of dust or dry snow blowing past the conductors, or by changes of line altitude.

Internal Causes.—These are mainly due to oscillations set up by sudden changes of circuit conditions or by resonance. Trouble of this character may be experienced both on overhead and underground systems.

1. Switching operations on unloaded line.

2. Sudden opening of loaded line, particularly under short-circuit conditions.

3. Resonance.

4. Arcing grounds.

It should be noticed that the external causes produce over-voltages between line and earth, whereas the internal causes may also produce over-voltages between conductors. Also any oscillating potentials set up in a line by the external causes have an enormously high frequency with an upper limit of about one million cycles per second, whereas those produced by the internal causes have usually a comparatively moderate frequency ranging between several hundred and several thousand cycles per second. Arcing grounds form an exception to this rule, inasmuch as they set up oscillations between line and earth, the oscillations having either moderate or high frequency depending on the distance of the fault along the line.

Experimental investigations into these pressure rises have been difficult on account of the high voltages and frequencies involved, but the development of such instruments as the high-voltage, cathode-ray oscillograph and the klydonograph has changed the outlook very considerably. The cathode-ray oscillograph renders it possible to secure oscillograms showing the voltage, current, and time relations of an electrical transient which may occupy a time of 1 / 100 000 000 of a second or less. The klydonograph is a surge recorder which gives a direct indication of the polarity, and the approximate magnitude of an

over-voltage, and an approximate indication of its wave front Both these and other types of instruments have been utilised in America for a comprehensive series of investigations carried out on actual transmission systems. As a result of this work much valuable data has been secured on the nature of pressure rises, and the protective value of lightning arrestors and other devices under these conditions.[1]

193. Direct Lightning Stroke.—Lightning discharges have been divided by Lodge[2] into two kinds, namely, A and B strokes.

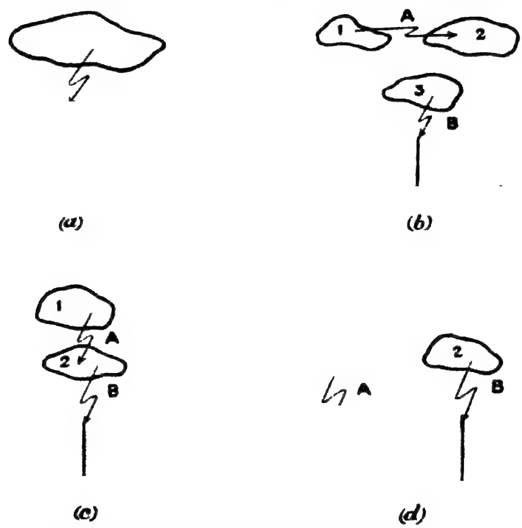

FIG. 129.—Types of lightning discharges.

The A stroke is illustrated in Fig. 129 (a). A charged cloud near the surface of the earth induces a charge of opposite sign on all upstanding objects, such as church spires, chimneys, etc. The electrostatic stress at the upper end of these objects is very great, and the air in the neighbourhood is rapidly ionised, streams of charged particles being repelled from all sharp corners and edges. This produces a gradual lowering of the resistance of the discharge path between the cloud and the conductor, until eventually a disruptive discharge occurs which tends to equalise the potentials of the two bodies. The distinguishing characteristic of this type of stroke is the length of time taken to

produce, and the fact that when it occurs it is usually directed towards the highest and most sharply pointed object in the neighbourhood.

The *B* stroke is an impulsive rush of electricity occurring when the potential difference between the cloud and the earth is established almost simultaneously. It is generally induced by a previous *A* stroke. Thus in Fig. 129 (*b*), if an *A* stroke takes place between the two charged clouds 1 and 2, the cloud 3 may be left with a greater potential than the air can withstand, and a *B* stroke thus occurs to earth. This type of stroke takes place with absolute suddenness and does not seem to obey any definite laws like the *A* stroke does. It may shatter a building, ignoring a neighbouring well-designed lightning conductor, or strike the ground even if there are tall objects such as trees or chimneys near by. The *B* stroke is in consequence far more dangerous than the *A* stroke. Figs. 129 (*c*) and 129 (*d*) show other ways in which a *B* stroke may be produced. In Fig. 129 (*c*) an *A* stroke occurs between clouds 1 and 2, the latter being near the earth's surface, and a *B* stroke is thereby induced between the lower cloud and earth. In Fig. 129 (*d*) the *A* stroke from 1 to earth causes the *B* stroke from 2 to earth.

The protection of a transmission line from the effects of a direct lightning stroke is a matter of some difficulty, as field investigations have shown that under these conditions pressure rises of the order of 10 000 kV. may occur. Lightning arrestors and similar devices are often installed at the ends of a line for the protection of directly-connected apparatus, but only protect a comparatively short section of the line itself. Hence, if a line is struck in the open country, the discharge flows along in both directions, shattering insulators and even wrecking poles until its energy is dissipated in the paths to earth thus formed. Efforts are now being made, however, to develop cheaper and more efficient types of lightning arrestors, which can be applied to the protection of individual insulator strings, thus preventing dynamic flash-over of the line. A new and simple protective device operating on the de-ion principle has also been produced for certain voltage ratings. It is considered that the resulting increase in reliability of operation would often justify the installation of this kind of apparatus at every tower position on sections of lines exposed to severe lightning storms.[3]

194. Side Stroke of Lightning.—Measurements have shown that pressure rises induced in a transmission line by side strokes of lightning may reach a value of 2 000 kV., but that about 90 % of these pressure rises are below 500 kV.[1] Since the actual voltage induced in the line is only a small percentage of the pressure of the discharge itself, the latter is probably of the order of one hundred million volts. Enormous power is developed as the current in the discharge may amount to from 10 000 to 100 000 amperes. On the other hand, as the duration of the phenomenon is apparently only a few micro-seconds, the actual quantity of electricity involved may be quite small.

In the past there has been considerable doubt as to whether lightning was oscillatory or unidirectional, but the high resistance of the discharge path due to radiation was considered to be a factor which would prohibit phenomena of the former character. The more recent experimental evidence seems to prove definitely that lightning discharges are unidirectional in character, consisting of impulses having a steep wave front. The equivalent frequency of these impulses may vary between 10 and 100 kilocycles per second. Occasionally, highly damped oscillations of one or two cycles are found on earthed-neutral systems, but these are apparently due to line flash-over following the lightning discharge, and these disturbances are really similar to unidirectional impulses in their effects.[1]

When a side stroke occurs near a transmission line, corresponding high-frequency currents are induced in the line conductors. If these currents are not diverted they travel along the line, and will set up destructive pressure rises on entering the windings of electrical machinery connected at the ends of the line. Convincing evidence of the high-frequency nature of these currents is found in the way the inductance of a very small bend or loop in the run of an ordinary lightning conductor will cause a discharge through an alternative path of much higher resistance. For instance, where the conductor is taken over the coping of a building a discharge will invariably pass through the masonry, and not along the copper conductor.

195. Electrostatically-induced Charges.—A cloud electrostatically charged, and lying above a transmission line, will induce in the adjacent section of the line a corresponding charge of opposite sign known as a bound charge. This charge has its

maximum value immediately below the cloud and then gradually tails away as shown in Fig. 130. At the far end of the line, charges of like sign to that in the cloud are located. So long as the cloud remains stationary this static distribution on the line persists. It should be noted that while this state of affairs holds, no oscillations will be set up, but the potential of the line immediately below the cloud may rise to such a value as to cause a 'spill-over' at the insulators. A static over-potential is therefore a rather dangerous phenomenon. If now the cloud is suddenly discharged, the charges of opposite sign induced on the conductors will rush toward one another as a travelling wave or impulse. The reactance of the line prevents the freed charges spreading instantly, with the result that they often find a path to earth over the adjacent insulators.

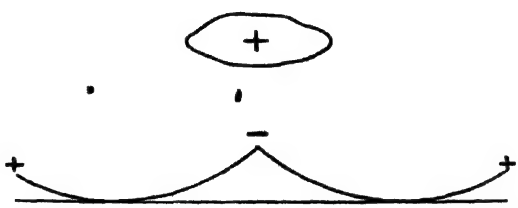

FIG. 130.—Transmission line with electrostatically-induced charge due to presence of thundercloud.

196. Electrostatic Charging of Line Due to Changes in Altitude, or Friction Effects.—The atmospheric equipotential surfaces have a tendency to crowd together in mountainous districts, the surfaces of high potential coming very much nearer the ground in these regions than over the plains. The result is that a gradually-ascending transmission line passes progressively from regions of low, to regions of high, potential. This leads to the conductors being gradually charged up and the potential to earth being increased. A pressure rise of the order of 40 000 volts due to this cause has been observed on a low-voltage line.[4]

The wind blowing past the conductors, particularly if carrying in suspension sand or dry snow, is also often responsible for charging the line electrostatically. A rise of 13 000 volts produced in this way has been measured on a telegraph line.

In both these cases the phenomenon is entirely static, and it is unlikely that travelling waves of any importance would be set up.

197. Switching Operations on Unloaded Line.—On connecting an open-circuited line to the terminals of a generator, travelling waves are set up which rapidly charge the line. On reaching the open end of the line, as previously shown, these waves are totally reflected, and overlapping with the incoming waves momentarily raise the voltage of the line to twice that of the impressed voltage at the instant of switching-in. Hence the maximum potential attained by the line at starting cannot be greater than twice the maximum generator voltage E_m. In practice, owing to the dissipation of energy, the potential attained by the line would be somewhat less than this theoretical maximum.

Analogous phenomena are produced on switching-out an open-circuited line. Travelling waves are set up which raise the potential of the line momentarily to a value not exceeding twice the generator voltage at the instant of disconnection.

The considerations thus far discussed do not take into account any characteristics of the arcing which always takes place at the contacts of a circuit-breaker, due to the finite speed of closing or opening. This phenomenon affects results considerably. For instance, suppose that the line is being opened. The arc between the circuit-breaker contacts first extinguishes at the zero point of the charging-current wave, which will, of course, be the crest of the voltage wave E_m. This leaves the line charged to a maximum of $+ E_m$. The circuit-breaker contacts are assumed to be separating continuously until at the middle of the next cycle, when the impressed voltage is $-E_m$, the voltage across the contacts is $2E_m$. There will be no restriking of the arc at this instant if the contacts are too far apart to break down at $2E_m$. If not, arc-over will occur, and there will be a resulting high-frequency oscillation about the point of the applied voltage $-E_m$ with an amplitude of $2E_m$, and the momentary rise of potential measured from zero will amount to $-3E_m$. The oscillation will rapidly damp down, leaving the line charged to a potential of $-E_m$. Arc-over may again take place during the next cycle, but the potential can never be higher than $3E_m$. This case is the commonest met with in practice, but it should be noted that when very small charging currents are involved it is possible for the arcing to be extinguished at the zero point of the *high-frequency* current wave, and to be re-established at the crest of

the 50-cycle voltage wave. This would lead to higher potential rises than $3E_m$.

Although circuit-breaker opening conditions have been discussed, it is obvious that closing operations accompanied by arcing lead to rises of the same magnitude.

A thorough experimental investigation of the subject has been made by Cox, McAuley, and Huggins,[5] and their chief conclusions are:—

1. Switching surges are usually unidirectional. When oscillatory they are highly damped and therefore of short duration.

2. Less than 50 % of all switching operations create a disturbance.

3. Pressure rises due to switching may reach 6 times normal operating crest voltage of the line, but 93 % of all produced are less than 3 times normal. On cable systems, 99 % are less than 3 times normal voltage.

4. No switching operations cause pressure rises high enough, or with a duration long enough to affect the safety of properly insulated lines.

198. Sudden Opening of Loaded Line.—If a transmission line is suddenly opened while carrying load, a transient pressure rise is produced of value

$$e = iZ_0, \qquad (213)$$

where i is the instantaneous value of the current at rupture, and Z_0 the natural impedance of the line. Thus, if the natural impedance of the line is 400 ohms, and the instantaneous current at rupture 200 amperes, the transient rise of pressure however rapidly the current is interrupted cannot possibly exceed 80 000 volts, because this is the maximum value of the pressure wave necessary to store in the electrostatic field the whole of the energy stored in the magnetic field at the instant of rupture. If E_m is the crest value of the operating voltage, the maximum potential to which the line may be subjected is $E_m + 80\,000$ volts.

Since the transient pressure rise produced by the sudden interruption of load is not a function of the line voltage, it follows that low-pressure transmission systems are liable to overvoltages of the same magnitude as high-pressure systems. The higher safety factor of the insulation on low-pressure systems is thus justified.

The worst case that could happen would be the interruption of the short-circuit current of the system at its crest value, when pressure rises of several hundred thousand volts might be produced. Fortunately, the use of oil circuit-breakers has practically eliminated all danger due to short-circuits, as these breakers possess the valuable property of opening the circuit when the current wave has its zero value.

199. Resonance.—If, in a circuit consisting of a condenser (of capacitance C farads) in series with an inductance and resistance (of value L henries and R ohms respectively), there is impressed an alternating voltage of E volts, the current is given by

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}, \qquad . \quad . \quad (229)$$

where $\omega = 2\pi$ frequency. The condensive reactance $\frac{1}{\omega C}$ thus tends to neutralise the inductive reactance ωL. Complete neutralisation is obtained when $\frac{1}{\omega C} = \omega L$, and the condition is known as electrical resonance. The current then becomes equal to $\frac{E}{R}$ and is in phase with the impressed voltage, hence, so far as the current is concerned, the circuit is equivalent to a simple resistance only.

From another point of view, however—as regards the voltage distribution in the circuit—the resonating circuit is by no means equivalent to a simple circuit of resistance R. For while in the latter the potential difference between any two points can never exceed the value of the impressed voltage, in the resonating circuit, potential differences occur which may be large multiples of the impressed voltage.

In commercial transmission circuits the capacitance is usually so small that resonance cannot occur at the fundamental supply frequency, but if the generator e.m.f. wave is distorted, trouble may be experienced due to resonance of one of the higher harmonics. This is owing to the fact that with a constant inductance, the value of capacitance required to produce resonance varies inversely as the square of the frequency.

An approximation to a circuit of massed constants is formed by an underground transmission line connected at one end to

generating and transforming apparatus, and open-circuited at the other end. The inductance of the cable is usually negligibly small in comparison with that of the rest of the system, so that the capacitance and inductance are localised to a great extent, the one in the cable, and the other in the terminal apparatus. The phenomena of resonance in such a circuit can be exhibited most clearly by taking a concrete example.

Consider, for instance, a 50-cycle system comprising a 33 000-volt, 0·20 sq. in., three-core cable connected to an alternator and transformer at one end. Let the leakage inductance of the alternator be 0·025 henries per phase and the resistance 0·5 ohms per phase, the corresponding quantities for the transformer being 0·015 henries per phase and 0·5 ohms per phase, all values being expressed in terms of the high-tension circuit. The total

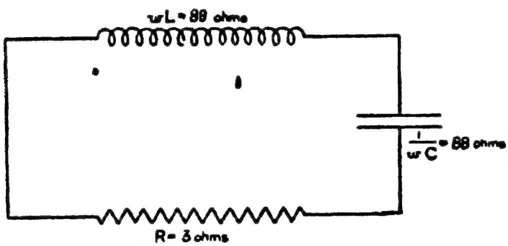

FIG. 131.—Resonating circuit formed by open-circuited cable and terminal apparatus.

inductance is 0·04 henries, and the cable capacitance necessary to produce resonance at the fundamental frequency of the supply is

$$C = \frac{1}{\omega^2 L}$$

$$= \frac{1}{4\pi^2 \times 50 \times 50 \times 0{\cdot}04}$$

$$= 253 \quad \text{microfarads,}$$

a value which would not be reached on an actual transmission.

For the seventh harmonic, however, the capacitance required for resonance would only be 5·2 microfarads (being one-forty-ninth of the value required at fundamental frequency), and would be provided by a cable about 18 miles long. Assuming the capacitance of the cable is localised at its mid-point, the additional

resistance introduced into the circuit is that of 9 miles of conductor or, say, 2 ohms per phase. The inductance of the cable can be neglected, so that the resonating circuit for the seventh harmonic consists of an inductance of 0·04 henries, a capacitance of 5·2 microfarads, and a resistance of 3 ohms, as shown in Fig. 131. For this harmonic $\omega L \quad \frac{1}{\omega C} \quad 88$ ohms.

Supposing now that the generator e.m.f. wave consists of a fundamental having an R.M.S. value E_1 volts to neutral and a seventh harmonic of R.M.S. value E_7 volts to neutral, the amplitude of E_7 being 10 °/ₒ that of E_1. Then, the following relationship holds :—

$$\sqrt{1 + (0{\cdot}1)^2}E_1 \qquad \frac{33\,000}{\sqrt{3}}$$

and

$$E_1 = 18\,960 \quad \text{volts,}$$

hence

$$E_7 = \frac{E_1}{10} \quad 1\,896 \quad \text{volts.}$$

The current produced in the circuit due to E_7 is

$$I_7 \quad 1\,896$$

$$632 \quad \text{amperes,}$$

and the potential difference across the terminals of the capacitance is

$$e_7 = \frac{I}{\omega C}$$

$$= \frac{632}{2\pi \times 50 \times 7 \times 5{\cdot}2 \times 10^{-6}}$$

$$= 55\,300 \quad \text{volts (R.M.S.).}$$

This is equivalent to a crest value of $\sqrt{2} \times 55\,300 = 78\,200$ volts which would be superimposed, in the worst case, on the crest value of the generator voltage.

The effect of a load at the receiving end of the line is to prevent such dangerous pressure rises due to resonance. For instance, assume that a non-inductive load corresponding to one-quarter of the carrying capacity of the cable is being

supplied. This state of affairs may be represented by a resistance R_1 of about 250 ohms connected across the terminals of the condenser standing for the capacitance of the cable, as shown in Fig. 132 (a).

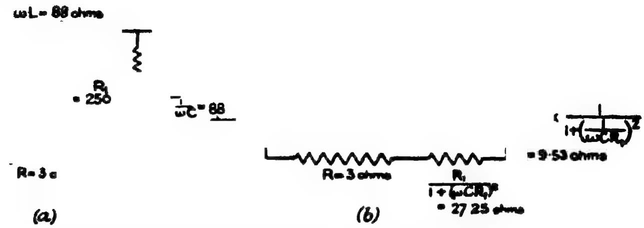

FIG. 132.—Effect of load in preventing resonance.

The equivalent series impedance of the circuit formed by the condenser and resistance in parallel is first determined. The joint admittance of this circuit is

$$\sqrt{\frac{1}{R_1^2} + (\omega C)^2}$$
$$\frac{1}{R_1}\sqrt{1 + (\omega C R_1)^2},$$

and the phase-angle is $\tan^{-1} \omega C R_1$.

The impedance of this parallel circuit is thus

$$\frac{R_1}{\sqrt{1 + (\omega C R_1)^2}}$$

and splitting into its components the resistance is

$$\frac{R_1}{\sqrt{1 + (\omega C R_1)^2}} \times \cos \tan^{-1} \omega C R_1$$
$$= \frac{R_1}{\sqrt{1 + (\omega C R_1)^2}} \times \frac{1}{\sqrt{1 + (\omega C R_1)^2}}$$
$$= \frac{R_1}{1 + (\omega C R_1)^2} \qquad (230)$$

and the condensive reactance is

$$\frac{\omega C R_1^2}{1 + (\omega C R_1)^2}$$
$$\frac{1}{\omega C} \times \frac{1}{1 + \left(\frac{1}{\omega C R_1}\right)^2} \qquad (231)$$

The effect of a non-inductive load is therefore equivalent to the insertion into the original circuit of an additional resistance of value

$$\frac{R_1}{1 + (\omega C R_1)^2},$$

while the condensive reactance has been decreased by multiplying by

$$\frac{1}{1 + \left(\frac{1}{\omega C R_1}\right)^2}.$$

This equivalent circuit is shown in Fig. 132 (*b*).

Substituting in the above formulæ, the resistance of the load (250 ohms) and capacitance (5·2 microfarads) in parallel for the seventh harmonic is

$$27{\cdot}25 \quad \text{ohms},$$

the condensive reactance is

$$9{\cdot}53 \quad \text{ohms},$$

and the impedance is

$$28{\cdot}87 \quad \text{ohms}.$$

Hence the resistance of the complete circuit is

$$27{\cdot}25 + 3 = 30{\cdot}25 \quad \text{ohms},$$

the reactance is

$$88 - 9{\cdot}53 = 78{\cdot}47 \quad \text{ohms},$$

and the impedance is

$$84{\cdot}1 \quad \text{ohms}.$$

The current produced in the circuit by the seventh harmonic in the impressed voltage is

$$I_7 = \frac{1\,896}{84{\cdot}1} = 22{\cdot}54 \quad \text{amperes},$$

and the potential difference across the condenser and load is

$$e_7 = 22{\cdot}54 \times 28{\cdot}87 = 651 \quad \text{volts (R.M.S.)},$$

which is equivalent to a crest value of 920 volts.

Thus if the system on open-circuit is liable to resonance, the addition of load removes the danger: (1) by destroying the equality between the condensive and inductive reactances of the circuit, and (2) by limiting the current by the introduction of

more equivalent resistance into the circuit. There is, of course, the possibility that a system which is non-resonant on open-circuit may be brought into resonance by a light load, but the increased resistance of the circuit is usually sufficient to prevent any dangerous pressure rises taking place.

Throughout the above discussion, the assumption has been made that the leakage inductance of the terminal apparatus is constant. In actuality, however, this condition is not fulfilled, owing to the variable permeability of the magnetic circuits of such apparatus and other causes, and it should be realised therefore that the above method is only an approximation of the complete solution.

The extreme case to the circuit of massed constants is provided by the transmission line itself, considered apart from any terminal apparatus. Such a line possesses uniformly distributed inductance and capacitance, and has various resonant frequencies corresponding to a fundamental natural period of oscillation and its overtones.

Suppose, for example, a long overhead transmission line connected at one end to the high-tension busbars of a large generating station containing many generating and transforming units working in parallel. The reactance of the terminal apparatus may then be neglected for a first approximation, and the line treated as short-circuited at this end. If the remote end of the line is open-circuited, resonance will occur at a fundamental frequency of $\frac{1}{4l\sqrt{L_0C_0}}$ and its odd harmonics. Since the quantity L_0C_0 is practically constant for all overhead lines, the resonant frequencies are thus determined solely by the length of the line.

Similarly, if the line is open-circuited at each end, resonance will occur at a fundamental frequency of $\frac{1}{2l\sqrt{L_0C_0}}$ and its overtones.

Steinmetz[6] has pointed out, however, that while a transmission line has quite definite resonant frequencies corresponding to the fundamental and its harmonics, it will oscillate with any frequency whatever, providing that this is very high. This phenomenon is due to the variation of the line constants at high frequencies. With these frequencies neither the inductance nor the capacitance

of a line is perfectly constant: the inductance varies with the frequency by the increased screening effect or unequal current distribution over the cross-section of the conductor; the capacitance increases due to corona formation on the insulator, and by the increase of the effective conductor diameter also due to corona. Hence the frequencies of the very high harmonics are to some extent variable, and since they are close to each other they overlap, so that at very high frequencies the transmission line has no definite frequency of resonance, but responds equally well with any frequency.

In the general case, a transmission circuit is complex, consisting of a line having uniformly distributed constants, connected to generating, transforming and receiving apparatus where the constants may be considered as massed. When resonance occurs in such a complex circuit the entire circuit oscillates, usually as a full-wave oscillation and its harmonics, and the fundamental frequency of resonance may be very much lower than that corresponding to the transmission line itself. The determination of the resonant frequencies of such a complex circuit is not a simple matter, but methods of attacking the problem have been given by Evans and Sels.[7]

200. Arcing Grounds.—Arcs and sparks in electric circuits produce oscillations in a similar manner to the production of such oscillations in a wireless transmitting circuit. Such a transmitting circuit consists of a spark gap shunted by a condenser and inductance in series. When an alternating e.m.f. is impressed on the circuit the condenser charges up each half-wave, and if the voltage is high enough to breakdown the spark gap one or more oscillating discharges occur from the condenser through the gap each half-period. When these oscillations are set up in transmission systems they are known as 'arcing grounds.'

For instance, consider an unearthed transmission system where the earth is brought within striking distance of one of the line conductors, as by the puncture of an insulator on an overhead line, or by the appearance of a weak spot in the insulation of an underground cable. A spark discharge then occurs to earth, and the arc following discharges the conductor by a transient oscillation. That is, the conductor is brought down to earth potential, and the other two conductors of the three-phase

system correspondingly rise in voltage from the star to the delta voltage. As soon as the conductor is discharged, the spark gap to earth opens, and the conductor then charges again from the power supply of the system, its voltage to earth rising until sufficient to jump to earth again and start a second transient oscillation. One or more series of oscillations may occur in each half-cycle of the voltage wave, thus forming a 'recurrent' oscillation. Occasionally, however, each transient oscillation persists up to the beginning of the next transient of the same half-wave of voltage, and the recurrent oscillations thus tend to run into each other and approach a 'continuous' oscillation. These phenomena have been shown clearly by oscillograms taken on artificial transmission lines.[8]

In an arcing ground, oscillations are set up on both sides of the faulty point, the frequency of these oscillations being the natural frequency of the section of line in which they occur. In themselves, such oscillations are not specially dangerous, but there is every probability of a pressure rise causing failure of the insulation at some other point of the system. If this happens on one of the other phases, a double earthing is formed, which is equivalent to a short-circuit of the system. Breakdown of transformers is also liable to occur, either due to resonance effects, or to localised pressure rises taking place in the windings. This latter danger will now be considered in more detail.

201. Pressure Rises in Transformers.—In high-voltage transformers the high-tension winding contains a large number of coils and a great length of conductor, so that its capacitance to earth is of considerable value. Each separate coil has resistance, inductance, and earth capacitance, so that the whole winding is a circuit with distributed electrical constants somewhat analogous to a transmission line. In addition to the earth or shunt capacitance, there is also a series capacitance between adjacent coils. Hence, neglecting resistance, the winding is electrically equivalent to the arrangement shown in Fig. 133 in which

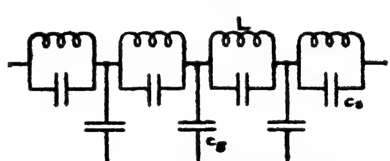

FIG. 133.—Transformer with distributed electrical properties.

L = inductance per unit length,
c_s = capacitance between adjacent coils per unit length,
c_g = capacitance to earth per unit length.

The resultant impedance of L and c_s in parallel is

$$\frac{\omega L}{1 - \omega^2 c_s L},$$

and it acts merely as an inductance at frequencies lower than that given by $1 - \omega^2 c_s L = 0$, and as a condenser at higher frequencies. In the former case the equivalent inductance is

$$L_e = \frac{L}{1 - \omega^2 c_s L}, \qquad (232)$$

and in the latter case the equivalent capacitance is

$$c_e = \frac{\omega^2 c_s L - 1}{\omega^2 L}. \qquad (233)$$

The electrical behaviour of the transformer therefore when subjected to an oscillating voltage, depends on whether the frequency of the oscillation is greater or less than f_k, the critical frequency which satisfies the equation $1 - \omega^2 c_s \mathrm{L} = 0$.

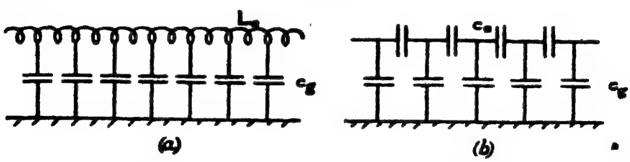

FIG. 134.—Equivalent transformer circuits.

At frequencies lower than f_k the transformer winding acts as L_e and c_g in Fig. 134 (a), that is, in the same way as a transmission line whose inductance and capacitance to earth are L_e and c_g respectively. For frequencies higher than f_k the winding can be considered to be a combination of c_e and c_g as in Fig. 134 (b), the whole arrangement being similar to the arrangement of the various capacitances of a suspension-type insulator.

If the transformer is connected to the end of a line in which oscillations of frequency $f < f_k$ are being set up, there is danger of resonance occurring in the windings with consequent failure of the insulation. With high-voltage power transformers trouble

of this character is most likely to be produced by frequencies of the order of 10 000 cycles per second, and it should be noted that the oscillations set up by arcing grounds are liable to fall within this range as their frequency is determined by the distance from the end of the line to the fault. Hence, arcing grounds may produce resonance in the transformer winding resulting in breakdown of those coils in the interior of the winding where the maximum stress is experienced.

If the incoming oscillations have a higher frequency than f_k, the transformer winding behaves as a series and shunt arrangement of capacitances, and the potential distribution across the winding is non-uniform, the highest stresses occurring on the end coils. A method of calculating the potential distribution in such cases has been developed previously in connection with suspension-type insulators (Art. 81). Berg[9] has also carried out an experimental investigation, and shown that arcing grounds of high frequency may set up pressure rises across the end coils ($2\frac{1}{2}$ °/₀ of the winding) which amount to 16 times the normal pressure.

The same phenomenon is also produced by a steep-fronted travelling wave or impulse. The two-fold effect of such an impulse in concentrating the pressure caused thereby across the end coils of the transformer winding, and also raising the potential of the winding with respect to earth, is made quite clear by the following analogy due to Lincoln.[10]

Consider the hydraulic transmission of a wave along a flexible piping system. The elasticity of the walls of the pipe represents the capacitance of the electrical system, and the inertia of the contained liquid the inductance of the system. Now either transformer or generator windings have two important differences from an equal length of transmission line in that both the inductance and capacitance per unit length is greatly increased. In the hydraulic analogy, the increase in capacitance per unit length may be represented by imagining a much thinner-walled pipe for the transformer or generator. The increase in inductance per unit length may be represented in the analogy by an increase in the specific gravity of the liquid, say, by substituting mercury for water.

In Fig. 135, therefore, b_1 is the transmission line with relatively heavy walls, although still flexible, and a light liquid contained therein, say, water; b_2 is the transformer or generator which has

relatively thin walls, containing a heavy liquid such as mercury. When a wave or surge that has been started out on the transmission line reaches the point P, which is the terminus of the line. it is obvious that the following will occur:—

1. A part of the energy of the incoming wave will be reflected and will therefore travel back through the line from the point P.

2. The remainder will begin to travel through part b_2, but the speed of its propagation will be much reduced because the

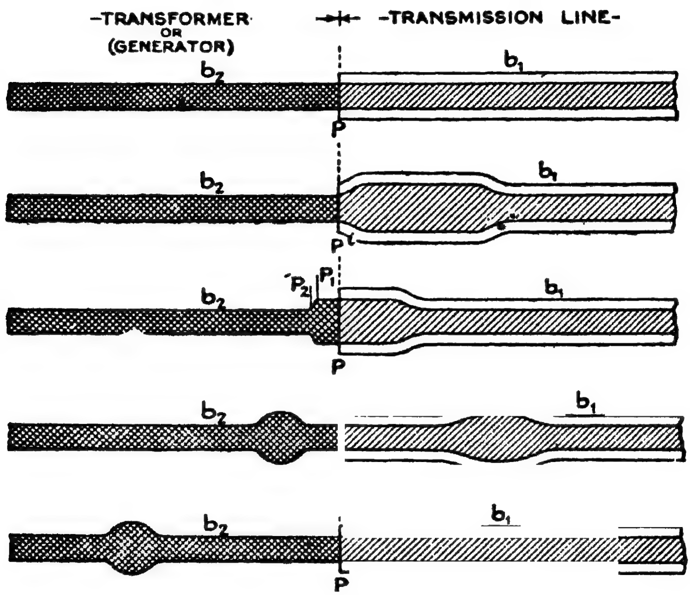

FIG. 135.—Hydraulic analogy of electric travelling waves.

liquid being set in motion is very much heavier, and also because the forces acting upon it through the elastic retaining walls are much smaller.

3. The steepness of the wave front during its propagation through b_2 will be much increased over that obtaining in b_1 by the above process.

At P_1 there is a tendency to burst the pipe, that is, to break down the insulation to earth, but there is also a heavy stress tending to break through from point P_1 to P_2. Suppose that

the portion b_2 of the pipe, instead of being straight as in the figure, were to be coiled upon itself so that a complete coil took place in the distance P_1P_2; there would then be a tendency for the liquid to break through the walls of the pipe into the neighbouring coil. This is exactly what occurs in the electrical system. An incoming wave in penetrating the turns of electrical apparatus causes an excessive voltage stress between turns. The momentary breakdown or snapping across of the surge from one turn to the next will do no particular damage unless such breakdown is followed by the power current and an arc is thereby established.

There is, due to the normal operation of the apparatus, an insulation stress from conductor to earth, and also one between adjacent turns. The stress to earth may be increased up to twice its normal value, the stress between turns may be increased up to fifty times.

In order to prevent failure of the windings due to these localised stresses, special designs of transformers have been developed, such as the non-resonating transformer, in which special capacitance shields have been fitted in such a way that the voltage distribution in the windings at high frequencies is similar to that which obtains at power frequencies.[11]

202. Attenuation of Travelling Waves.—High-frequency travelling waves and impulses of steep wave front caused by switching operations on complex circuits, or due to atmospheric disturbances, are of common occurrence in transmission systems. Fortunately, such impulses or waves are rapidly dissipated by energy losses in the line, so that their effect is more or less local. Furthermore, as the steepness of the wave front of such transients increases, so does the rate of attenuation or decay of the transient so that a wave shape containing many harmonics is smoothed out and gradually becomes sinusoidal in the course of its travel along the line.

This increase in the rate of decay at high frequencies is due to the variable character of the electrical constants of the line. The influence of skin effect on the resistance and reactance of the conductors has already been dealt with. A more important factor, according to Steinmetz, is the electromagnetic and electrostatic radiation, which though negligible at low and medium frequencies becomes of dominating importance at frequencies between 10^5 and 10^7 cycles. This radiation has the effect of rapidly increasing the

effective resistance of the conductor, and also introduces a further source of loss dependent on the voltage and represented by a shunted conductance, so that the energy of the higher frequencies of a complex wave is quickly dissipated.

For example, consider a travelling wave produced in an overhead transmission line by a suddenly liberated electrostatic charge. Such a wave, in order to be sinusoidal in shape, would require a sinusoidal distribution of charge on the line at the instant discharge, which is not probable, and the wave will usually complex, consisting of a fundamental and many harmonics. question of the change in shape of the front of the wave as it travels along the line is thus one of the attenuation or decay of its component frequencies. Steinmetz[12] has calculated the attenuation of a rectangular travelling wave of fundamental frequency 60 kilocycles during its passage over a typical overhead line. At the start, the equivalent frequency of the wave front is infinite, after travelling 0·3 kilometres the equivalent frequency is 1 110 kilocycles, after 3 kilometres 325 kilocycles, after 30 kilometres 103 kilocycles, and after 108 kilometres only 63 kilocycles. It thus appears that the danger zone of steep wave fronts is limited to relatively short distances.

If the amplitude of the travelling wave is such that corona is formed on the conductors, the attenuation of all the component frequencies of the wave will be increased, and the general effect will be to smooth out a steep wave front much more rapidly than would occur on a corona-free line. Thus, taking the 60-kilocycle rectangular wave adopted by Steinmetz as an example, Whitehead[13] shows that for a moderate amount of corona the wave will be approximately sinusoidal after travelling only 0·3 kilometres. Actual measurements on overhead systems have confirmed the existence of this rapid attenuation and smoothing-out effect in the case of steep-fronted waves.[1]

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CHAPTER XIV.

APPARATUS FOR PROTECTION AGAINST DANGEROUS PRESSURE RISES.

203. Phenomena against which Protection is Required.—In considering the installation of apparatus to protect the transmission line and terminal apparatus against the effect of abnormal pressure rises, a distinction should be made between over-voltages that are dangerous and those that are not. Many of those met with in the normal operation of the system are not dangerous, being provided for by the voltage margin represented by the factor of safety on the line insulators. Phenomena which experience has proved to be of a dangerous character are shown in Table 25.

TABLE 25.—*Dangerous Pressure Rises and Chief Types of Protective Apparatus.*

Phenomena.	Protective Apparatus.
1. Direct stroke of lightning.	Ground wire and lightning arrestors.
2. High-voltage impulses of steep wave front.	Ground wire and lightning arrestors.
3. Low-voltage high-frequency oscillations.	Surge absorbers.
4. Arcing grounds.	Arcing ground suppressor. Petersen coil.
5. Static overpotential.	Water jet earthing resistances. Earthing choking coils.

Of course, the simplest method of eliminating trouble due to (4) and (5) is to earth the neutral point of the system. The question of earthing the neutral, however, has to be considered from other points of view, and the advantages and disadvantages entailed thereby will be discussed later on in this chapter.

204. Ground Wire.—Overhead ground wires have been in extensive use since the construction of the earliest transmission lines. One wire or stranded conductor is usually employed, run

above the transmission line and earthed at many points, preferably at every pole or tower. It is generally recognised that this method of protecting against direct strokes is very efficient, and this is confirmed by experiments on model lines.

The objections to the ground wire are the additional cost, and the possibility of the wire breaking and falling across the line conductors, thus causing a direct short-circuit. Failure due to the latter cause, however, is rare, as substantial galvanised stranded steel conductors are usually adopted. Indeed, such a steel conductor joining the tops of the towers generally adds greatly to the mechanical strength and stability of the line, particularly if the line is of the flexible type.

Besides taking the brunt of a direct stroke, the ground wire reduces the voltage electrostatically or electromagnetically induced

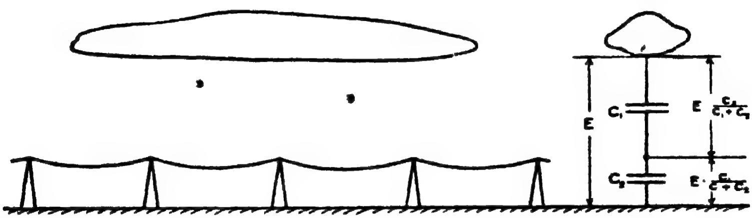


FIG. 136.—Reduction of induced voltage due to ground wire.

in the conductors by the discharge of a neighbouring thundercloud. For example, referring to Fig. 136, the voltage induced on the line due to the cloud depends on the ratio of the capacitances cloud-transmission line / transmission line-earth. The effect of a ground wire run above the conductors is to bring the equipotential surface corresponding to zero potential from the surface of the earth to the immediate vicinity of the conductors, thus increasing the value of the capacitance between line and earth. The pressure rise may be considerably less than in a line not protected by an overhead ground wire. The function of the ground wire is thus preventive in character, whilst that of the lightning arrestor is merely curative.

If a sufficient number of parallel grounded wires were provided to surround the conductors, all pressure rises due to external causes could be entirely removed. On account of the heavy capital expense involved such a solution would of course be impracticable. Hence one ground wire is generally used in

practice, which allows the intensity of over-voltages to be considerably reduced, and lessens the work demanded from the lightning arrestors connected to the system. In exposed situations, or on lines subject to severe lightning disturbances, it would probably be good practice to install one or two additional wires.

Peek [1] has made a laboratory study of the protective effect of ground wires and finds that a single ground wire reduces the induced lightning voltage to one-half of that without ground wire, for two ground wires the reduction is to one-third, while for three ground wires it is to one-fourth. These results were obtained under the favourable condition of good earths and low impedance for the earth connections. In a dry country with poor earths the protective value of a ground wire would be low. Its value would also be low even in a damp country if earths were made very infrequently, since then the impedance of the wire between successive earth connections would be considerable.

Another protective action of the ground wire is its damping effect on any wave or other disturbance travelling along the line by its action as a short-circuited secondary. The ground wire must consume a considerable part of the energy of a travelling wave and thus greatly increase the energy dissipation of the wave, especially when it is considered that the wire is often made of steel, and thus possesses a large high-frequency resistance.

205. Horn-gap Arrestors.—The term 'lightning arrestor' is commonly, but inaccurately, used to cover all devices which are employed to protect a transmission system from the effects of over-voltages, no matter whether these are due to external or internal causes. The horn-gap arrestor was one of the first types to be developed, and is still used to a certain extent on lower-voltage lines on account of its great simplicity. It consists of two horn-shaped pieces of metal separated by a small air-gap, and connected in shunt between each conductor and earth. The distance between the two electrodes is such that the normal potential difference between line and earth is insufficient to jump the gap, but abnormally high voltages will break down the gap and so find a path to earth. The arc following the initial discharge constitutes a flexible conductor and rises owing to the upward tendency of the heated air. This forms a bend in the current path, and following the principle that an electric circuit always tends to enlarge the area it encloses, the arc

is drawn up by electromagnetic action towards the top of the horns. Here it is extinguished owing to the line pressure being unable to maintain an arc of such length. The time taken for the complete operation is usually from 3 to 5 seconds.

As regards the gap setting, the ideal arrangement would be to by-pass all voltage waves exceeding normal amplitude, but this is impracticable. On the other hand, the setting must be for a voltage less than the test voltage of the line insulation or transformers, so as to ensure that over-voltages will discharge to earth through the appointed device, and not elsewhere. Since all equipment is designed for a test voltage of at least twice normal it is generally deemed sufficient if the arrestor comes into operation somewhat below this figure.

The simple connection of the arrestor between line and earth would be insufficient to relieve the system from the effects of high-frequency waves of moderate amplitude. It is very probable that such waves would pass straight on to the transformers or other terminal apparatus, and there pile up the voltage in such a manner as to puncture the insulation. The end turns of transformer windings are specially heavily insulated to withstand this trouble, and sometimes the impedance of the end turns alone is relied upon to reflect the voltage wave back to the lightning arrestor, where it is discharged to earth. More usually a choking coil, consisting of several turns of bare copper wire, is connected in the line between the arrestor and the apparatus to be protected. The reactance of this coil is sufficiently high to ensure that all high-frequency waves are reflected back to the horns, while at the supply frequency it is negligible. In practice, twenty or thirty turns on a diameter of about 12 inches are found to be quite effective for the purpose.

Fig. 137 (*a*) illustrates the method of installing a simple horn-gap arrestor in conjunction with choking coils. For the utmost protection to the terminal apparatus the arrestor should be located as close to the end of the transmission line as possible. Also the connections from choking coil to arrestor, and arrestor to earth, should be as short and direct as possible.

The commonest form of choking coil is wound in a simple helix, the turns of which are usually braced between clamps to prevent movement when heavy short-circuit currents traverse the coil. For outdoor work a suspension-type coil is used, the coil

being wound round a wooden tension insulator which can be inserted in the run of the conductor as it is sufficiently robust to take the dead-end pull of the line if need be. Continental practice favours a pancake coil, the turns being wound with flat strip. This design has the advantage of ease in winding and supporting, but the insulation between turns is apt to be weak. Naturally very great electric and mechanical stresses are liable to come into existence between the turns of a protective choking coil, and it is desirable to have each turn separated by a good air space. The solenoid type of coil lends itself better to this construction.

Regarded simply from the point of view of relieving the line

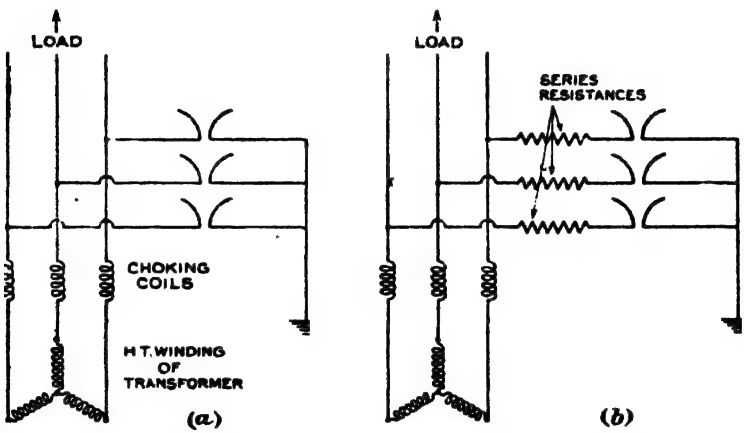


FIG. 187.—Installation of horn-gap arrestor and choking oils on a three-phase line.

of excess voltages, a simple horn gap without series resistance is the best. In this case, however, a discharge across one gap in the case of a system with earthed neutral, or on two gaps simultaneously on an insulated-neutral system, constitutes a dead short-circuit for the dynamic current of the system. This introduces operating difficulties, as a horn gap is only capable of rupturing quite small currents, certainly not in excess of 10 amperes. It therefore becomes necessary to provide a resistance in series with the gap to limit the flow of dynamic current to a value which can be safely interrupted by the arrestor. On the other hand, such a resistance very materially reduces the efficiency of the arrestor.

The selection of a value for the resistance is therefore a matter for compromise, the value generally adopted being such as will limit the dynamic current to between 1 and 5 amperes with normal voltage impressed. The resistance should be connected on the line side of the horns as in Fig. 137 (*b*) and not between horns and earth, since should the arcs from adjacent horns be blown together a short-circuit would occur in the latter case.

The resistances for use with horn gaps may be of the liquid metallic, or carborundum type. Resistances of the liquid type are usually placed in U-shaped earthenware vessels. The water is mixed with about its own weight of glycerine to prevent freezing, and it is advisable also to cover each surface of liquid with a layer of oil so as to avoid losses through evaporation. The advantages of the liquid resistance are its cheapness and simplicity, but it requires more attention than the other types. Carborundum rods or built-up resistances of a similar nature are cheap and in fairly common use, but are rather apt to disintegrate under the passage of heavy currents. The aim in selecting any resistance is to obtain a high thermal capacity, since this will enable the arrestor to discharge through a longer period of time. Wire-wound resistances, oil immersed, are good from this point of view, and furthermore their value does not vary with temperature like that of a liquid resistance. If a resistance of this type is used, however, it should be wound as non-inductively as possible.

In some installations an effort to secure protection has been made by employing two or more sets of horn gaps. Thus there might be horns without resistances set to arc over at 200 °/₀ of normal voltage, and horns with series resistances set at 160 °/₀ normal volts. The function of the horns without resistances is to give a direct path to earth for the severest over-voltage waves. When they operate, it is recognised that power current will follow and cause the protective circuit-breakers to operate. However, as the over-voltage conditions would constitute a serious menace to the system, this disadvantage is accepted as unavoidable. The function of the horns with series resistances is to operate on the less serious, but still undesirable, over-voltage waves falling between 160 °/₀ and 200 °/₀ of normal voltage. Incidentally they also come into action when the other horns operate, but owing to the former being connected direct to earth they will take the major portion of the discharge. Since these over-voltages are less

dangerous, more time can be allowed for their passage from the system if, by so doing, a power arc can be prevented, and a system shut-down avoided.

It is difficult to believe that a horn gap having a sufficient resistance in series to make it self-extinguishing has any protective value. An analogous case is met with in hydraulic pipe-lines which are fitted with a pressure-compensation pipe as a relief valve against water-hammer. This pipe can only protect the line against water-hammer when it has a section equal to that of the main pipe-line; if the section is much smaller, water will overflow (like sparking-over of a horn-gap arrestor), and the presence of water-hammer is indicated, but no appreciable relief is given. A similar state of affairs exists with electrical phenomena: the surge impedance of an overhead line usually falls between 400 and 600 ohms, that is, a travelling voltage wave has associated with it a current wave of value roughly 2 amperes for every kilovolt potential of the wave. Thus a travelling wave of 100 kilovolts would have an accompanying current of 200 amperes. It is fairly evident that an arrestor which can only by-pass 10 amperes from the 200 amperes as the wave rushes along into the transformer winding will have a correspondingly small influence in reducing the amplitude of the voltage rise across the apparatus.

Theoretical formulæ for calculating the breakdown voltage of a cylindrical horn gap have been worked out by Grütter[2] and others, but are not found to be of much use in practice. For one reason, the formulæ give the 50-cycle breakdown voltage of the gap, but the voltage at which they will breakdown when subjected to high-frequency lightning voltage may be very much different, depending on the impulse ratio of the gap. Also the state of the atmosphere and roughness of the horn surface make a difference, and the horns should preferably be set experimentally, being set as close together as circumstances will permit. Table 26, given

TABLE 26.—*Horn-gap Settings at Sea-level.*

Normal Volts.	Gap.	Normal Volts.	Gap.
10 000	½ in.	30 000	2¼ in.
15 000	1 ,,	40 000	3⅛ ,,
20 000	1⅜ ,,	50 000	4 ,,
25 000	1¾ ,,	60 000	4¾ ,,

by Coates,[3] will enable a first approximation to be made. The figures are average settings for horns at sea-level, corresponding to a spill-over voltage of 150 °/ₒ normal. At higher altitudes a longer gap is necessary, the arc-over voltage between fixed electrodes being very closely proportional to the air density.

It is seen that on low-voltage installations the gap has to be set very closely, with consequent danger of accidental discharge by insects or other objects bridging the gap. Alteration of calibration also is liable to occur, due to corrosion or formation of small globules of melted metal. To permit of a longer gap being used the arrangement shown in Fig. 138 can be employed The main gap is set at a good distance, say, 1 inch for 5 000 volts. The auxiliary gap, which is provided with an electrode having an adjustable platinum point is, however, set to discharge, say, at 25 °/ₒ above the normal voltage. If a foreign body gets into the auxiliary gap the resulting discharge quickly dies out owing to the high resistance in series therewith. On the other hand, an arc formed by a pressure rise persists, and ionises the air in the main gap, causing the same to arc across and relieve the line.

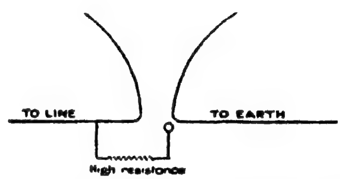


FIG. 138.—Diagram of horn gap with auxiliary electrode.

Another modification of the simple horn gap is the Burke arrestor which embodies as a characteristic feature a triangular choking coil wound from copper strip, one side of the coil being used as one of the horns. The arrangement of the device is as shown in Fig. 139. It will be seen that the line current passes round the triangular reactance coil and from the centre of the latter goes away to the load. The main gap has a series resistance, and there is also an auxiliary gap which is connected direct to earth. Very severe over-voltages will jump the double gap and go direct to earth in preference to passing through the resistance, while discharges of less severity pass to earth *via* the main gap and the resistance. The resistance itself consists of one or more rods of a material similar in appearance to carborundum. Its ohmic value is such as to limit the dynamic current with normal voltage to from 4 amperes at 3 300 volts, to 2 amperes at 66 000 volts.

It is claimed that the choke coil in this arrestor has a magnetic blow-out effect, causing the arc to rise rapidly up the horns. Furthermore, a wave travelling along the line meets its first impedance at the first sharp upward bend of the triangle opposite to which the earthed horn is mounted. This piles up the voltage and allows a considerably wider gap to be used than with other forms of horn arrestor, without, however, diminishing the sensitiveness of the device.

There are several other forms of arrestor which are modifications in some degree of the horn and resistance device. The graded-resistance horn arrestor, for example, has a tapered or stepped gap. A moderate discharge takes place across the narrowest portion of the gap which has a high resistance in

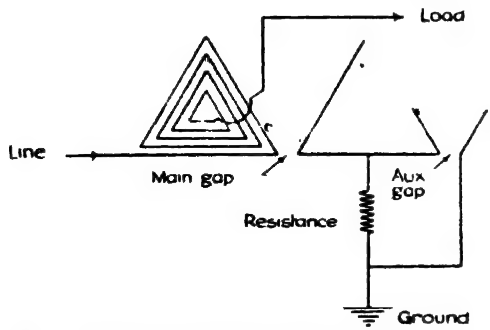


FIG. 189.—Diagram of Burke horn-gap arrestor.

series therewith. If the current flow is not sufficient to relieve the line the arc jumps across to the next wider gap, and so on, the successive gaps having lower and lower resistances in series. Under the severest conditions the discharge will be taken at the last gap where no resistance at all is in series.

Space does not permit of a description of these various modifications in detail, as they only enjoy a limited popularity. More efficient types of protective apparatus can now be obtained at a reasonable cost, and the greatest value of the horn gap at the present time is as an adjunct to some of these modern arrestors

206. Multiple-gap Arrestor.—This type of arrestor, first developed by Wurts,[4] has, like the horn gap, been in use for a considerable time. It consists of a series of small metal cylinders insulated from one another, and separated by airgaps

of about $\frac{1}{32}$ of an inch wide. The first of the series is connected to the line, and the last to earth, and the number of gaps necessary depends upon the line voltage. To assist in the suppression of the arc and thereby prevent dynamic current from the line flowing through the arrestor, the cylinders are made of an alloy of zinc called a non-arcing metal. The vapour formed by an arc between these cylinders has a rectifying effect similar to mercury vapour, thereby suppressing the arc every other half-cycle. It is common practice to knurl the surface of the cylinders so as to present as large a cooling surface as possible, this also helping to extinguish the dynamic current.

The theory of the arrestor is briefly as follows. There is a certain capacitance between consecutive cylinders, and also between each of these cylinders and earth, and this leads to a non-uniform distribution of the potential between the various gaps. The phenomenon has been previously discussed in connection with the analogous case of the suspension insulator (Art. 81). It is sufficient here to state that the potential per gap is greatest at the line end of the arrestor, and gradually diminishes towards the gaps at the earthed end. The result is, that when the voltage across the arrestor reaches a certain critical value, breakdown occurs between the first and second cylinders. The second cylinder is then connected to the first by an arc so that its potential rises accordingly until breakdown occurs between the second and third cylinders; and so on. The dynamic current then follows the discharge and in so doing produces a sensibly uniform fall of potential along the line of cylinders, with the result that the maximum potential difference between cylinders is considerably less than that required for the initial breakdown. The dynamic current continues to flow until the line voltage passes through zero to the next half-cycle, when the arc-extinguishing quality of the metal electrodes comes into action. Some of the zinc is vaporised by the heat of the discharge, and when the current in the gap passes through the zero point of the wave the zinc vapour prevents its re-establishment in the opposite direction. Before the voltage again reverses, the arc vapour in the gap has cooled to a non-conducting state, cutting off further flow of current. As in the case of the horn arrestor there is a current limit, although a higher one, beyond which the arc will be maintained despite the quenching action, so that limiting

resistances must be employed in most instances. These resistances are usually of graphite mixture or metallic, and are connected in series with the group of gaps as shown in Fig. 140. Their value is such as to limit the dynamic current to less than 20 amperes.

The multiple-gap arrestor is not satisfactory for use on systems with line pressures exceeding, say, 33 000 volts, due to the fact that the necessary increase in the number of gaps to prevent arcing over by the normal voltage is out of all proportion to the increase in voltage. There is also much uncertainty as to the number of gaps required, this depending on the position of the arrestor relative to surrounding earthed objects. With the earth potential brought very near to the arrestor, the potential gradient at the end near the line frequently becomes high enough to ionise the air between the cylinders, thus carrying the line potential to lower cylinders until the remaining gaps are so few that a discharge occurs.

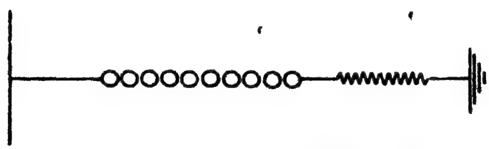


FIG. 140.—Diagram of multiple-gap arrestor.

In order to obtain a more equal distribution of potential over the arrestor, and so allow of a reduction in the number of gaps necessary, a metal earth-shield or antenna is sometimes placed near the gaps at the high-potential end of the arrestor and connected to the line wire.[5] Other modifications also have been proposed, but are not in general use, as the cost of the arrestor for the high voltages at which such special designs would be necessary is so great as to make other types of arrestor a better commercial proposition.

As with other types of lightning arrestor, the multiple-gap arrestor loses its efficiency as a discharge path for intense disturbances when it is provided with a series resistance. Thomas[6] has shown, however, that the difficulty can be largely surmounted by shunting some of the gaps with a resistance and using a smaller series resistance than with the unshunted arrangement. The arrestor with shunted gaps is known as the 'low-equivalent arrestor,' and the principle of its construction is shown in Fig. 141.

It will be seen that the point *B* is at earth potential when conditions are normal, so that a discharge will take place when the pressure is sufficient to break down the series gaps between *A* and *B*. The impulsive rush of current following the break-down, being of high frequency, will choose the straight through path to earth *via* the shunted gaps between *B* and *C*, instead of the alternative path through the shunt resistance, finally passing to earth *via* the small series resistance. As soon as the impulsive rush of current is over, the arcs in the shunted gaps die out, with the result that any dynamic current attempting to follow will have to flow through both resistances, which are now in series. This current will be so reduced by the high resistance in its path that it will not be sufficient to maintain the arcs in the series gaps, which therefore go out as well.

The effect of shunting some of the gaps is therefore to give the arrestor a certain amount of selective action, since it interposes

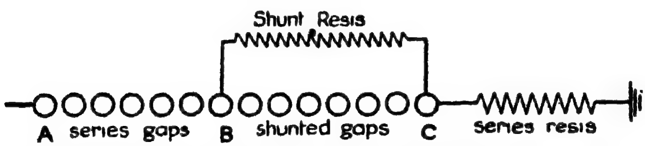


FIG. 141.—Diagram of 'low-equivalent' multiple-gap arrestor.

a low resistance in the path of high-frequency disturbances, and a high resistance in the path of low-frequency discharges. If wire resistances are used this selective action will be improved since they cannot be wound absolutely non-inductively, and so their impedance will increase to some extent with the frequency. Thus the higher the frequency of the discharge, the greater the impedance of the shunted resistance, and therefore the greater the tendency for such a discharge to bridge all the gaps and to bypass the shunted resistance. In one form of construction the series resistance is dispensed with altogether, the arrestor consisting simply of a large number of gaps some of which are shunted and some not.

As compared with the horn arrestor, the multiple-gap type has the advantage of greater sensitivity to minor pressure rises. Furthermore, it has the property of extinguishing any dynamic current following the discharge at the zero value, instead of in the neighbourhood of the maximum value as with a horn gap. There

is therefore no danger of the arrestor becoming the seat of high-frequency oscillations after it has operated. On the other hand, the multiple-gap type is more costly, especially for the higher line voltages.

207. Lightning Arrestor Spark Gaps.—During recent years there have been evolved types of lightning arrestor such as the electrolytic, oxide film and autovalve arrestors which can be designed to give satisfactory service no matter what the voltage of the system on which they are installed. These modern arrestors have a very high discharge capacity owing to the fact of their possessing a valve characteristic. In other words, they act like a counter e.m.f. equal to the line voltage, and the energy to be dissipated on discharge is limited to the energy of the over-voltage.

In general, these arrestors consist of two parts, first, a spark gap for discharging an abnormal voltage, and second, a means for preventing the normal line voltage from maintaining a power arc across the gap. Much research work has been carried out to determine the best form of spark gap for this purpose, and the simple horn gap formerly used has now been practically superseded.

In order to understand the *raison d'etre* of these later types of spark gap the fundamental phenomena relating to spark-over must be briefly reviewed. Consider, first of all, the case of a needle gap. When a 50-cycle voltage is applied to such a gap and gradually increased, spark-over occurs at some definite value of the voltage. This is the minimum voltage required for the gap to discharge, and the phenomenon requires a finite period of time. When a steep wave front or high-frequency voltage is applied to the same gap, however, spark-over does not occur at the instant the minimum or 50-cycle voltage is reached but at a higher point on the rapidly rising voltage wave. The 'slower' the gap, the higher the voltage rises before breakdown. This dielectric spark lag is due to the fact that in the non-uniform field of the needle gap balls of corona first form around the electrodes, and grow in size until the stress between them is sensibly uniform and great enough to break down the remaining air. Hence both time and energy are required before spark-over can take place. In Fig. 142 are shown a series of typical spark-over curves relating to needle gaps in air, from which it will be noted that changes in frequency cause a considerable change in spark-over voltage.

A sphere gap, on the other hand, has the characteristic of possessing a spark-over voltage approximately independent of the frequency, *i.e.* voltages of all frequencies and wave fronts discharge across the gap at the same value. In this case the field between the electrodes is practically uniform. The only energy required prior to breakdown is the extremely small amount necessary to charge the spheres, and consequently the time-lag is practically zero. The curves of Fig. 143, which exhibit the results of tests on sphere gaps, show no variation of spark-over voltage with frequency at spacings less than, or comparable with, the sphere diameter.

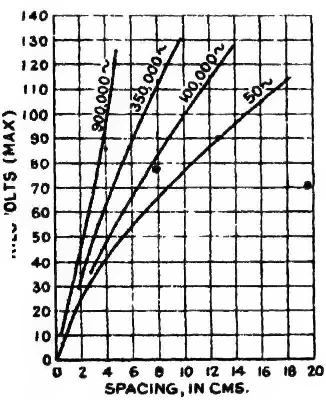


FIG. 142.—Spark-over voltage for needle gaps.

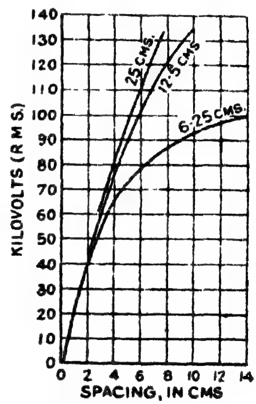


FIG. 143.—Spark-over voltage for sphere gaps.

The following points therefore should be noted :—

1. Two gaps with equal 50-cycle spark-over voltages may have extremely different lightning or impulse spark-over voltages because of time-lag.

2. This time-lag is the greater in a non-uniform field, or for electrodes where corona precedes spark-over, and is a minimum for a uniform field.

3. The spark-over voltage increases, and the time-lag decreases, with increasing steepness of wave front.

With any particular set of electrodes the ratio between the voltage at impulse frequency, say, 500 kilocycles per second, required to cause spark-over, and that necessary at line frequency, is termed the impulse ratio of the gap. The impulse ratio

between needle points is of the order of 1·5 to 2·3, depending on the spacing to some extent. A sphere gap at close settings has, of course, an impulse ratio of unity. With regard to horn gaps, these may have either sphere or needle-gap characteristics, depending upon the ratio of the diameter of the electrodes to their distance apart. If the setting is very small, as with low voltage, the gap will have sphere-gap properties and therefore a unity impulse ratio. If the gap is wide, as with high voltages, then its properties will be more nearly those of a needle gap.

It is important to utilise these principles in designing transmission systems. That is, lightning arrestor spark gaps should have an impulse ratio of unity or as low as possible, while insulators, and insulation generally, should have a high impulse ratio. In this connection it should be noted that the impulse ratio for the usual types of suspension insulator varies between 1·2 and 1·6, while for the pin-type insulators the figures are about 1·3 to 2·3. With solid dielectrics the impulse ratio is generally higher than with air. Thus varnished cloth or oiled presspahn between flat discs and immersed in oil shows an impulse ratio of from 2·2 to 2·5.

In comparing the protective values of lightning arrestor spark gaps it is necessary to take into account, not only the impulse ratio, but also the gap setting imposed by operating conditions. Thus, if the spark gap is placed out of doors without any protection from the weather, it must be set so that it will withstand normal line voltage with a factor of safety under the most unfavourable conditions. The 50-cycle spark-over voltage of a gap is very much decreased if the surfaces of the electrodes are covered with moisture due to rain or fog. The decrease in spark-over voltage due to moisture differs greatly with the shape of the electrodes, and, speaking generally, the more uniform the gap the greater the reduction. Thus the influence is a minimum for points and a maximum for plane surfaces. For spheres the wet spark-over voltage is 40 to 50 °/ₒ of the dry spark-over voltage, and for horns the corresponding ratio is about 60 to 70 °/ₒ, varying with the spacing to some extent.

On the other hand, the impulse spark-over voltage of any form of gap is practically unaffected by moisture on the electrodes. Denoting by a the impulse ratio of a spark gap, and by β the setting factor or ratio of dry spark-over voltage

to wet spark-over voltage, the relative protective values of two gaps for high-frequency voltages vary inversely as $\frac{a_1\beta_1}{a_2\beta_2}$.

To illustrate this point consider the following types of gap :—

1. Horn gap installed out of doors,
2. Sphere gap out of doors, and
3. Sphere gap covered.

The figures in Table 27 give representative values of a and β for each case, and the superiority of the covered sphere gap is clearly demonstrated. Calling the protective value of the latter 100 %, the protective value of the outdoor sphere gap is 50 %, and that of the simple horn gap only about 36 %.

TABLE 27.—*Relative Protective Values of Spark Gaps.*

	a.	β.	$a\beta$.	Relative Protective Values.
Outdoor horn gap	1·5	1·5	2·25	36
Outdoor sphere gap	1·0	2·0	2·0	50
Covered sphere gap	1·0	1·0	1·0	100

It is therefore seen that in order to fully realise the high protective value of a sphere gap it must be covered, or the surfaces of the electrodes kept fairly dry. Of course a simple sphere gap is not possible for use with all types of arrestor since a horn is often necessary to assist in breaking the dynamic arc. Wide horn gaps are often provided with spherical or hemispherical electrodes at their lower parts so as to increase the speed of the gap.

A more recent development in high-speed gaps is the 'impulse protective gap' due to Allcutt.[7] This device has a selective action so that it operates at a lower voltage with impulse frequency than it does at the supply frequency. In other words, the apparent impulse ratio of the gap is less than unity. In view of the comments previously made on the covered sphere gap such added facility may at first glance appear unnecessary. It should be borne in mind, however, that even with spheres, the arrestor will not discharge at any frequency until a voltage in excess of the gap setting is reached. Connected apparatus will therefore be

subjected to all high-frequency waves up to perhaps 150 % of normal voltage, and is exposed to trouble due to localised pressure rises or resonance caused by these waves.

In principle the impulse protective gap is very simple, as shown in Fig. 144. The gap proper consists of two sphere-horn electrodes S_1 and S_2 which are connected respectively to the line and to the arrestor, the latter being usually of the electrolytic type. In addition to those two main electrodes an auxiliary electrode A is provided, which is connected to one of the horns through a condenser K_2, and to the other horn through a condenser K_1 and an unbalancing resistance UR. At the normal line frequency the impedance of UR is negligible compared to that of the condensers, and if the two latter are equal in value the potential of the auxiliary electrode will be about half-way between the potential of the two main electrodes. If, then, the auxiliary electrode is mounted half-way between the main electrodes, it will have practically no influence on the distribution of the electrostatic field between them, nor on the 50-cycle discharge voltage of the gap.

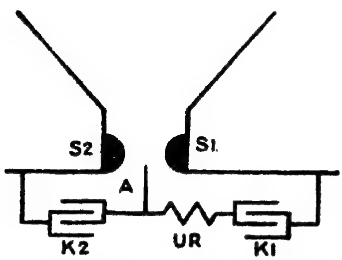


Fig. 144.—Diagram of impulse protective gap.

Suppose now that a high-frequency voltage is impressed on the gap. The distribution of potential is then disturbed, due to the difference in the impedance of the circuits (UR alone offers appreciable impedance in this case), with the result that practically the whole of the voltage is concentrated across the gap between A and S_1. This gap at once breaks down, the rest of the gap between A and S_2 immediately following. In effect, the length of the gap is halved the instant a high-frequency wave occurs. On the other hand, a slight time-lag is introduced due to the fact that, in the initial breakdown, one of the electrodes is pointed. However, an apparent impulse ratio of 0·6 to 0·7 can be obtained with the commercial forms of the gap.

208. Electrolytic Arrestor.—This arrestor has been undoubtedly one of the most effective lightning protective devices brought out, and is in very extensive use on the high-voltage

systems of America. The action of the arrestor depends on the characteristics of a film of aluminium hydroxide, which is deposited on aluminium plates immersed in electrolyte. Such a film presents a very high resistance to the passage of current if the impressed voltage is comparatively low. Above a definite limit, however, known as the critical voltage, the film punctures and permits current to pass freely. The magnitude of this breakdown voltage varies between 350 and 450 volts, depending on the composition of the electrolyte employed.

The volt-ampere characteristic of an electrolytic cell having aluminium electrodes is shown in Fig. 145. Suppose that such a cell is connected to a circuit whose voltage can be varied. Then,

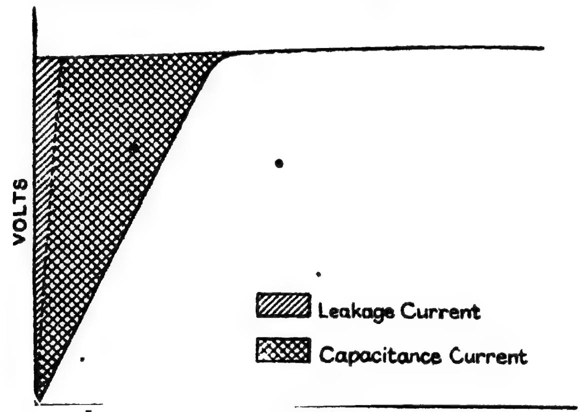


FIG. 145.—Volt-ampere characteristic of electrolytic cell.

if a low voltage is applied, a film of thickness corresponding to the voltage will form. If now the voltage is increased, there will be a rush of current for a few seconds until the thickness of the film has increased to an amount corresponding to this higher voltage. If this is repeated, the same phenomena will be observed until the critical voltage of the cell has been reached, when the film will not be able to increase its thickness any further, with the result that there will be a free flow of current.

It will be readily seen that this property of the aluminium cell can be made use of in order to relieve the line of excess pressures. Stacks of cells can be connected between each line and earth, the number of cells being such that the breakdown

voltage is greater by any desired amount than the voltage set up across the stack by the ordinary line pressure. It is not necessary that the number of cells should be such that, on charging, the film is built up to the voltage corresponding to the critical voltage. For, supposing the film is built up with a voltage below critical, then any high-frequency discharge over this voltage will break it down, since the time element required for this is much less than the time required at commercial frequency to build the film further. As long as conditions are normal no current will flow through the arrestor (except capacitance current which will be considered later), but if the pressure rises the films break down, thus allowing a free path to earth for the discharge. The films re-form immediately this discharge has passed because the normal line pressure is not sufficient to enable dynamic current to flow. Any desired degree of protection can be obtained by having a sufficient number of cells per stack, and the arrestor has an extremely high discharge capacity as no series resistance is necessary.

The construction adopted for the arrestor is as shown in Fig. 146. Spun aluminium trays of double-conical section are nested one within the other, electrical contact between them being prevented by small porcelain spacers. Electrolyte is poured into each tray so as to make contact from one to the next, and the whole stack of trays is immersed in transformer oil. The oil serves the double purpose of increasing the thermal capacity of the arrestor, and of preventing evaporation of the electrolyte. Under extremely severe lightning discharges the electrolyte may splutter a little, and it is therefore usual to interpose a cylindrical micarta lining between the trays and the steel containing tank.

Each pair of trays in the electrolytic arrestor are normally separated by an insulating film, and thus the whole apparatus can be looked upon as a series arrangement of condensers. For this reason it is impossible to have the stacks permanently in circuit, otherwise continual capacitance and leakage currents would flow to earth, raising the temperature of the electrolyte and lowering the discharge capacity. It is therefore necessary to interpose a spark gap in the connection between the line and each stack.

For this purpose the Westinghouse Co. employ the impulse

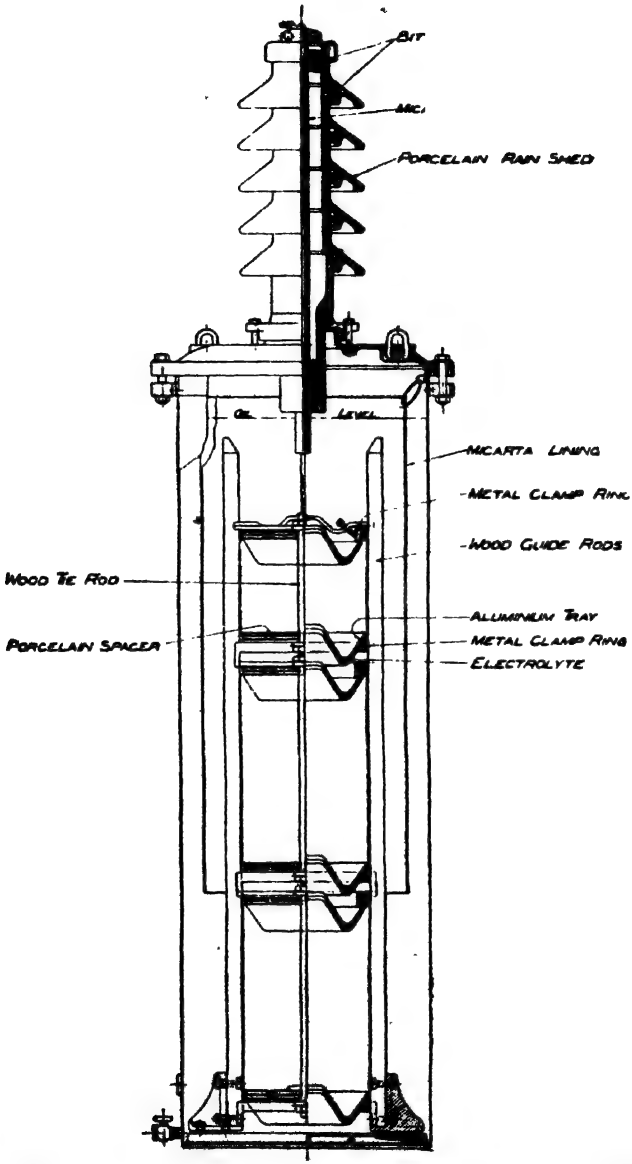


FIG. 146.—Construction of electrolytic arrestor. (Metropolitan-Vickers Electrical Co.)

protective gap, and Fig. 147 shows the application of this gap to an electrolytic arrestor. In the actual construction of the gap ordinary insulators are used for the condensers K_1 and K_2, and the unbalancing resistance *UR* is mounted on the upper insulator. One property of the arrestor which is of great importance is that when standing the film slowly dissolves in the electrolyte, and requires re-forming by the momentary passage of current. This charging process, as it is termed, is generally carried out once each day, or in very hot weather when electrical storms are frequent, twice daily. This is done by bridging the two horns of the gap for a few seconds, thus connecting the arrestor direct to the line. When this is done there is an initial rush of current which re-forms the film, and the operator can tell whether this reforming is complete by the appearance of the arc on breaking the contact at the horns. If, by any chance, the film has been allowed to dissolve entirely, there is a very heavy current rush on charging. To reduce this rush of current, and to eliminate the danger of pressure rises which might be caused by oscillations set up thereby, it is usual to charge through a limiting resistance. It will be seen in Fig. 147 that the earth-side horn is split, so that when the horns are bridged above this split the charging current has to flow through the resistance *CR*. When a discharge from the line occurs, however, it takes place across the spheres, so that only the resistance of the electrolyte is in circuit, thus giving maximum freedom of discharge. The resistance of commercial arrestors varies from about 60 ohms for a 22-kilovolt installation to 320 ohms for a 110-kilovolt installation, so that the discharge capacity is high.

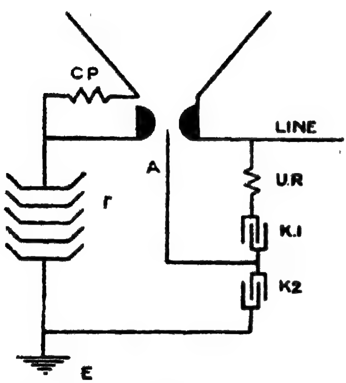


FIG. 147.—Arrangement of electrolytic arrestor and impulse protective gap.

The chief disadvantages of the electrolytic arrestor are the freezing of the electrolyte at low temperatures (which makes the arrestor practically inoperative), and the necessity of daily charging. This latter consideration considerably increases the

maintenance expense, particularly in the case of arrestors located on the more remote parts of the line.

209. Oxide Film Arrestor.—This type of arrestor has not been in use for many years but appears, in some respects, to be superior to the electrolytic arrestor. Its action depends upon the fact that certain dry chemicals change very rapidly from good conductors to almost perfect non-conductors when heat is applied. Thus, lead peroxide which has a specific resistance of about 1 ohm per inch cube at ordinary temperatures becomes converted to red lead at about 150° C., its specific resistance being then of the order of 24 million ohms per inch cube. A further increase in temperature reduces the red lead to litharge, which is practically an insulator.

The actual arrestor working on this principle consists of a number of cells held together under slight pressure in sections or stacks. Each cell is made up of sheet-metal electrodes about 7½ inches in diameter dipped in an insulating varnish, which covers them with a film, and separated by a porcelain ring about 0·5 inch thick. The space between the electrodes inside the ring is filled with lead peroxide powder. The number of cells used in an arrestor is such that the voltage per cell is approximately 300 volts.

Fig. 148 shows the volt-ampere characteristic of a cell compared with that of an aluminium arrestor cell. It will be seen that right up to the critical point the leakage current is very small, and then the cell breaks down with absolute suddenness. When the insulating film on the electrodes breaks down it is punctured in very fine pinholes, and therefore any dynamic current following the discharge has to flow through these pinholes. Thus there is intense local concentration of current, which produces local heating to such an extent that the neighbouring lead peroxide becomes converted to the insulating litharge. Hence the pinholes are almost immediately re-sealed, thus preventing any flow of dynamic current. According to Field,[8] the time taken for this re-sealing is certainly less than one four-thousandth of a second. The ohmic resistance of an arrestor during discharge varies between 0·01 and 0·1 ohms per cell, depending on the path followed by the discharge, so that the impedance offered to the flow of a lightning discharge is very small. In the course of time all the lead peroxide must be used

up, but the quantity employed in the cells appears to suffice for several years' normal service.

Although the oxide film arrestor does not take such a large capacitance current as the electrolytic type, it is necessary to place a spark gap in series therewith because of the deteriorating effect of the alternating current on the sheet-metal plates. The General Electric Co. employ for this purpose covered hemisphere gaps, set at about 170 °/₀ of the normal voltage of the system.

The great advantage possessed by the oxide film arrestor is that it does not require daily charging, and it may thus be installed at points on transmission systems where daily attendance

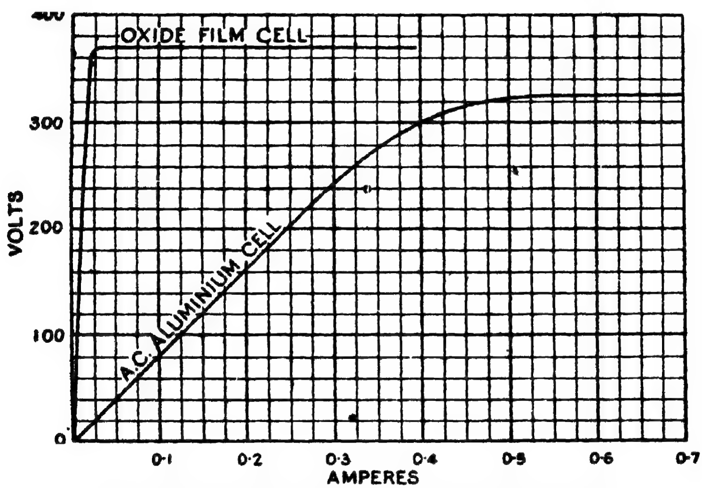


FIG. 148.—Volt-ampere characteristic of oxide film cell.

is difficult or expensive to provide. On the other hand, varnish has an impulse ratio of above unity, so that as the action of the arrestor first depends on the puncture of a varnish film, it appears as though the device as a whole would be less sensitive to steep-fronted waves than to low-frequency disturbances.

210. Autovalve Arrestor.—The autovalve arrestor, originated by Slepian,[9] is one of the most modern developments for the protection of transmission lines from the effects of abnormal pressure rises. In principle it is essentially a multiple spark-gap type of arrestor, but makes use of a glow discharge instead of an arc discharge.

The lower curve in Fig. 149 shows the characteristic of an arc discharge in air. If, however, the electrodes of a gap are made of material of considerable resistivity, any discharge is distributed over the faces of the electrodes, and local heating with formation of an arc is not possible. In this case, therefore, the discharge is maintained as a glow with volt-ampere characteristics, as shown in the upper curve of Fig. 149. Studies have shown that when flat electrodes are separated by insulating spacers of the thickness to give gap lengths of 3 to 5 mils the glow-discharge voltage is practically constant at 350 volts (instantaneous value) over a wide

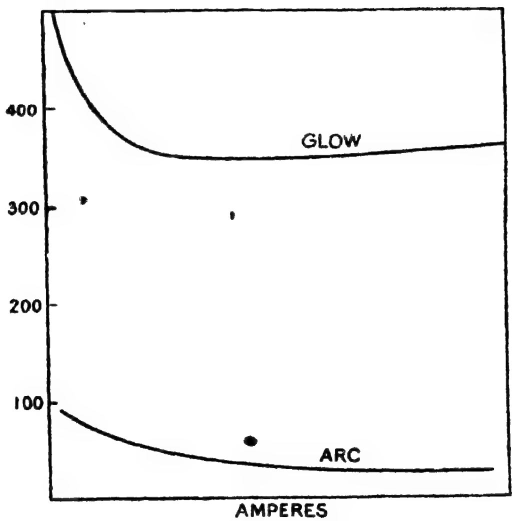


Fig. 149.—Volt-ampere characteristics of arc and glow discharge.

range of current values. The breakdown voltage is also of the same magnitude. Thus a structure composed of a column of flat electrodes suitably spaced will operate as a valve, the critical voltage being approximately 350 volts per gap. The characteristics of the arrestor over this critical voltage will depend on the thickness, area, and resistivity of the electrodes.

In the commercial form, the arrestor is built up of a series of disc electrodes made of a composition material of suitable resistivity, each pair of discs being separated by mica spacers. Each column of gaps is connected between line and earth through

a series gap. The number of gaps is chosen so as to give one gap for each 200 volts (R.M.S.) between line and earth, thus providing a voltage margin of approximately 25 % before breakdown occurs. The disc area is selected to give a resistance rather lower than the electrolytic. For example, arrestors for a 22-kilovolt installation have a resistance of about 45 ohms, and those for use on a line voltage of 110 kilovolts have an approximate resistance of 225 ohms.

The autovalve arrestor has the advantage over the electrolytic type in that the initial cost and maintenance charges are lower, and there is no necessity for daily attendance. Hence it has a wider field of application than the electrolytic, as it can be used to protect isolated transformer stations, automatic substations, or similar plant where the installation of electrolytic arrestors would not be justified from an economic point of view.

211. Later Types of Arrestors.—In the latest design of autovalve arrestor the construction has been radically altered, and the arrestor is now built up of cylindrical, porous blocks, incorporating in them small amounts of powdered metal or other conducting material. These cylindrical blocks or elements are assembled in a porcelain casing which also includes a totally enclosed gap structure, the number of elements depending on the voltage rating of the arrestor.[10]

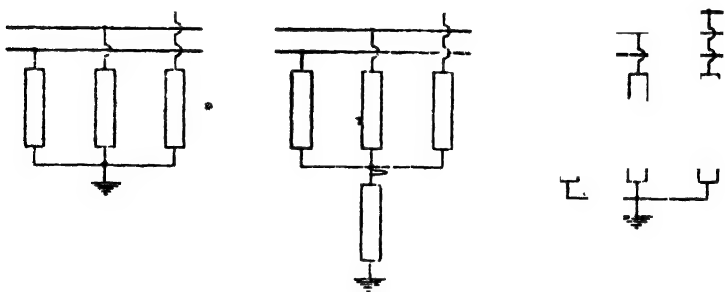
Another recently-developed type of arrestor is very similar in many respects to the new autovalve arrestor, but is built up of discs or elements made from a synthetic material known as thyrite. This material has the useful property of decreasing in resistance as the current increases. For example, each time the voltage is doubled the current increases about thirteen times.[11]

Commercial development of apparatus on the above lines has resulted in the production of arrestors with a greatly improved performance, and only one-half the size and weight of the older types.

212. Arrangement of Lightning Arrestors.—The arrangement of electrolytic and other valve-type arrestors on a three-phase circuit depends on whether or not the neutral point of the circuit is earthed. In Fig. 150 (*a*) the three arrestor stacks or columns are star connected and the neutral point connected direct to earth. This is suitable for use on a system with earthed neutral, because if one of the lines becomes earthed the other lines will

still be at the star voltage above earth, and will not therefore cause the arrestor to discharge.

If the neutral point of the system is not earthed, then four stacks are necessary, as shown in Fig. 150 (*b*). The reason for this is, that in the event of an earth on one line, the other two lines will be raised to the full line voltage above earth instead of the normal star voltage. As a result, if the arrangement of Fig. 150 (*a*) were used with an unearthed neutral, an earth on one line would immediately cause the arrestors on the other two lines to operate, unless they were set to operate at a pressure between line and earth greater than the line pressure of the system, *i.e.* more than 173 °/。 of the normal voltage across them. The protection afforded in such a case would obviously be too coarse.



(*a*) (*b*) (*c*)

FIG. 150.—Arrangement of lightning arrestors on a three-phase circuit.

With the neutral point of the system connected to earth through a resistance, the precise voltage impressed across each stack of the arrestor depends on the earth current. To guard against mishap, however, it is customary to use the four-stack or multiplex arrangement in all cases except where the neutral point is earthed solid.

An arrangement of stacks recommended for use with the autovalve arrestor is shown in Fig. 150 (*c*). In this method of connection three columns or stacks are used, each designed for line or delta voltage, and an interconnection between stacks is provided at the star-voltage point in each stack. The arrangement is based on the idea that the requirements from the standpoint of discharge-current capacity are fixed by the most severe conditions, which are those imposed by disturbances of external or

atmospheric origin. Discharges due to these external causes pass through the three line elements in parallel and then through the earth element. Thus, in the multiplex connection, the IR drop in the arrestor resistance of the line and earth elements in series is four times that across a line element. With the method of connection shown in Fig. 150 (c) the resistance is reduced to that of an arrestor for line voltage only. In other words, the addition of 29 % in the material used increases the discharge capacity by about 132 %.

213. Condensers.—In order to relieve the line of those high-frequency disturbances which are not of sufficient amplitude to discharge over an arrestor, electrostatic condensers are often used. These have the useful property of possessing an impedance $\frac{1}{\omega C}$ which is inversely proportional to the frequency of the discharge. For example, supposing that a 50-cycle transmission system had a battery of condensers of value 0·05 microfarads installed between each line and ground. Assuming that the wiring connections to the condensers were of negligible impedance, the apparent resistance to the passage of the 50-cycle current would be 63 500 ohms. If, however, an oscillation having a frequency of 500 kilocycles per second occurred in the line, the apparent resistance of the condenser battery thereto would only be about 6·35 ohms.

It is almost the same, as far as these high-frequency discharges are concerned, as though the line were connected direct to earth at the point where the condensers are installed. It is this property of the condenser of presenting a very small obstacle in the path of high-frequency currents, while allowing practically no current to pass at line frequency, which makes it so valuable as a protective device against those high-frequency oscillations always accompanying lightning phenomena.

The first satisfactory design of condenser for this purpose was due to Moscicki,[12] and consists of a long thin glass tube, provided inside and out with a coating of silver deposited chemically and then electroplated. This method of construction obviates any danger of air bubbles being enclosed between the coating and the glass, which would be a source of danger. The thickness of the tube must be kept as low as possible in order to get a high value of capacitance, and special precautions must also be taken to prevent 'edge' trouble due to concentration of the electrostatic stress

at the edges of the metallic coatings. Each condenser is mounted inside a long metal tube, the space between being filled with a mixture of water and glycerine to give the apparatus a large thermal capacity, and thus prevent a dangerous rise of temperature on passing a heavy discharge. The glycerine also prevents freezing in cold weather. The tubes are finally assembled in metallic frames in the form of batteries, a sufficient number being connected in parallel to give the requisite capacitance.

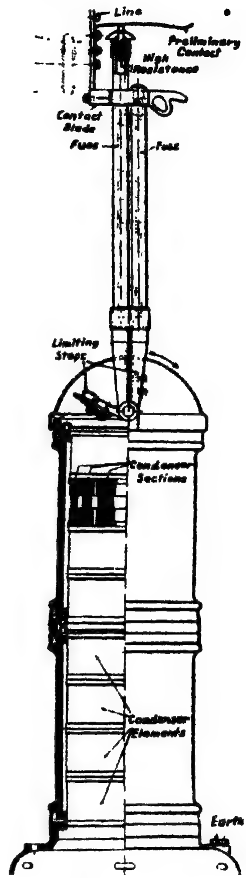


FIG. 151. — High-tension condenser with disconnecting fuse-switch. (Isenthal & Co.)

The Moscicki condenser met with extensive application, but proved rather difficult to develop for the high voltages now in more general use, and its fragile character due to the use of glass also hindered its commercial exploitation. Other materials such as impregnated paper and mica have been tried out for high-tension condensers, but the latest type uses as dielectric strips of acetyl cellulose known under the name of 'Cellon.' This material is remarkably suitable for the purpose, as it has good electrical properties, and can be obtained in large sheets or strips of very uniform quality. Most important of all, it is free from fissures and air pockets which have been the cause of much trouble with condensers using other dielectric materials. In the manufacture of cellon condensers, strips of the material are silver plated on each side so that the plating forms the metallic conductors of the condenser. This plating is carried out by chemical means by passing the band of dielectric through appropriate chemical baths. A high-resistance film is then applied to the edges of the metal with the object of producing a gradual tailing-off of the potential at these points, and avoiding trouble due to local stress concentration.

After treatment, the strips are wound up into rolls which are fitted into short insulating tubes provided with metal end-caps making cartridge elements which can be connected in series. Several of these cartridges are connected together and mounted inside a porcelain tube container, which is fitted with metal end-caps to provide terminal connections for the condenser. Any number of these porcelain-covered condenser units can be connected in series by screwing the metal caps together, and Fig. 151 illustrates such an arrangement with a disconnecting fuse-switch mounted on the top. This fuse-switch is of a special design and introduces a high resistance into circuit just prior to disconnection, so that the arcing is minimised, and there is no trouble set up by pressure rises due to this cause. The condenser can be fitted with rain shields and is then very suitable for outdoor installation, or it may be arranged for suspension mounting.

Although condensers are permanently in circuit, the power taken by them is exceedingly small because the current at supply frequency is small, and furthermore it leads the applied voltage by nearly 90 degrees. The departure of the current from true quadrature depends upon the magnitude of the dielectric losses and these are quite low. When installing condensers, choking coils are connected between them and the apparatus to be protected. These choking coils act as an impassable barrier to the high-frequency waves, which have then to find their way to earth through the condenser.

214. Surge Absorbers.—It must not be understood from the above remarks that condensers are able to dissipate the energy of a discharge as though the discharge had taken place through a resistance. The action of the condenser is really to shunt the high-frequency discharge away from the apparatus to be protected, and the dissipation of the energy is due to the resistance of the line conductors and the earth connections. It thus appears desirable to place in series with the condensers a non-inductive resistance, so as to rapidly absorb the energy of a disturbance. Such a combination of resistance and capacitance is known as a surge absorber.

Another type of surge absorber is provided by an arrangement of a non-inductive resistance in parallel with an inductance coil. Both of these devices were first proposed by Campos,[13] and undoubtedly have a certain protective value. The best conception

of their action is obtained by considering them as resistances to which a condenser or inductance is connected in series or shunt to act as a switch. This imaginary switch automatically connects resistance in circuit when a high-frequency discharge of energy occurs, and automatically disconnects it when conditions are normal and resistance is no longer required.

It is now standard practice on some transmission systems to lead the current from an overhead line into terminal stations through a length of underground cable. Such a cable forms a type of surge absorber capable of dissipating considerable quantities of high-frequency energy, owing to the combined effect of dielectric and sheath losses. However, a considerable length of cable is required to obtain adequate attenuation and smoothing-out effects.

Another type of apparatus which has come into use comparatively recently is the Ferranti surge absorber, which consists of an air-core inductance connected in series with each line conductor and having a high-resistance metallic sheet, termed the dissipator in close electrostatic and electromagnetic association. This sheet is connected to ground. When a steep-fronted wave or impulse enters the absorber, the energy required to charge the condenser formed by the coil and earthed sheet is abstracted from the energy in the wave. More important still, the eddy currents set up in the dissipator reduce the energy of the high-frequency components of the wave, so that the absorber acts as a filter circuit for currents of high frequency, or steep-fronted impulses. It thus possesses similar characteristics to a long length of cable. It is claimed that, by installing this apparatus, the pressure rise across the end turns of a transformer subjected to incoming high-frequency waves can be reduced to about 15 °/ₒ of its original value.[14]

215. Arcing Ground Suppressors.—On fully-insulated transmission systems, arcing grounds are always liable to give trouble. Furthermore, should a spill-over occur on a line insulator, there is the possibility of the arc continuing during an appreciable length of time sufficient to do serious damage to the insulator, even if it should not totally destroy it. In order to protect against trouble due to these causes, the arc suppressor due to Creighton[15] has been occasionally used. This is an automatic device for momentarily short-circuiting the arc through a switch. By providing a metallic connection between the conductor and earth the arc is

suppressed, and it will usually not re-form when the switch is again opened because the air in the path of the arc has had time to cool, and the line pressure, which was sufficiently high to maintain the arc once started, is not able to break down the insulation of the new layers of cooler air.

The diagram of Fig. 152 illustrates the principle of action of the apparatus. The chief feature is the selective relay which energises the solenoid operating the switch on the faulty line. The relay operates on the principle of a leakage detector. For high-pressure systems it is of the electrostatic type, but for low

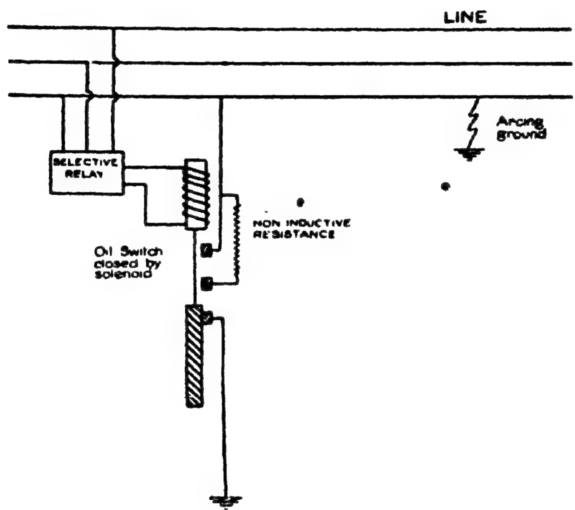


FIG. 152.—Diagram of arcing ground suppressor.

voltages it works on the electromagnetic principle. When the relay operates, it picks out that phase which is brought near to earth potential by a fault, and through the intermediary of the trip coil and oil switch connects the faulty phase to earth for a fraction of a second, after which it is automatically disconnected again. Of course three earthing oil switches are employed, one in each phase, but only one is shown in the figure. Owing to the increasing practice of earthing the neutral point of three-phase systems the need for the arc suppressor has almost disappeared and at the present time it is chiefly of historical interest.

216. Petersen Coil.—The neutral-earthing reactor proposed by Petersen [16] is another device sometimes employed to protect against the effect of arcing grounds on those transmission systems for which earthing is not desirable for one reason or another. For example, the terminal apparatus may not be able to safely withstand short-circuit, or a single transmission circuit only being in use frequent interruptions to the supply cannot be tolerated. The device itself consists of an air-core reactor connected between the neutral of the circuit and earth. The reactance is of such a value as to neutralise the capacitance of the circuits when an accidental earthing with arc formation occurs. Under this accidental

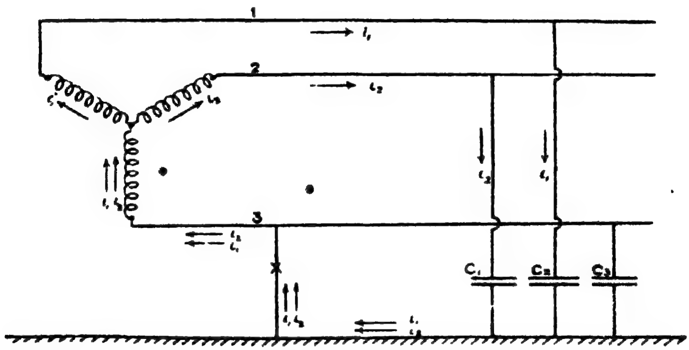


FIG. 153.—Flow of currents in three-phase system with arc from one phase to earth.

condition the reactor is electrically in parallel with the active capacitances, and the residual current is of very small magnitude.

The theory of operation can be understood by referring first to Fig. 153, which represents the high-voltage side of the transformer bank, and the transmission circuit on which there is an arc from phase 3 to earth. The charging currents will flow through the capacitances c_1 and c_2 from phases 1 and 2 to earth, through the arc back on conductors 3 to the transformer, and thence to conductors 1 and 2 as shown. Capacitance c_3 is short-circuited by the arc. Now if a reactor is inserted between the neutral and earth, as in Fig. 154, a current i_l will flow through the reactor and the arc as shown. i_l is opposite in phase to $i_1 + i_2$, and if a suitable value of reactor is chosen the current through the arc will be neutralised, and the final flow of current will be as shown. In actual practice,

there is an active component of current due to the losses in the reactor and lines, and the current cannot be balanced out but exists as a small residual current through the arc. This, however, is soon extinguished, as it is insufficient to support a permanent arc.

It will be obvious that the conditions are entirely changed if the fault is a definite leak, which has not the re-sealing features of an earth arc on extinction, and there is nothing in the conditions outlined which allows of discriminating protection. It is convenient therefore to install a time-lag relay, which on a sustained discharge through the reactor will operate to close a switch, shunting the reactor and thereby solidly earthing the neutral, and

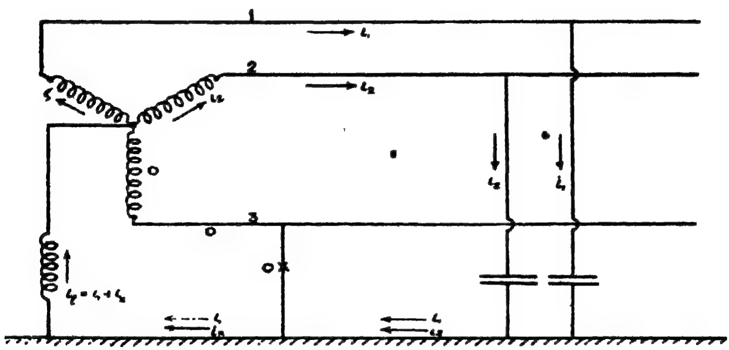


FIG. 154.—Flow of currents in three-phase system with neutral-earthing reactor, and arc from one phase to earth.

throwing the business of clearing the fault on to the other protective gear installed.

Although there are a large number of these coils in use on the Continent, the device does not appear to be of general application. In particular, it is not quite satisfactory for installing on transmission systems operating at the highest voltages, or even on moderate-voltage systems if the length of transmission is very great. For in this case the charging current is large, and the reactance required for the Petersen Coil is correspondingly low. The reactance of the transmission line, on the other hand, is high, so that it is difficult to maintain a balance for arcs at different points on the line, and there may be enough residual current to maintain the arc.

217. Water-jet Earthing Resistances.—These devices have been much employed on the Continent to relieve overhead lines of dangerous electrostatic charges. They consist essentially of permanent leaks to earth through continuous jets of water. The diagrams in Fig. 155 show two common types. In the first, the jets of water issue from earthed nozzles and impinge against inverted metallic cups connected by means of isolating switches to the line. In the second type, the jets are at the top and the water flows into insulated troughs or funnels each connected to one pole of the system, and thence into an earthed tank. In

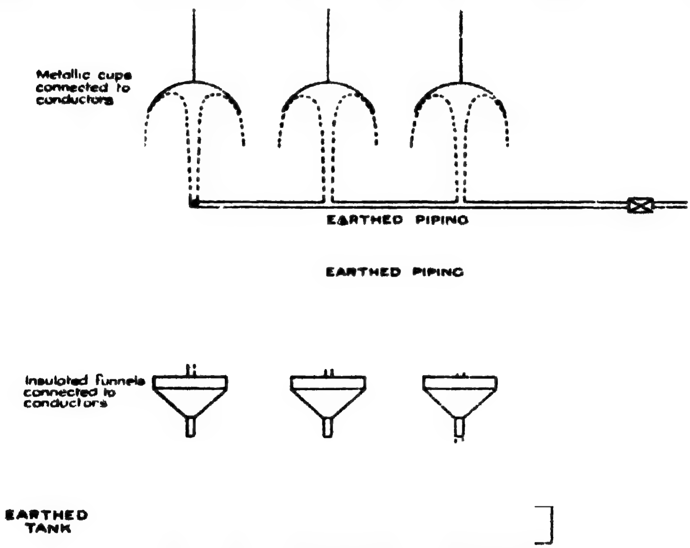


Fig. 155.—Water-jet earthing resistances.

this case, each pole has two leakage paths to earth in parallel, so that it is often used in places where the water is very pure and therefore of high resistance.

Since these water jets constitute a permanent leak to earth they are very efficient in relieving the line of gradual accumulation of static charges, but the resistance is naturally very high in order to prevent the loss of too great an amount of energy. For example, the Allgemeine Elektricitäts Gesellschaft recommend that the leakage current for a 77 000-volt system should have a value of about 0·08 amperes, corresponding to a resistance

for each jet of 555 000 ohms.[17] It is obvious that a resistance of this value is utterly incapable of giving any protection against steep-fronted travelling waves due to lightning effects. Furthermore, the safeguard against static over-potential is very dearly bought, especially if the water jets are permanently in circuit. Taking the 77 000-volt case again, and assuming that the cost per unit of energy at the station is 0·5 pence, the value of the wasted energy would be about £200 per annum.

To reduce the loss of energy it is common practice to cut the water jets out of circuit unless thunderstorms are expected, but this reliance on the human element renders the apparatus less valuable than an equally effective device which is always ready to act.

From the above it will be seen that water-jet earthing resistances are able to deal only with static charges on the line, and only then if the accumulation of charge is slow. Since there are other types of arrestor which will safeguard the line against this effect in addition to travelling waves, the installation of water-jet earthing resistances can hardly be justified.

218. Earthing Choking Coils.—For large systems, where it is necessary to provide means to dissipate static charges, the water jet has often too high a resistance and earthing choking coils have to be resorted to. These have the advantage of dissipating very little energy because the current through them is practically wattless. Also there is no waste of water which, in any but a hydro-electric station, might be a serious item.

In order to have a sufficiently large reactance and prevent too much current flowing to earth these coils have a laminated iron core, so that there are small iron losses in addition to small copper losses. But since the inductance of the coil is the main factor in limiting the permanent earth current, the resistance can be made so small that even rapidly accumulating static charges can be successfully dealt with.

These choking coils are of no value in relieving the line of travelling wave effects because, their reactance being proportional to the frequency of the current flowing through them, they will offer an enormous impedance to high-frequency waves. Their function, like that of water-jet earthing resistances, is to relieve the line of static over-potentials only.

Earthing choking coils are connected between each line and

earth, so that their insulation will normally have to withstand the star voltage of the system. They have to be insulated, however, for the full line voltage, since that is the voltage the insulation will have to withstand if one of the lines should become earthed.

Of course, if the neutral point of the transmission system is earthed solidly, or through a resistance, a path to earth is provided for any static charges on the conductors, thereby rendering unnecessary any special apparatus for this purpose.

219. Modern Tendencies.—The more important types of apparatus for the protection of transmission lines and terminal stations from the effects of pressure rises have now been described. There are many devices to choose from, each of which has its own particular field of usefulness, and it is now possible to provide reasonable protection against discontinuity of service during atmospheric disturbances.

For the highest voltages, *e.g.* over 110 kilovolts, there is a tendency to do away altogether with lightning-arrestor equipment under normal conditions, and to rely mainly on the insulation backed-up by ground wire, and possibly surge-absorber, protection. The pressure of the system is already so exceedingly high that if the insulation of the line and the transformers is designed with a reasonable factor of safety it will stand up to anything in the nature of an over-voltage, with the exception of a direct lightning stroke, or exceptionally severe induced stroke.

On the other hand, a line traversing districts exposed to severe lightning storms requires special protective measures to reduce interruptions of supply to the minimum. To this end, small and compact forms of arrestor and other devices for the prevention of flash-over on individual insulator strings have been developed, and are now being tried out on various installations. If successful these devices may have a fairly wide field of application.[10, 11, 18].

It is also probable that the near future will see some developments with the idea of making the line more or less self-protective. For example, it has been suggested that the properties of the corona might be utilised as a protection against lightning and other similar types of disturbance. The spacing and sizes of conductor of transmission lines could often be arranged so that the normal operating voltage was only just below the corona-forming voltage. If an over-voltage then occurred, the energy

of the disturbance would be quickly dissipated in the corona itself.

For low- or medium-voltage lines the spacing necessary for corona formation would be so small as to constitute a serious danger of the wires swinging together, or of flash-over. In these cases it would be possible to equip lines of standard spacing with some form of studs or points, which would produce a corona-discharge curve equal to or better than the normal corona power-loss curve. Experiments have been made by Nagel[19] and Whitehead,[20] using barbed wires, and also wires having mounted on them corona-forming studs, with satisfactory results. Apparently sufficient protection for the station apparatus could be obtained by arranging for this artificial corona formation on the two terminal miles at each end of the transmission line.

220. The Potentials of a Three-phase Transmission System.—Before going on to discuss the subject of earthing, the potential relations to earth will be considered of a three-phase system on which no deliberate earthing has taken place.

On a three-phase system with solidly-earthed neutral the potential difference between any conductor and earth is equal to the phase-to-phase voltage divided by $\sqrt{3}$. On the other hand, if the neutral is not earthed, the position of the earth-potential point is unstable, and varies with the relative condition of the insulation on the three phases. Its position also depends on the magnitude of the capacitance current of the system. A simple means of determining the potential of such a system has been given by Field.[21]

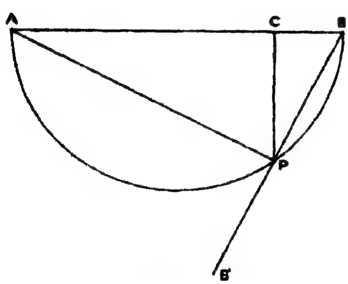


FIG. 156.—Single-phase system with leakage and capacitance.

First of all consider the case of a non-earthed single-phase system whose voltage is represented by the rotation of the vector in Fig. 156. If the pole A be connected to earth the diagram may be considered as rotating round the point A as centre. Similarly, if the pole B is earthed instead of A, the vector would rotate round B as centre. If, however, the sole connections of A and B to earth are by the leakage conductances

of the insulation, the point at earth potential will be in some position between A and B, depending on the relative magnitude of these leakage conductances.

Assume now that there is a susceptance to earth of B farads in connection with pole A, and a path to earth of conductance G mhos in connection with pole B. The current to earth through the susceptance at A will be 90 degrees ahead of the pressure producing it, whereas the current through the conductance at B will be in phase with the pressure. The position of the zero-potential point must be such that these two currents are equal and opposite at every instant. It will be found that the locus of the zero-potential point is a semicircle described upon the line AB as diameter, as shown in Fig. 156.

The diagram will then rotate round a point P as centre, where PA represents in phase and magnitude the pressure acting on the susceptance between pole A and earth. This will produce a current 90 degrees in advance of the pressure in the direction PB' and of amount $B \times \overline{PA}$ amperes. On the other hand, the vector PB represents in phase and magnitude the pressure acting upon the conductance G and producing in it the current $G \times \overline{PB}$ amperes in the direction PB.

The condition that P is at earth potential is that these currents are equal and opposite, that is

$$B \times \overline{PA} = G \times \overline{PB}$$

or

$$\frac{\overline{PA}}{\overline{PB}} = \frac{G}{B}.$$

If a perpendicular PC is dropped from the point P the following geometrical relations are obtained:

$$\frac{\overline{CA}}{\overline{CB}} = \frac{\overline{PA}^2}{\overline{PB}^2} = \left(\frac{G}{B}\right)^2.$$

The combined effect of capacitance and leakage in the case of a three-phase system may now be considered. Taking first the effect of leakage, a point P' can be found on the diagram of Fig. 157 which will represent the position of earth potential due to this cause. It can be readily shown that the position of P' is the same as the centre of gravity of three masses placed at A, B, and C, and proportional to the conductances of the leakage paths to earth from these respective phases.

For example, assume that the system comprises a transformer and overhead line, the leakage conductances of each phase of the complete system being

$$\begin{aligned} \text{Phase } A &= 2 \times 10^{-4} \text{ mhos,} \\ B &= 4 \times 10^{-4} \text{ mhos,} \\ C &= 6 \times 10^{-4} \text{ mhos,} \end{aligned}$$

and the sum of the conductances G' thus being 12×10^{-4} mhos.

Neglecting capacitance currents, the currents from A and B alone would be equal if the line AB revolved round the point D, the lengths BD and AD being such that $4\overline{BD} = \overline{2AD}$. The total

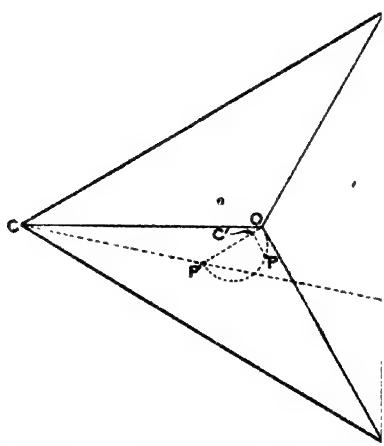


FIG. 157.—Effect of leakage and capacitance currents on a three-phase system.

leakage path from A and B may now be considered as concentrated at D, and as in this particular instance the leakage conductance from C has an equal value, the point P' midway along the line CD will represent the point at earth potential.

The effect of capacitance currents must now be taken into account. The same value of susceptance usually applies to each conductor owing to the practice of transposing any unsymmetrical circuits. Hence capacitance effects can be taken into consideration by assuming a susceptance equal to three times the conductor susceptance connected at O, the central point of the diagram. The total conductance of the leakage paths may similarly be considered as concentrated at the point P'.

Upon OP' describe a semicircle and divide OP' at a point C such that

$$\frac{\overline{OC'}}{\overline{C'P'}} = \left(\frac{G'}{3B}\right)^2.$$

From the point C' draw a perpendicular to meet the semicircle at the point P. The point P will then be the zero-potential point in the diagram.

It is important to note the considerable stability given to the neutral point of a three-phase transmission system by the capacitance currents flowing in the conductors.

221. Earthed *versus* Isolated Neutrals.—The conditions arising out of operation with earthed or isolated neutrals have to be borne in mind when installing all protective devices, whether for the prevention of dangerous currents, or potentials. There has been for many years a divergence of opinion on the subject, but modern practice strongly inclines towards earthing for the following reasons:—

1. On a solidly-earthed system, the voltage to earth at all points is definitely limited to the star voltage of the system. This often permits of considerable economies in the matter of insulation. For example, transformers for the highest voltages can be built with the neutral connected directly to the core, and only one high-tension bushing. Such a transformer would be insulated for 2·73 times the star voltage, instead of 3·46 times, which is standard for transformers with isolated neutral.

2. All trouble due to arcing grounds or static pressure rises is eliminated.

3. Many of the most important relay protective devices and lightning-arrestor equipments can only operate to full advantage if the neutral is earthed.

The chief argument in favour of the isolated neutral is the possibility of maintaining continuity of service in case one phase becomes earthed until it is convenient to disconnect and repair the faulty line. This has been done in the early days of power transmission, when mileage was small and voltage low. With the more extensive systems of the present day this method of operation is impracticable, owing to the fact that most earths are not of a solid nature but are accompanied with severe arcing. Thus, not only are the unearthed conductors subject to a pressure to earth equal

to $\sqrt{3}$ times the normal value, but high-frequency disturbances are set up which place the whole system under severe strain, and often result in breakdown of apparatus at different points of the system.

In the case of an earthed-neutral system, there will occur practically as many primary line failures, such as breakdown of insulators or other apparatus, but as soon as one phase becomes earthed a short-circuit is produced on one leg of the earthed-neutral transformer. The current in this short-circuit flows over definite and known paths which renders possible the use of automatic protective devices to disconnect the faulty line. The selective action of relay apparatus is thus made more positive, and secondary breakdowns due to pressure rises are almost unknown. Furthermore, as most systems feed their important stations by more than one circuit, the loss of a line due to trouble does not necessarily involve any stoppage of the supply.

The trend of opinion is clearly evidenced by the result of an inquiry held by a Sub-committee of the American Institute of Electrical Engineers on the conditions prevailing in that country.[22] Out of 235 systems operating at pressures between 11 and 220 kV., and representing a total mileage of 56 860, from which replies were received, it was found that a mileage of 51 550 or 91 % was earthed either solidly or through an impedance.

As regards methods of earthing, this is usually done at the sending station, a neutral point on the high-voltage circuit being obtained by using delta-star raising transformers. Where solid earthing is resorted to, transformer neutrals at the receiving stations or substations are often earthed as well. The only difficulty introduced by having several earths on one network is in the calculation of the short-circuit current. Such calculations are comparatively simple with only one neutral earthed, but where several are involved each contributes to the earth current, the amount and direction depending upon the location of these earth points. It is thus sometimes very difficult to predetermine the earth current with sufficient definiteness as to allow of accurate selection and adjustment of protective relays.

The question of the desirability of limiting the earth current in transmission systems is one on which opinion varies. There is little doubt that the greatest protection from high-voltage strain comes from the use of a solidly-earthed neutral, and this is also the best arrangement from the point of view of relay protection.

Providing, then, that the terminal apparatus is designed to withstand the magnetic and thermal effects of absolute short-circuit currents for the time necessary for the relays to clear the circuit, no disadvantage is entailed by earthing solidly. This is also the more popular method, as is shown by the fact that of the 56 860 miles of line reported upon in the above inquiry, 42 300 miles or 75 % of the total mileage was dead-earthed.

In cases where from one reason or another a current-limiting resistance between neutral and earth is requisite, this should be of comparatively low value. The maximum voltage which such a resistance would be called upon to bear is the voltage to neutral of the system, and the maximum current to be passed is that required to operate the circuit-breaker on the heaviest line. It is obvious that this current will be much heavier if the lines are only protected with overload devices and not leakage or balanced protective gear. Even if the latter be used, however, overload protection will in most cases be used as well, so that the earthing resistance should in all cases, if possible, be of capacity enough to carry for the requisite time the current necessary to trip the overload gear on the heaviest line.

Earthing resistances are usually built up of cast-iron grids, and must be insulated for the voltage to neutral excepting only at the end where they are permanently connected to earth. Their time rating at maximum current usually varies between 30 to 60 seconds, during which interval their temperature rise may be about 400 degrees C. When used on high-voltage systems they are very expensive, and take up a large amount of room. They are also liable to be burnt out in service.

Another type of earthing resistance is the Brazil carbon-powder, or, in its later form, the carbon-slab resistance. Cast-iron grids increase in resistance with temperature, but the carbon type possesses a negative temperature coefficient, so that when red-hot the resistance is only a small fraction of its cold value. It is thus possible to arrange the initial resistance to be much higher than with other types of apparatus. If the fault current which first passes is not sufficient to open the circuit-breakers it continues to flow until, the resistance having gone down, the current has increased by the required amount. The shock to the system due to a fault is thus considerably minimised.

Earth connections used in transmission work fall into two

categories: those used solely for lightning arrestors, and those used in conjunction with relay protective apparatus. In the case of the former, the ohmic value of the earth connection itself is not particularly important. Fifteen or twenty ohms is probably good enough, since it is usually in series with protective resistances of relatively much greater value, and when called into action there will be an extremely high potential impressed across the combination. On the other hand, the earth contact resistance for relay protection must be as low and as constant as possible, since the series resistance is relatively small or non-existent, and the close setting of relays depends on an accurate knowledge of the circuit conditions as a whole.

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CHAPTER XV.

POWER LIMITS OF TRANSMISSION SYSTEMS.

222. Power Limits of Short Lines.—With constant voltage E_s at the sending end of a transmission line, there is a physical limit to the amount of power that can be transmitted. For example let Z_1 represent the impedance of an inductive load of constant power factor $\cos \phi_r$. As Z_1 is gradually reduced in magnitude the current I in the system continuously increases, and the receiving-end voltage E_r continuously diminishes, so that at one particular value of Z_1 the power $E_rI \cos \phi_r$ in the receiving-end circuit is a maximum for the value of $\cos \phi_r$ under consideration. Variations in the value of $\cos \phi_r$ affect the amount of power that can be transmitted, and the absolute power limit of the line is reached when $\cos \phi_r$ has such a value that the power factor $\cos \phi_s$ at the sending end is equal to unity.

Formulæ for the calculation of power limits in various cases can easily be derived.

Let the line impedance

$$Z = R + jX \quad \text{ohms},$$

and the load of power factor $\cos \phi_r$ be represented by the impedance

$$\begin{aligned} Z_1 &= R_1 + jX_1 \\ &= R_1 + jR_1 \tan \phi_r \quad \text{ohms}. \end{aligned}$$

Then the current in the circuit

$$I = \frac{E_s}{\sqrt{(R + R_1)^2 + (X + R_1 \tan \phi_r)^2}}, \tag{234}$$

and the power delivered at the receiving end of the line is

$$P = \frac{I^2 R_1}{1\,000}$$

$$= \frac{E_s^2 R_1}{1\,000\{(R + R_1)^2 + (X + R_1 \tan \phi_r)^2\}} \quad \text{kW. per phase.} \tag{235}$$

If R_1 is varied, all the other factors being kept constant, the maximum power delivered is found from

$$\frac{dP}{dR_1} = 0 = \frac{d}{dR_1}\left(\frac{E_s^2 R_1}{1\,000\{(R + R_1)^2 + (X + R_1 \tan \phi_r)^2\}}\right)$$

$$= \frac{E_s^2\{(R+R_1)^2+(X+R_1\tan\phi_r)^2 - 2(R+R_1)R_1 - 2\tan\phi_r(X+R_1\tan\phi_r)R_1\}}{1\,000\{(R + R_1)^2 + (X + R_1 \tan \phi_r)^2\}^2}$$

hence

$$(R+R_1)^2+(X+R_1 \tan\phi_r)^2 = 2(R+R_1)R_1 + 2\tan\phi_r(X+R_1\tan\phi_r)R_1$$

from which

$$R^2 - R_1^2 + X^2 - R_1^2 \tan^2 \phi_r = 0$$

and

$$R^2 + X^2 = (1 + \tan^2 \phi_r)R_1^2 \qquad . \quad . \quad (236)$$

so that

$$Z = Z_1, \qquad . \quad . \quad . \quad . \quad . \quad (237)$$

i.e. at maximum output the impedance of the line is equal to the impedance of the load, or in other words, the receiving-end voltage and the impedance drop in the line are numerically equal.

To investigate the effect of varying the receiving-end power factor $\cos \phi_r$, it should be noted from (236) that

$$R_1 = \frac{Z}{\sqrt{1 + \tan^2 \phi_r}}, \qquad . \quad . \quad . \quad (238)$$

and substituting this value of R_1 in (235) gives

$$P_{max} = \frac{E_s^2}{2\,000\{Z\sqrt{1+\tan^2\phi_r}+(R+X\tan\phi_r)\}} \text{ kW. per phase.} \quad (239)$$

The corresponding value of receiving-end voltage is

$$E_r = IZ_1 = IZ,$$

and substituting for I from (234)

$$E_r = \frac{E_s Z}{\sqrt{(R + R_1)^2 + (X + R_1 \tan \phi_r)^2}}$$

whence by substitution for R_1 from (238)

$$E_r = \frac{E_s Z}{\sqrt{\left(R + \frac{Z}{\sqrt{1 + \tan^2 \phi_r}}\right)^2 + \left(X + \frac{Z \tan \phi_r}{\sqrt{1 + \tan^2 \phi_r}}\right)^2}}$$

$$= \frac{E_s Z}{\sqrt{2Z\left(Z + \frac{R + X \tan \phi_r}{\sqrt{1+ \tan^2 \phi_r}}\right)}}$$

or at maximum power output the ratio of voltages is

$$\frac{E_r}{E_s} = \frac{1}{\sqrt{2\left(1 + \frac{R + X \tan \phi_r}{Z\sqrt{1 + \tan^2 \phi_r}}\right)}} \quad . \qquad (240)$$

The following cases are of interest:—

1. *Load Non-inductive.*—In this case $\tan \phi_r = 0$, and substituting in (239) gives

$$P_{\max} = \frac{E_s^2}{2\,000(Z + R)} \text{ kW. per phase.} \quad . \qquad (241)$$

The ratio of voltages from (240) is

$$\frac{E_r}{E_s} = \frac{1}{\sqrt{2\left(1 + \frac{R}{Z}\right)}} \quad . \quad . \quad . \qquad (242)$$

and the efficiency of transmission is

$$\eta = \frac{R_1}{R_1 + R} = \frac{Z}{Z + R} \quad . \quad . \quad . \qquad (243)$$

2. *Load Non-inductive and Line Non-inductive.*—In this case $Z = R$, and substituting in (241) gives the maximum power output as

$$P_{\max} = \frac{E_s^2}{4\,000R} \text{ kW. per phase,} \quad . \quad . \qquad (244)$$

and from (242) and (243) it is seen that $E_r = \frac{E_s}{2}$, the efficiency of transmission being 50 per cent.

3. *Regulated Line.*—In this case, by adjustment of synchronous phase modifiers installed at the receiving end of the line, the receiving-end power factor $\cos \phi_r$ may be given any desired value. The absolute power limit of the line is then found by differentiating equation (239) in respect of $\tan \phi_r$ as

$$\frac{dP_{\max}}{d \tan \phi_r} = 0 = \frac{d}{d \tan \phi_r}\left[\frac{E_s^2}{2\,000\{Z\sqrt{1 + \tan^2 \phi_r} + (R + X \tan \phi_r)\}}\right]$$

$$= \frac{E_s^2\{Z \tan \phi_r(1 + \tan^2 \phi_r)^{-\frac{1}{2}} + X\}}{2\,000\{Z\sqrt{1 + \tan^2 \phi_r} + (R + X \tan \phi_r)\}^2}$$

hence

$$\frac{Z \tan \phi_r}{\sqrt{1 + \tan^2 \phi_r}} = -X,$$

and

$$\tan \phi_r = -\frac{X}{R},$$

but by assumption $\tan \phi_r = \frac{X_1}{R_1}$,

so that $\frac{X_1}{R_1} = -\frac{X}{R}$.

Since the condition that $Z_1 = Z$ must also be satisfied,

$$R_1 = R \quad \text{and} \quad X_1 = -X,$$

or the absolute power limit of the line is reached when the resistance of the line and receiving-end circuit are equal, and the inductive reactance of the line also equals in magnitude the condensive reactance of the receiving-end circuit.

The value of this maximum output is obtained by substitution in (239) as

$$P_{max} = \frac{E_s^2}{4\,000R} \text{ kW. per phase,}$$

and the efficiency of transmission is again 50 per cent. as in the preceding case. E_r, however, does not now equal $\frac{E_s}{2}$, but is found by substituting in (240) as

$$E_r = \frac{E_s Z}{2R}. \qquad (245)$$

223. Power Limits of Long Lines.—Analogous formulæ to the above can be derived in the case of long lines, but it is usually more convenient to solve problems graphically by means of the power circle diagram introduced in Art. 105. With this diagram a whole range of results showing the effect of varying the sending- or receiving-end voltages, the influence of synchronous phase modifiers of varying capacities, etc., may be obtained with a fraction of the labour involved in analytical solutions.

The co-ordinates of the centres of receiving-end power circles are a and b, and the radius is c where

$$a = -\frac{3E_r^2(A_1B_1 + A_2B_2)}{1\,000B^2}$$

$$b = \frac{3E_r^2(A_1B_2 - A_2B_1)}{1\,000B^2}$$

$$c = \frac{3E_rE_s}{1\,000B},$$

these formulæ, as before, being specially adapted to a three-phase system as they show the relation between the *total* reactive and active power at the receiving end of the line. The centres of the circles are in the second quadrant of the diagram, and lie on a straight line drawn from the origin and making an angle $\tan^{-1}\dfrac{A_1B_1 + A_2B_2}{A_1B_2 - A_2B_1}$ with the vertical axis.

It is readily seen that if the receiving-end voltage E_r is maintained constant, the position of the centres is fixed, and a family of concentric circles can be drawn about this point, each of which will represent a definite value of the sending-end voltage E_s.

On the other hand, if the sending-end voltage is constant, and the receiving-end voltage variable, the position of the centres is not fixed but depends on the chosen value of the receiving-end voltage. Keeping the sending-end voltage constant, and assuming various ratios of $\dfrac{E_r}{E_s}$ such as 1·0, 0·9, 0·8, etc., a family of circles can be drawn as shown in Fig. 158, the distance of the various centres from the origin being proportional to E_r^2.

A variable inductive load of constant power factor $\cos \phi_r$ can be represented by a straight line in the fourth quadrant of the diagram, such line being drawn from the origin at an angle ϕ_r with the horizontal axis. Hence the maximum power transmitted at this particular power factor by a non-regulated line is given by the intersection of the load line with the envelope to the power circles, this envelope being shown by the thick line in Fig. 158.

In the case of a regulated line, providing that sufficient synchronous phase modifier capacity is present, the output at any particular value of receiving-end voltage may be increased until it corresponds to the point of tangency of the power circle with a line parallel to the vertical axis of the diagram. A curve joining these various points of tangency is thus the locus of the maximum loads which can be transmitted at the various receiving-end voltages, this curve being shown dotted in Fig. 158. At any particular value of receiving-end voltage E_r the maximum output is

$$P_{\max} = c + a$$
$$= \frac{3E_r}{1\,000\,B}\left\{E_s - \frac{E_r(A_1B_1 + A_2B_2)}{B}\right\} \text{ kW.} \qquad (246)$$

When the absolute power limit of the line is reached

$$\frac{dP_{\max}}{dE_r} = 0 = \frac{d}{dE_r}\left[\frac{3E_r}{1\,000B}\left\{E_s - \frac{E_r(A_1B_1 + A_2B_2)}{B}\right\}\right]$$

$$\frac{\quad}{1\,000B}\left\{E_s - \frac{2E_r(A_1B_1 + A_2B_2)}{B}\right\},$$

from which $$E_r = \frac{E_sB}{2(A_1B_1 + A_2B_2)}, \qquad (247)$$

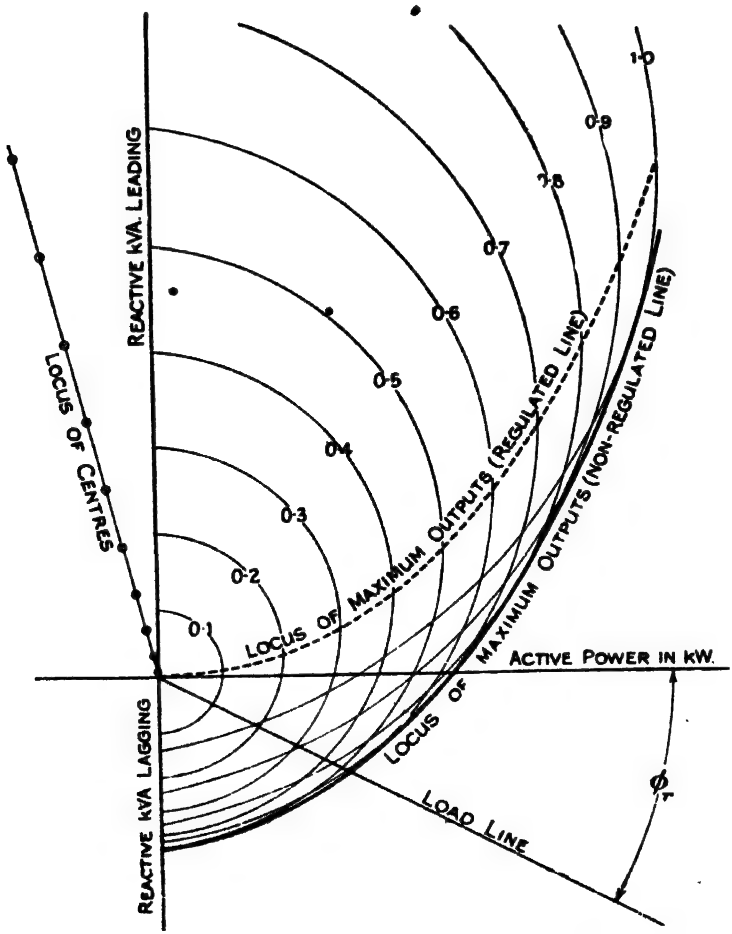

FIG. 158.—Receiving-end power circles (E_s constant, E_r variable).

and the power output is obtained by substitution in (246) as

$$P_{max} = \frac{3}{1\,000}\left\{\frac{E_s^2}{2(A_1B_1 + A_2B_2)} - \frac{E_s^2}{4(A_1B_1 + A_2B_2)}\right\}$$

$$= \frac{3\,E_s^2}{4\,000(A_1B_1 + A_2B_2)}. \qquad (248)$$

It is interesting to compare equations (247) and (248) with the corresponding equations for the short line, derived earlier in the chapter.

224. General Circuit Constants.—In the determination of power limits it is often necessary to take into account the effect of sending- and receiving-end transformers, that is, to calculate the performance of the system between the generator busbars and the low-tension busbars of the receiving-end station. Several more or less approximate ways of doing this have been put forward, but undoubtedly the best method, as suggested by Evans and Sels,[1] is to utilise what are known as general circuit constants. Having found the values of these constants for the composite circuit of line and transformers, analytical or graphical solutions can be made with the same facility as those for the line itself.

In any complicated network of circuits containing but one source of power, the voltage and current at any given point can always be expressed as a linear function of the voltage and current at any other point, provided that the various impedances and admittances forming the network have definite fixed values. Thus if E_s and I_s are respectively the voltage and current at the sending end of the network, and E_r and I_r the corresponding quantities at the receiving end, the following equations can be written—

$$E_s = E_rA_0 + I_rB_0, \qquad (249)$$

$$I_s = I_rD_0 + E_rC_0. \qquad (250)$$

A_0, B_0, C_0, and D_0, the general circuit constants of the network, are complex quantities whose values depend on the particular arrangement of the circuits. In all cases, however,

$$A_0D_0 - B_0C_0 = 1, \qquad (251)$$

and making use of this relationship the receiving-end voltage and current can be expressed in terms of the sending-end quantities as

$$E_r = E_sD_0 - I_sB_0, \qquad (252)$$

$$I_r = I_sA_0 - E_sC_0. \qquad (253)$$

To obtain expressions for A_0, B_0, C_0, and D_0, the values of the general circuit constants A, B, C, and D for each element of the complete network is first written down, and these are then combined by means of simple rules to be derived later. In the following, the values of these general circuit constants will be determined for some of the elements into which a transmission system can be divided.

1. *Lumped Impedance.*—For the lumped impedance (Fig. 159 (*a*)), the following equations evidently hold:—

$$E_s = E_r + I_r Z,$$
$$I_s = I_r.$$

Hence for this circuit

$$A = 1 \qquad B = Z \qquad C = 0 \qquad D = 1. \qquad (254)$$

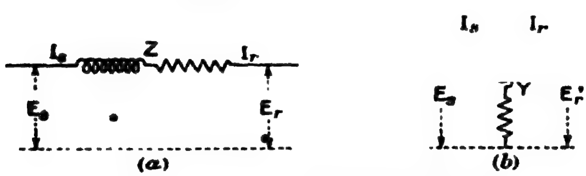

FIG. 159.—Lumped impedance and admittance.

2. *Lumped Admittance.*—For the lumped admittance (Fig. 159 (*b*)) the following equations can be written:—

$$E_s = E_r,$$
$$I_s = I_r + E_r Y,$$

from which

$$A = 1 \qquad B = 0 \qquad C = Y \qquad D = 1. \qquad (255)$$

3. *The Transformer.*—As previously shown, the equivalent network of a transformer is a T circuit. The impedances of the two arms are equal to the separate impedances of the two windings, but in the absence of evidence to the contrary these are taken to be equal when referred to the same side. Hence if Z_t is the impedance of the transformer, the impedance of each arm of the T is $\frac{Z_t}{2}$ as shown in Fig. 160. The admittance Y_t of the transformer is not, strictly speaking, a constant, as, due to saturation of the core, the value of Y_t depends to some extent on the impressed voltage. However, the effect of such changes on the general performance of the transformer is insignificant, and Y_t can be taken to be fixed and

equal in value to the apparent open-circuit admittance of the transformer at rated impressed voltage.

Considering then the T circuit (Fig. 160), the following relations hold:—

$$E_s = E_r\left(1 + \frac{Z_tY_t}{2}\right) + I_r\left(Z_t + \frac{Z_t^2Y_t}{4}\right),$$

$$I_s = I_r\left(1 + \frac{Z_tY_t}{2}\right) + E_rY_t,$$

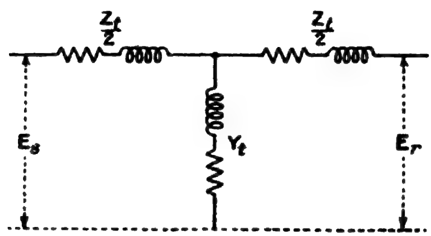

FIG. 160.—Equivalent T circuit of transformer.

from which

$$\left.\begin{aligned} A &= 1 + \frac{Z_tY_t}{2} & B &= Z_t\left(1 + \frac{Z_tY_t}{4}\right) \\ C &= Y_t & D &= 1 + \frac{Z_tY_t}{2} \end{aligned}\right\} \quad (256)$$

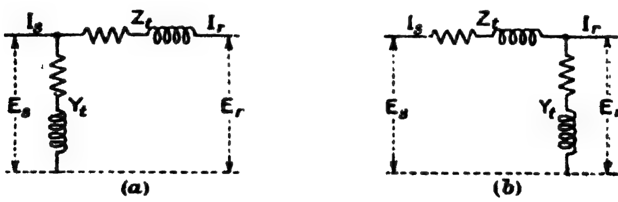

FIG. 161.—Transformers represented by cantilever circuits.

Sometimes it is more convenient to represent the transformer by a *cantilever* circuit, *i.e.* the admittance Y_t is assumed to be connected at one of the terminals as shown in Fig. 161.

In the case where the admittance is at the sending end of the circuit (Fig. 161 (*a*)), the following equations can be written:—

$$E_s = E_r + I_rZ_t,$$
$$I_s = I_r(1 + Z_tY_t) + E_rY_t,$$

hence

$$\left.\begin{array}{ll} A = 1 & B = Z_t \\ C = Y_t & D = 1 + Z_t Y_t. \end{array}\right\} \quad (257)$$

When the admittance Y_t is connected at the receiving end of the circuit (Fig. 161 (*b*)), the following relations hold:—

$$E_s = E_r(1 + Z_t Y_t) + I_r Z_t,$$
$$I_s = I_r + E_r Y_t,$$

from which

$$\begin{array}{ll} A = 1 + Z_t Y_t & B = Z_t \\ C = Y_t & D = 1 \end{array} \quad (258)$$

4. *Long Transmission Line.*—The general circuit constants (previously referred to as auxiliary constants) of a long transmission line were derived in Chapter V. They are

$$\begin{array}{ll} A = \cosh \sqrt{ZY} & B = \sqrt{\frac{Z}{Y}} \sinh \sqrt{ZY} \\ C = \sqrt{\frac{Y}{Z}} \sinh \sqrt{ZY} & D = \cosh \sqrt{ZY}. \end{array} \quad (259)$$

It will be noticed that the constants A and D are equal, which is the case with all symmetrical circuits.

225. Networks in Series.—Any number of networks connected in series may be represented by a single set of general circuit constants between any two points, provided that power is supplied or received at these points only.

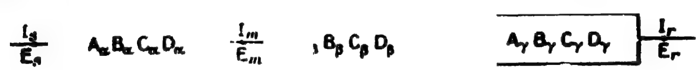

FIG. 162.—Networks in series.

1. *Three Networks in Series.*—Fig. 162 shows the case of three networks in series, and the following equations can be immediately written down:—

$$E_s = E_m A_\alpha + I_m B_\alpha \quad . \quad . \quad . \quad (260)$$
$$I_s = I_m D_\alpha + E_m C_\alpha \quad . \quad . \quad . \quad (261)$$
$$E_m = E_n A_\beta + I_n B_\beta \quad . \quad . \quad . \quad (262)$$
$$I_m = I_n D_\beta + E_n C_\beta \quad . \quad . \quad . \quad (263)$$
$$E_n = E_r A_\gamma + I_r B_\gamma \quad . \quad . \quad . \quad (264)$$
$$I_n = I_r D_\gamma + E_r C_\gamma. \quad . \quad . \quad . \quad (265)$$

Substituting equations (264) and (265) in equations (262) and (263) gives

$$E_m = E_r(A_\beta A_\gamma + B_\beta C_\gamma) + I_r(A_\beta B_\gamma + B_\beta D_\gamma) \quad . \quad (266)$$

$$I_m = I_r(C_\beta B_\gamma + D_\beta D_\gamma) + E_r(C_\beta A_\gamma + D_\beta C_\gamma). \quad . \quad (267)$$

Substituting equations (266) and (267) in equations (260) and (261) and collecting terms gives

$$\begin{aligned} E_s = E_r\{A_\alpha(A_\beta A_\gamma + B_\beta C_\gamma) + B_\alpha(C_\beta A_\gamma + D_\beta C_\gamma)\} \\ + I_r\{A_\alpha(A_\beta B_\gamma + B_\beta D_\gamma) + B_\alpha(C_\beta B_\gamma + D_\beta D_\gamma)\} \quad (268) \end{aligned}$$

$$\begin{aligned} I_s = I_r\{C_\alpha(A_\beta B_\gamma + B_\beta D_\gamma) + D_\alpha(C_\beta B_\gamma + D_\beta D_\gamma)\} \\ + E_r\{C_\alpha(A_\beta A_\gamma + B_\beta C_\gamma) + D_\alpha(C_\beta A_\gamma + D_\beta C_\gamma)\}. \quad (269) \end{aligned}$$

Hence the general circuit constants of three networks in series are

$$\left.\begin{aligned} A_0 &= A_\alpha(A_\beta A_\gamma + B_\beta C_\gamma) + B_\alpha(C_\beta A_\gamma + D_\beta C_\gamma) \\ B_0 &= A_\alpha(A_\beta B_\gamma + B_\beta D_\gamma) + B_\alpha(C_\beta B_\gamma + D_\beta D_\gamma) \\ C_0 &= C_\alpha(A_\beta A_\gamma + B_\beta C_\gamma) + D_\alpha(C_\beta A_\gamma + D_\beta C_\gamma) \\ D_0 &= C_\alpha(A_\beta B_\gamma + B_\beta D_\gamma) + D_\alpha(C_\beta B_\gamma + D_\beta D_\gamma) \end{aligned}\right\} (270)$$

2. *Two Networks in Series.*—The general circuit constants for two networks in series may be similarly obtained, or they may be derived from the above expressions by supposing that the impedances and admittances of one of the networks in Fig. 162 are reduced to zero, in which case the limiting value of its general circuit constants are $A = D = 1$ and $B = C = 0$. Assuming then that network γ vanishes, and substituting these values for A_γ, B_γ, C_γ, and D_γ in equations (270), the general circuit constants for two networks in series are found to be

$$\left.\begin{aligned} A_0 &= A_\alpha A_\beta + B_\alpha C_\beta \\ B_0 &= A_\alpha B_\beta + B_\alpha D_\beta \\ C_0 &= C_\alpha A_\beta + D_\alpha C_\beta \\ D_0 &= C_\alpha B_\beta + D_\alpha D_\beta \end{aligned}\right\} (271)$$

226. General Circuit Constants for Composite Transmission Circuit.—The values of the general circuit constants will now be determined for the combination of line with both sending- and receiving-end transformers, and also line with transformers at one end only. The exact expressions for the constants, employing the equivalent T circuits for the transformers, have been worked out by Evans and Sels and are given in the paper previously referred to.[1] Unfortunately, they are rather complicated, and in

practice it is better to represent the transformers by means of cantilever circuits. The errors introduced by the use of these cantilever circuits are insignificant, but the calculation of the general circuit constants is greatly simplified.

1. *Transmission Line with Sending- and Receiving-end Transformers.*—Fig. 163 shows the composite circuit, and substituting from (257) and (258) in the general formulæ for the combination of three networks in series, and remembering that for the line, $A = D$, the general circuit constants are obtained as

$$\left.\begin{aligned}
A_0 &= A(1 + Z_{tr}Y_{tr} + Z_{ts}Y_{tr}) + BY_{tr} + CZ_{ts}(1 + Z_{tr}Y_{tr})\\
B_0 &= A(Z_{ts} + Z_{tr}) + B + CZ_{ts}Z_{tr}\\
C_0 &= A\{Y_{ts}(1 + Z_{tr}Y_{tr}) + Y_{tr}(1 + Z_{ts}Y_{ts})\} + BY_{ts}Y_{tr}\\
&\qquad + C(1 + Z_{ts}Y_{ts})(1 + Z_{tr}Y_{tr})\\
D_0 &= A(1 + Z_{ts}Y_{ts} + Z_{tr}Y_{ts}) + BY_{ts} + CZ_{tr}(1 + Z_{ts}Y_{ts}).
\end{aligned}\right\}(272)$$

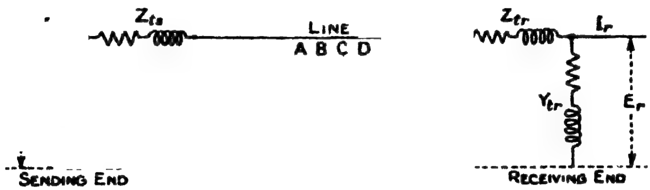

FIG. 163.—Composite circuit of transmission line with sending- and receiving-end transformers.

If the sending- and receiving-end transformers are identical, $Z_{ts} = Z_{tr}$, and $Y_{ts} = Y_{tr}$, and the general circuit constants simplify to

$$\left.\begin{aligned}
A_0 &= A(1 + 2Z_tY_t) + BY_t + CZ_t(1 + Z_tY_t)\\
B_0 &= 2AZ_t + B + CZ_t^2\\
C_0 &= 2AY_t(1 + Z_tY_t) + BY_t^2 + C(1 + Z_tY_t)^2\\
D_0 &= A(1 + 2Z_tY_t) + BY_t + CZ_t(1 + Z_tY_t).
\end{aligned}\right\}(273)$$

2. *Transmission Line with Sending-end Transformers.*—By substituting from (257) in the formulæ for the combination of two networks in series, or perhaps more readily by assuming that Z_{tr} and Y_{tr} in expressions (272) are equal to zero, the general circuit constants are obtained as

$$\left.\begin{aligned}
A_0 &= A + CZ_t\\
B_0 &= AZ_t + B\\
C_0 &= AY_t + C(1 + Z_tY_t)\\
D_0 &= A(1 + Z_tY_t) + BY_t
\end{aligned}\right\}(274)$$

3. *Transmission Line with Receiving-end Transformers.*—By an analogous process to the preceding case the general circuit constants are found to be

$$\begin{aligned} A_0 &= A(1 + Z_tY_t) + BY_t \\ B_0 &= AZ_t + B \\ C_0 &= AY_t + C(1 + Z_tY_t) \\ D_0 &= A + CZ_t. \end{aligned} \tag{275}$$

227. Numerical Example.—In order to illustrate the use of the formulæ derived in the preceding articles, the following problem will be taken:—

Problem 12.—A 300-mile transmission line with raising and lowering transformers is operated on the constant-voltage system, the voltage on the low-tension side of the transformers at both ends being held constant at 154 000 referred to the high-tension circuit. Determine the maximum load at 80 per cent. power factor lagging that can be transmitted, the synchronous phase modifier installation having a total capacity of 40 000 kVA. The following data are given:—

Line.

$R = 52{\cdot}5$ ohms.

$X = 188{\cdot}4$ ohms.

$G = 0.$

$B = 0{\cdot}001411$ mhos.

Transformers.

Sending-end transformers 11 000/154 000 volts and receiving-end transformers 154 000/33 000 volts are identical

Total installation at each end = 50 000 kVA.

Resistance	= 0·5 per cent.
Reactance	= 10 per cent.
Iron loss	= 0·6 per cent.
Copper loss	= 0·5 per cent.
Exciting current	= 5 per cent.

First of all the auxiliary constants of the line are calculated by one of the standard methods, their values being

$$A = 0{\cdot}869780 + 0{\cdot}035418j.$$

$$B = 47{\cdot}9370 + 180{\cdot}786j.$$

$$C = -0{\cdot}000017 + 0{\cdot}001349j.$$

The transformer constants are then determined as follows :—

$E_s = E_r = 88\,912$ volts.

Capacity $= 16\,667$ kVA. per phase.

$$I = \frac{16\,667 \times 1\,000}{88\,912} = 187{\cdot}45 \quad \text{amperes.}$$

$$R_t = \frac{0{\cdot}005 \times 88\,912}{187{\cdot}45} = 2{\cdot}3716 \quad \text{ohms.}$$

$$X_t = \frac{0{\cdot}1 \times 88\,912}{187{\cdot}45} = 47{\cdot}432 \quad \text{ohms.}$$

$$G_t = \frac{0{\cdot}006 \times 16\,667 \times 1\,000}{(88\,912)^2} = 1{\cdot}2650 \times 10^{-5} \quad \text{mhos.}$$

$$Y_t = \frac{0{\cdot}05 \times 16\,667 \times 1\,000}{(88\,912)^2} = 10{\cdot}541 \times 10^{-5} \quad \text{mhos.}$$

$$B_t = \sqrt{(10{\cdot}541)^2 - (1{\cdot}2650)^2} \times 10^{-5} = 10{\cdot}465 \times 10^{-5} \quad \text{mhos.}$$

$$Z_t = R_t + jX_t = 2{\cdot}3716 + 47{\cdot}432j \quad \text{ohms.}$$

$$Y_t = G_t - jB_t = (1{\cdot}2650 - 10{\cdot}465j)\,10^{-5} \quad \text{mhos.}$$

$$\begin{aligned} Z_t^2 &= (2{\cdot}3716 + 47{\cdot}432j)^2 \\ &= -2\,244{\cdot}2 + 224{\cdot}98j. \end{aligned}$$

$$\begin{aligned} Y_t^2 &= (1{\cdot}2650 - 10{\cdot}465j)^2 \times 10^{-10} \\ &= (-107{\cdot}92 - 26{\cdot}476j)\,10^{-10} \end{aligned}$$

$$\begin{aligned} Z_tY_t &= (2{\cdot}3716 + 47{\cdot}432j)(1{\cdot}2650 - 10{\cdot}465j)\,10^{-5} \\ &= (499{\cdot}39 + 35{\cdot}181j)\,10^{-5} \end{aligned}$$

$$1 + Z_tY_t = 1{\cdot}004994 + 0{\cdot}0003518j.$$

$$1 + 2Z_tY_t = 1{\cdot}009988 + 0{\cdot}0007036j.$$

The general circuit constants for the composite circuit of line and transformers can now be calculated from expressions (273) as

$$\begin{aligned} A_0 &= A_{01} + jA_{02} \\ &= A(1 + 2Z_tY_t) + BY_t + CZ_t(1 + Z_tY_t) \\ &= (0{\cdot}860780 + 0{\cdot}035418j)(1{\cdot}009988 + 0{\cdot}0007036j) \\ &\qquad + (47{\cdot}9370 + 180{\cdot}786j)(1{\cdot}2650 - 10{\cdot}465j) \\ &\qquad + (-0{\cdot}000017 + 0{\cdot}001349j)(2{\cdot}3716 + 47{\cdot}432j) \\ &\qquad\quad (1{\cdot}004994 + 0{\cdot}0003518j) \\ &= 0{\cdot}83362 + 0{\cdot}036036j. \end{aligned}$$

$$
\begin{aligned}
B_0 &= B_{01} + jB_{02} \\
&= 2AZ_t + B + CZ_t^2 \\
&= 2(0\cdot 869780 + 0\cdot 035418j)(2\cdot 3716 + 47\cdot 432j) \\
&\qquad + 47\cdot 9370 + 180\cdot 786j + (-0\cdot 000017 + 0\cdot 001349j) \\
&\qquad\quad (-2\,244\cdot 2 + 224\cdot 98j) \\
&= 48\cdot 437 + 260\cdot 43j.
\end{aligned}
$$

$$
\begin{aligned}
C_0 &= C_{01} + jC_{02} \\
&= 2AY_t(1 + Z_tY_t) + BY_t^2 + C(1 + Z_tY_t)^2 \\
&= 2(0\cdot 869780 + 0\cdot 035418j)(1\cdot 2650 - 10\cdot 465j)10^{-5} \\
&\qquad (1\cdot 004994 + 0\cdot 0003518j) + (47\cdot 9370 + 180\cdot 786j) \\
&\qquad (-107\cdot 92 - 26\cdot 476j)10^{-10} + (-0\cdot 000017 + 0\cdot 001349j) \\
&\qquad (1\cdot 004\cdot 994 + 0\cdot 0003518j)^2 \\
&= 0\cdot 0000114 + 0\cdot 001178j.
\end{aligned}
$$

$$
\begin{aligned}
D_0 &= D_{01} + jD_{02} = A_0 \\
&= 0\cdot 83362 + 0\cdot 036036j,
\end{aligned}
$$

and these values should be checked by means of the relation $A_0D_0 - B_0C_0 = 1$.

The co-ordinates and radius of the receiving-end power circle are now found to be

$$
\begin{aligned}
a &= -\frac{3E_r^2(A_{01}B_{01} + A_{02}B_{02})}{1\,000\,B_0^2} \\
&= -\frac{3(88\,912)^2(0\cdot 83362 \times 48\cdot 437 + 0\cdot 036036 \times 260\cdot 43)}{1\,000\{(48\cdot 437)^2 + (260\cdot 43)^2\}} \\
&= -16\,818.
\end{aligned}
$$

$$
\begin{aligned}
b &= \frac{3E_r^2(A_{01}B_{02} - A_{02}B_{01})}{1\,000\,B_0^2} \\
&= \frac{3(88\,912)^2(0\cdot 83362 \times 260\cdot 43 + 0\cdot 036036 \times 48\cdot 437)}{1\,000\{(48\cdot 437)^2 + (260\cdot 43)^2\}} \\
&= 72\,108.
\end{aligned}
$$

$$
\begin{aligned}
c &= \frac{3(88\,912)^2}{1\,000\sqrt{(48\cdot 437)^2 + (260\cdot 43)^2}} \\
&= 89\,528,
\end{aligned}
$$

and Fig. 164 drawn from these data is the receiving-end power circle diagram for the composite circuit. The load line can now be drawn in the fourth quadrant of the diagram, making an angle

$\cos^{-1} 0{\cdot}8$ with the horizontal axis. To determine the maximum load that can be transmitted under the given conditions, measure from the origin upwards on the vertical axis a distance representing the 40 000 kVA of the synchronous phase modifiers, and draw from this point a line (shown dotted in Fig. 164) parallel to the

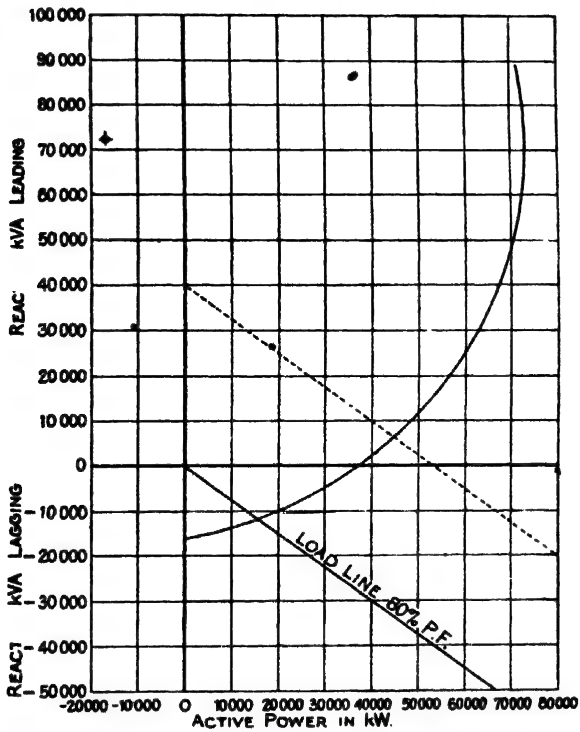

FIG. 164.—Receiving-end power circle diagram (Problem 12).

load line. The intersection of this dotted line with the power circle determines the maximum load that can be transmitted, and this is read off from the diagram as 44 300 kW.

228. Stability.—The various diagrams and formulæ derived above give correct results for the conditions assumed, *viz.* constant terminal voltages, dead loads, etc. It should be clearly realised,

however, that these are ideal conditions, and a complete study of the problem must take into account not only the characteristics of the line and transformers, but also those of all the rotating plant forming part of the system. The limits of output previously discussed are thus conventional values which, although of great utility particularly in comparative work, are rather greater than the actual limits of the system.

This leads to a conception of stability which may be defined briefly as the capacity of a system to respond to power demands upon it. Attempts to put too great a load on a system lead to loss of synchronism between the two ends although the actual value of the load at which breakdown occurs varies with circumstances. If the load is applied gradually, loss of synchronism occurs when the steady-state stability limit is reached, and, providing that sufficient synchronous phase modifier capacity is available, the power limit of the system may be closely approached. If the load is thrown on suddenly, due to a short-circuit or switching operation, the system breaks down at a lower value, the transient-state stability limit. This latter term, however, has a rather indefinite value depending on the magnitude and rate of application of the load and the previous circuit conditions.

The performance of a transmission line is analogous to that of a synchronous motor. When such a machine is loaded its rotor drops back in phase position, and the phase difference between the excitation voltage and the voltage impressed on the motor is increased, stability of operation being secured when an increase in the phase-angle results in an increase of torque. At a certain maximum load, defined by the condition that the phase-angle $= \tan^{-1} \frac{\text{Reactance}}{\text{Resistance}}$, the machine falls out of step. Similarly, as shown by Shand,[2] the phase difference between the receiving- and sending-end voltages of a transmission circuit increases until the power limit is reached, which occurs when the angle of phase difference $= \tan^{-1} \frac{X}{R}$ (or $\tan^{-1} \frac{B_2}{B_1}$ in the case of a long-distance line). Efforts to increase the load past this point lead to breakdown of the system.

Very intimately bound up with the question of stability is that of voltage regulation. Consider, for instance, the case of a generator supplying a synchronous motor over a long line. If additional

load is suddenly thrown on, its first effect is to slow down the motor momentarily, making the load-end voltage lag behind the generator voltage by an additional amount. If the increased lag is sufficient to carry the machine past its pull-out angle loss of synchronism occurs and the motor comes to rest. While the machine is drifting out of step, however, the generator voltage, which has dropped temporarily, is being rapidly restored to normal by the action of the automatic voltage regulators, and providing that sufficient additional power can be supplied from the generator end, the motor re-accelerates before the danger point is reached. The rotor of the machine then oscillates about its position of kinetic equilibrium, but these oscillations are rapidly damped out by the action of the amortisseur winding. Before the voltage regulators can exert their full effect a short but definite time interval is necessary, this sluggishness of action being due, not so much to the operation of the regulators themselves, as to the time constants of the field windings of generator and exciter. On the other hand, following a short-circuit or sudden application of load, the voltage regulation of the generator is determined by its leakage reactance which is much smaller than its synchronous reactance. After the first instant of the transient, the reactance begins to build up to its steady state or synchronous value, but during this short space of time an opportunity is afforded for the voltage regulators to make their presence felt.

In the case of short lines the steady-state stability limit is not an important factor, because this limit is usually not approached except with prohibitively high values of line regulation and low transmission efficiencies. For instance, with a non-inductive load it is seen from (242) that when giving maximum output the ratio $\frac{E_r}{E_s}$ varies from 0·5 to 0·707. Taking as a typical case $Z = 2R$ the ratio $\frac{E_r}{E_s} = 0{\cdot}577$, the line regulation being 73·2 per cent. The transmission efficiency at this output is 66·7 per cent. There is, however, always the risk of instability being caused by transient disturbances, particularly on heavy power transmissions (where R is small in comparison with Z). Naturally, with short distances of transmission the characteristics of the terminal machinery assume greater importance than those of the line itself.

In the case of long-distance lines circumstances are entirely

different. High voltages must be employed, and due to the necessarily wide spacing of the conductors the reactance is so great that synchronous phase modifiers are essential to maintain constant voltage, and to enable sufficient amounts of power to be transmitted. In order to be successful from a commercial point of view such lines must transmit very large blocks of power per circuit, which involves the use of heavy conductors. Now at wide spacings the reactance of the conductors is only slightly affected by their size, so that the power limit of the line—which is determined chiefly by the reactance—is not increased in proportion to the increased current-carrying capacity. An increase in the size of conductors by 100 per cent., for example, may only raise the power limit of the line by about 10 per cent. The result is that the stability limit of a long line may be reached with a load at which the transmission efficiency is still of the order of 90 per cent. As a matter of fact, the peak load on some of the large American systems is so near to the stability limit that many instances have occurred of lines dropping their load, and this has focussed much attention on the subject in that country. A great deal of theoretical and experimental work has been done in order to determine the best methods of operating existing systems, and to provide data for the design of the more extensive and higher-voltage systems that may be required in the future.

Once a system has been designed for a certain voltage there are two general methods by which the stability limit may be increased. One is to decrease the reactance (or reactance drop) of the transmission circuit, and the other is to improve the means taken to maintain the terminal voltages of the circuit. Thus the reactance drop may be decreased by using a number of lines in parallel, installing low-reactance transformers, or operating at a lower frequency. The latter method, however, would usually involve the use of frequency changers at the sending and receiving ends and thus be rather costly. Another proposed solution is the periodic loading of the line with series condensers whose effect would be to neutralise the reactance more or less completely. This scheme also would be expensive and does not appear justifiable from an economic view, although it might be found advantageous with the advent of higher line pressures. As regards methods of improving the control and maintenance of constant terminal voltages, this may be accomplished by the use of machines of lower leakage reactance

and generators have also been contemplated with which some form of rectifier could be used to rectify a portion of the main current and utilise it for assisting the excitation. The most promising development, however, and one not requiring the use of generators of special design, is the introduction of high-speed excitation systems. In the case of extremely long lines the use of intermediate synchronous phase modifier stations would also increase the amount of power that could be transmitted.

The above discussion is necessarily only a brief treatment of the subject, and must be regarded as being in the nature of an introduction to the many original papers which have appeared in recent years. Some of the more important of these papers are noted below.[3]

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[3] E. B. Shand, *loc. cit.*

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